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Short Note

Extending Stolt migration to general dispersion relations

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INTRODUCTION

Nowadays, most data processing methods use depth dependent parameters that can not be determined in a direct way but have to be estimated indirectly. On the other hand, data processing in time requires only the knowledge of parameters that can be measured on the surface such as time, offset and horizontal velocity (Muir, 1993). This is of advantage especially in the case of anisotropy as the vertical velocity does not need to be estimated any longer.

In an attempt to implement an example of time processing, an extended Stolt migration algorithm was developed for general dispersion relations and an exact interpolation scheme (Rosenbaum and Boudreaux, 1981). Based on Fourier transformation both in time and in space, the Stolt migration (**?**) is able to handle the sometimes triplicating behavior of real elastic anisotropy as can be expected, for example, for shear waves in certain shales. Therefore, Stolt migration may be a useful tool in determining the amount of anisotropy in a medium.

As an example, we implement in this study the dispersion relation for the case of anelliptic anisotropy into the migration algorithm which was designed for the use of any dispersion relation. Anelliptic anisotropy is an extension of elliptic anisotropy which retains the convenient symmetry properties of elliptic anisotropy in approximation (Muir and Dellinger, 1985; Karrenbach, 1991; Dellinger et al., 1993). It is especially useful when more independent parameters than vertical and horizontal velocity are needed, but the full complexity of transverse anisotropy is unnecessary. The effect of the anelliptic anisotropy is shown on impulse responses for different grades of anellipticity.

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EXTENDED STOLT MIGRATION

The normal Stolt migration algorithm for the dispersion relation of the scalar wave equation

$$\frac{\omega^2}{v^2} = k_x^2 + k_z^2 \tag{1}$$

is implemented using the following equation to map from the "data" to the "image" space:

$$image(x,z) = \int\int \frac{v|k_z|}{\sqrt{k_x^2 + k_z^2}} P(k_x, \omega(k_z)) e^{ik_x x} e^{ik_z z} dk_x dk_z \tag{2}$$

where $P(k_x, w(k_z))$ is the fourier transformation of the data recorded at the surface. This equation can easily be transformed into the time domain by using the relationship $k_\tau = v_z k_z$ between two-way traveltime τ, depth z and vertical velocity v_z and can be extended into:

$$image(x,\tau) = \int\int J(k_x, k_\tau) P(k_x, \omega(k_\tau)) e^{ik_x x} e^{i\gamma(k_x,\omega)\tau} dk_x dk_\tau \tag{3}$$

where

$$k_\tau = \gamma(k_x, \omega)$$

is a function of k_x and ω and $J(k_x, k_\tau)$ is the new Jacobian depending now on k_τ instead of k_z. Any desired dispersion relation can be put into this new generalized equation by simply deriving the appropriate Jacobian and k_τ which can be substitited in equation (3).

The Stolt migration algorithm for the extended imaging equation (3) can then be represented as:

$$p(x, z=0, t) \rightarrow P(k_x, w) \rightarrow Q(k_x, k_\tau) = J(k_x, k_\tau) P(k_x, \omega(k_\tau)) \rightarrow q(x, \tau)$$

In order to determine the value $Q(k_x, k_\tau)$ from $P(k_x, \omega(k_\tau))$ a mapping from the ω to the k_τ axis has to be performed. In calculating $Q(k_x, k_\tau)$, we require the value of $P(k_x, \omega(k_\tau))$ which is calculated by a frequency-domain interpolation. In this study, an exact interpolation scheme by Rosenbaum (1981) is implemented in the migration algorithm:

$$C'(n+\delta n) = \sum_{m=0}^{N-1} C(m) e^{-\pi i[(n+\delta n)-m]} sinc[(n+\delta n) - m] \tag{4}$$

where N is the number of given points and the point to be interpolated lies at the point $n + \delta n$. In this interpolator the weights are the product of a sinc function and a corkscrew function $e^{-\pi i[(n+\delta n)-m]}$. Popovici et al. (1993) compare Rosenbaums technique with a slow Fourier transformation in time for an irregular range of frequencies followed by an inverse Fourier transformatio. This has been implemented on a parallel computer by Blondel and Muir (1993).

