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Short Note

Elements of an exact kinematic theory for wavefronts traveling through layered anisotropic media

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INTRODUCTION

Most conventional seismic processing steps assume either that the velocity function, $v(z)$, is known, or that it can be determined from the data. A further, related, assumption is that the earth is isotropic. This is a major convenience and allows the use of simple, ray-based arguments for developing processes such as **DMO**. This **Isotropic Paradigm** is clearly useful where it is backed up by well control, but is unnecessary and may be quite misleading in frontier areas where surface seismic is the sole information source.

In case of surface seismic data all processing steps up to and including image reconstruction can be accomplished using a model space not involving depth, and in this Short Note general kinematic relations are presented which provide a basis for such a methodology. An important feature of the relations is that the parameters involved are "addable" and thus allow data to be decomposed into intervals.

DEVELOPMENT

Layered impedance

As a start, it is convenient also to limit impedance to a layered model, and to impose axisymmetry on the velocity functions. Conventional first-pass velocity analysis often makes a like assumption.

The surface kinematics of reflection wavefronts in such a (not necessarily isotropic) layered earth may be exactly and completely described by two functions of the ray parameter, p. These are the **travel-time function**, $t(p)$, and the **offset function**, $x(p)$. The kinematics of a set of such layers is simply and exactly formed because

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these functions are elements of Abelian groups. That is, the reflection response from the base of n such layers arranged in any order is given by

$$t(p) = \sum_{j=1}^{n} t_j(p)$$

and

$$x(p) = \sum_{j=1}^{n} x_j(p)$$

If we now define $v^2(p) = x(p)/pt(p)$ then $v^2(p)t(p) = x(p)/p$. But this latter is addable for constant p and so then must be $v^2(p)t(p)$. That is,

$$v^2(p)t(p) = \sum_{j=1}^{n} v_j^2(p)t_j(p)$$

where $v(p)$ is some as yet unspecified velocity.

The isotropic layer

Although the formalism extends to velocity anisotropy, it is informative to look at the isotropic case. Most of the results will not be new, but it is a check on the general theory and a link to the familiar. Starting with the basic **time/offset** equation $t_x^2 = t_0^2 + x^2/v^2$, the equations for **time**, **offset**, and **velocity** as functions of p are

$$t(p) = \frac{t_0}{\sqrt{1 - v^2p^2}}$$

and

$$x(p) = \frac{v^2pt_0}{\sqrt{1 - v^2p^2}}$$

and

$$v(p) = v$$

A bunch of isotropic layers

From these and previous results it follows that we have these relationships for **travel-time**, **offset**, and **velocity** of a stack of n layers as a function of p.

$$t(p) = \sum_{j=1}^{n} \frac{t_j(0)}{\sqrt{1 - v_j^2p^2}}$$

and

$$x(p) = \sum_{j=1}^{n} \frac{v_j^2pt_j(0)}{\sqrt{1 - v_j^2p^2}}$$

and

$$v^2(p)t(p) = \sum_{j=1}^{n} \frac{v_j^2 t_j(0)}{\sqrt{1 - v_j^2 p^2}}$$

This last relation is particularly useful, since it gives us the means for distinguishing whether non-hyperbolic move-out is due to anisotropy or heterogeneity. A simple scheme might look like this:

1. determine $v(p)$ for each reflector.

2. Decompose these velocity functions into their interval counterparts.

3. Test each interval $v(p)$ for closeness to isotropy.

Simply put, those layers whose interval velocities are p-independent are isotropic (or maybe elliptic).

A diffraction theory

Notice that this is not a ray theory nor a plane-wave theory, and certainly not a dynamic elastic wave theory. It does correspond to notions with which explorationists are familiar, and only uses (no $v(z)$!) parameters that can be estimated at the surface. Also note that this is not, despite appearances, just a horizontal reflector theory. A kinematic diffraction theory develops by combining half reflections from the same layer set but with differing ray parameters for the up- and down-going waves. In other words, there are now two independent ray parameters, p_s and p_g, and development follows most simply by considering a source-to-scatterer-to-receiver model.