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Short Note

Measuring and modeling attenuation in rocks

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INTRODUCTION

Cross-well seismic methods have been most discussed and perhaps found widest application in the context of velocity estimation as an aid to finding, delineating, and monitoring hydrocarbon pools. One reason for this is that it has called upon procedures and processes which are well understood by the exploration community. At the same time rock physicists have known for some time that under certain conditions fluids in porous rocks lead to quite unusual levels of low-frequency seismic attenuation. Biot (1956) provides the basic theory behind this notion. More recently Berryman and others (1988) and Norris (1993) have specialized the theory to partially saturated rocks, where liquid sloshing is much facilitated by the presence of gas, with the possibilities for quite dramatic levels of attenuation in passing seismic waves. While it might be possible to base experimental design and analysis on some implementation of these theories, as a practical matter it seems more expedient to develop schemes based on a simple and self-contained heuristic, and this is what is done here.

A TOMOGRAPHIC INVERSION SCHEME

A tomographic data acquisition and inversion scheme for seismic loss might look very like its velocity counterpart with the principal difference that log(amplitude) replaces travel-time as the dependent variable. Also, since amplitudes contain no travel-path information, a linear inversion scheme based on a straight-path assumption is used. Another major difference in the suggested implementation is that the dependent variable is no longer a scalar but a 2nd rank tensor. Loss at some location depends on the direction of the path through that position.

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The algorithm

A numerical algorithm might consist of the following steps.

1. Evaluate the logarithm of the amplitude of the first arrival.

2. Correct this value to first order for geometric spreading.

3. Insert this value into the leading location of an otherwise null 2nd rank loss tensor, and rotate by the angle that the source/receiver line makes with the horizontal.

4. Distribute this tensor value over the straight path connecting source and receiver.

5. When all paths have been adjusted, model the total loss by integration along all the paths, compare, and iterate if necessary.

Problems

While source initiation times do not present a problem in cross-hole seismic studies (although defining arrival times might), absolute source amplitude information may be unavailable and there may be fluctuations from shot to shot, caused by linear and non-linear interaction between the source and borehole. One solution is to assume homogeneity of loss in the subject subsurface and solve for the source amplitudes, with the possibility of some high-pass filtering of these results to lessen medium effects. An alternative method which has the merit of simplicity would be to treat the sources as an extra column of material; a similar column at the other side would account for receiver discrepancies.

Extensions

Since most loss mechanisms are frequency-dependent in some simple way, it might be reasonable to divide the log-spectrum of the first arrival into chunks and solve for them independently. Alternatively this log-spectrum could be characterized by the form $K_0 + K_1 \log(\omega)$ where K_0 and K_1 are solved for. If velocity and loss inversion schemes share a common dataset, then they will also have the same structure for inversion. Apart from time savings, there would also be some technical benefit. First, the schemes could share the same definition of first arrival, and second, they could share the same, now no longer linear, back-projection path. Although most seismic tomography schemes have used a scalar representation of time-delay and its implied isotropy of the velocity field, Michelena et al. (1993) have described a scheme for inverting for velocities under an elliptic anisotropic constraint which could be simply implemented by using the scheme outlined in this paper, viz. replacing the scalar time-shift with a 2nd rank tensor representation. If this is done then there is no particular hardship in including elliptic forms that do not necessarily have a vertical symmetry axis.

AN ELASTIC MODELING SCHEME

Conventionally, loss is introduced into the elastic wave equation by making the elements of the stiffness tensor frequency-dependent, with constraints imposed by energy conservation and causality. A problem with this approach is that it is not clear how or even why a loss effect should be represented by a 4th rank tensor. If we are thinking of "slosh" as the loss mechanism, then it seems more reasonable to think of loss as an inertial effect, closely related to the permeability which is properly represented by a 2nd rank tensor, since, quite generally, permeability is not isotropic. This being the case it seems reasonable to leave the stiffness tensor frequency-independent, and modify the inertial term in two ways. In private discussion Berryman has pointed out that this is precisely what his paper (Berryman et al., 1988) argues for, although in a more restrictive, isotropic environment. The modifications to the wave equation have been previously discussed by Muir (1992) and lead to the following result.

1. Replace the scalar mass density ρ with a tensor mass density R_{ij}.

2. Replace the constant elements of R_{ij} with functions of s, the Fourier representation of the time differential, so that the full inertial term is now $R_{ij}(s)\ddot{u}_j$.

3. Replace the displacement variable, u, with an energy-flux variable defined

$$w = R(s)^{1/2} u$$

With this definition the lossy wave equation has the following canonical form

$$R(s)^{-T/2} \nabla^T C \nabla R(s)^{-1/2} w - \ddot{w} = 0$$

and can be cast in Christoffel form for plane-wave solution.

CONCLUSIONS

In summary, I have tried to make the following points:

- Attenuation may be an important lithologic indicator.
- Loss has direction and is best viewed as a 2nd rank tensor.
- Tomographic inversion for loss parallels its velocity counterpart.
- Modeling loss due to slosh with a frequency-dependent inertial term is intuitive, theoretic and computationally reasonable.

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