ANELLIPTIC ANISOTROPY

The extended Stolt algorithm is applied to the case of anelliptic anisotropy. Anelliptic anisotropy is an extension of elliptic anisotropy that retains the convenient symmetry properties of the elliptic anisotropy in approximation. Muir and Dellinger (1985) showed that the equations for elliptic anisotropy of the form

$$f = (z - \text{term})^2 + (x - \text{term})^2 \tag{5}$$

can be generalized by adding an anelliptic factor q:

$$f = \frac{z^4 + (1+q)z^2x^2 + x^4}{z^2 + x^2} \tag{6}$$

The anelliptic factor q gives the deviation of true horizontal velocity and paraxial (near vertical, short spread) NMO velocity and perturbs the behavior away from ellipticity in between the coordinate axes. q=1 reduces equation (6) back into the elliptic form. The dispersion relation for the anelliptic case is given by

$$\omega^2 = \frac{(v_z^2k_z^2)^2 + (1+q)v_z^2k_z^2v_x^2k_x^2 + (v_x^2k_x^2)^2}{v_z^2k_z^2 + v_x^2k_x^2} \tag{7}$$

Using again the relation

$$k_\tau = v_z k_z$$

equation (7) becomes

$$\omega^2 = \frac{k_\tau^4 + (1+q)v_x^2k_x^2k_\tau^2 + (v_x^2k_x^2)^2}{k_\tau^2 + v_x^2k_x^2} \tag{8}$$

It is obvious that the transformation into the time domain removes any dependence of the unknown vertical velocity in the dispersion relation. It is now a function of parameters that can all be measured on the surface. Equation (8) can be transformed to

$$k_\tau^2 = -\frac{(1+q)v_x^2 - w^2}{2} + \frac{1}{2}\sqrt{(4 - (2(1+q))w^2v_x^2k_x^2 + ((1+q)^2 - 4)v_x^4k_x^4 + w^4} \tag{9}$$

The Jacobian for the Stolt migration is given by

$$\frac{\partial\omega}{\partial k_\tau} = \frac{1}{2} + \sqrt{\frac{k_\tau^2 + v_x^2k_x^2}{k_\tau^4 + (1+q)v_x^2k_x^2k_\tau^2 + v_x^4k_x^4}}\,\frac{2k_\tau(k_\tau^4 + 2k_\tau^2v_x^2k_x^2 + qv_x^4k_x^4)}{k_\tau^4 + 2k_\tau^2v_x^2k_x^2 + v_x^4k_x^4} \tag{10}$$

Setting q=1 results in the well-known Jacobian for the elliptic case.

After implementation of the dispersion relation and Jacobian in the Stolt migration algorithm, the effect of different q values is tested on an input consisting of three spikes. The result can be seen in Figure 1.

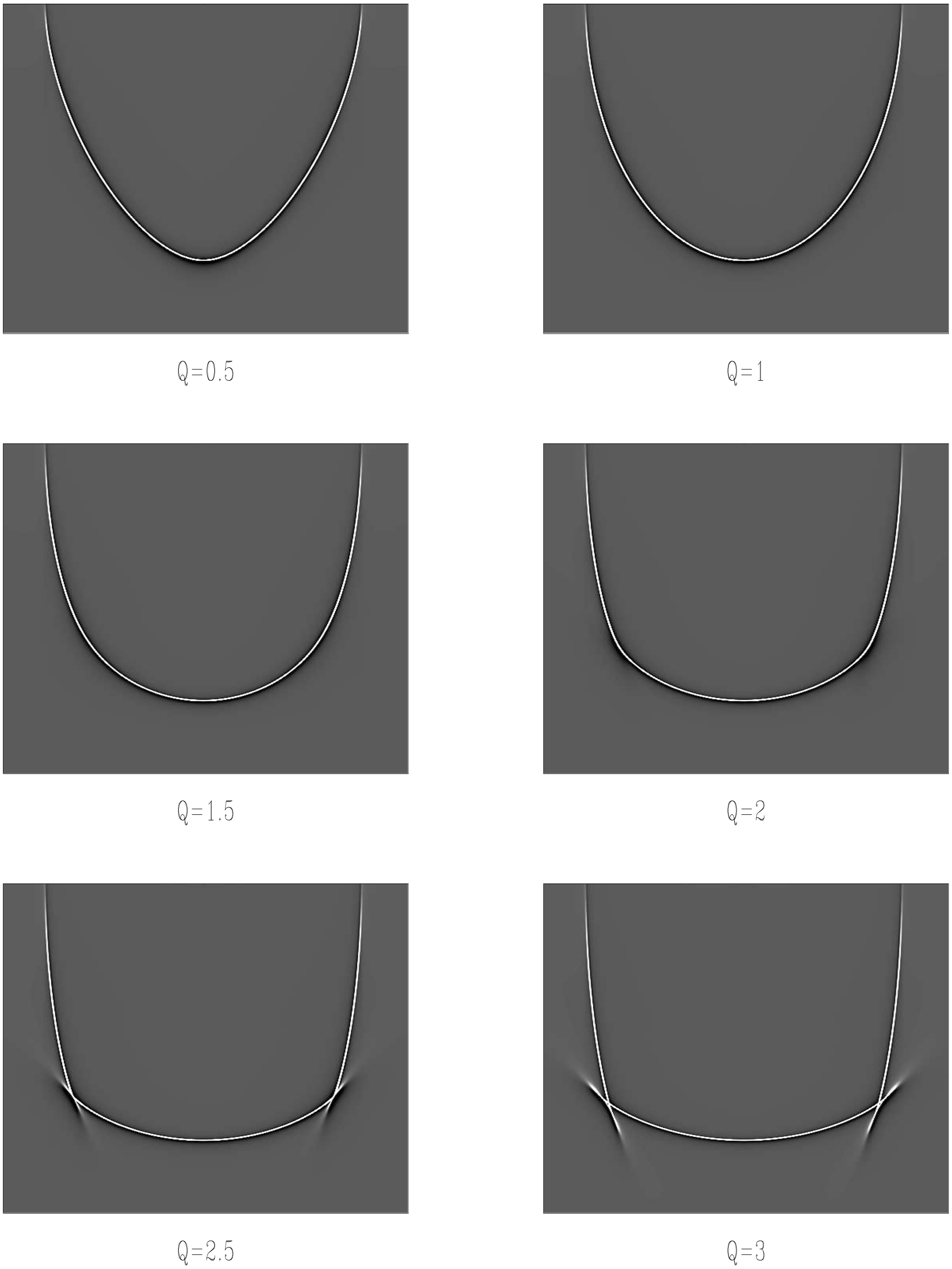

Figure 1: Impulse responses of the Stolt migration for different values of anelliptic anisotropy. Click the button to see a resulting movie intime3-stolt [ER]

Going from q=0.5 to q=3, the effect of the anisotropy can be clearly observed. The v-shaped impulse response at q=0.5 becomes an ellipse for q=1 and changes slowly into triplications for larger q values. The triplications result from extreme velocity variations in different directions. This triplicating behavior is shown in more detail for q=4 in Figure 2.

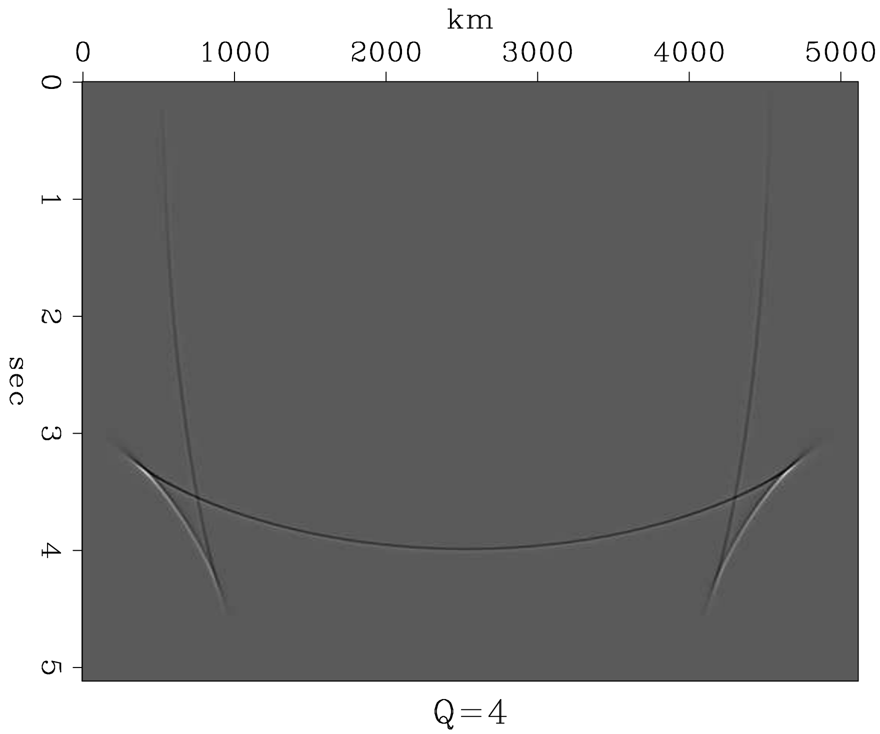

Figure 2: Migration result for q=4 $\boxed{\text{intime3-stolt4}}$ [ER]

Most of the energy is concentrated in the triplications and along the apex of the semi-circle. It diminishes along the sides of the semi-circle.

CONCLUSIONS

An extended Stolt migration algorithm in time was developed for general dispersion relations and an exact interpolation method. Implementation of the dispersion relation for anelliptic anisotropy showed clearly the effect of an increasingly anisotropic medium on impulse responses. The triplications that are generated after a certain grade of anisotropy is reached could be accurately processed. The next step will be the implementation of the inverse method to produce a modeling algorithm.

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