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Short Note

A trace interpolation scheme

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INTRODUCTION

Interpolation continues to occupy thought at SEP, and in this issue alone there are contributions from Balog, Claerbout, Ji, Karrenbach, and Zhang. It is a major topic in Claerbout's new book (1991,1992), and Spitz (1991) has recently introduced a new scheme with a useful reference list. Why should I add yet another paper? The answer is that this method is quite different and remarkably simple: *Interpolate the unaliased, low-frequency data by classical linear means, and then reconstruct the high frequencies from the low.* The computation is straightforward, using one-dimensional modules and well-known techniques. Indeed, the method is simple enough that it seems unlikely that it is new. What may be novel are the pointers to other work that suggest where the method will work, and where it will fail.

THE PRESCRIPTION

For purposes of discussion this paper assumes that the data are evenly sampled in space and time, and that the higher temporal frequencies are aliased in space. The task is to interpolate in space so as to remove the aliasing.

Step 1: Time resampling

One of the early steps is to extend the time-frequency bandwidth of the data. Although there may be no intention of going out beyond Nyquist, it is prudent to begin by resampling the data in time at least by a factor of two, and by an amount such that the new Nyquist is greater than three times the temporal frequency seperating the aliased and non-aliased regions. If this resampling is carried out in the frequency domain, then the traces should be predicted out from both ends for several cycles, or tapered in amplitude at each end.

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Step 2: Frequency partitioning

Each trace is now split into two; unaliased, low-frequency (LF), and aliased high-frequency (HF). As usual, the frequecy cut-off should not be too sharp, and in the time domain the LF filter might be a triangularly-weighted *sinc* function with the HF filter its identity complement. The choice of cut-off may be made either in terms of the lowest velocity event and the trace spacing, or by looking at f-k power spectra tiled to point up aliased energy. At this point the LF data is available for linear spatial filtering.

Step 3: Creating the pseudo-trace

The next step is to cube the LF traces (in the time domain). The effect of this is to broaden the spectrum of the trace by a factor of three, since multiplication in time corresponds to convolution in frequency. This process is applied both to the original and interpolated LF data.

Step 4: Shaping the pseudo-trace

These pseudo-traces will have approximately the correct phase spectrum (cubing preserves arrival times), but the original HF amplitude information is lost when the data are partitioned in frequency. In the case of data traces though, we can find for each trace the filter which best shapes the amplitude spectrum of the pseudo-trace to the HF data. This filtering can be zero-phase, but in case the reflection wavelets are not themselves zero-phase, some phase information can be modeled in the shaping filter.

Step 5: Interpolation and reconstruction

Although the shaping filters may be quite rich in HF, and thus subject to aliasing if they are to be interpolated, the zero- or near zero-phase constraint discussed in the last section should remove this possibility. So, both the LF data and the shaping filters can be linearly interpolated, and these can then be combined, together with the cubing operation, to generate the wide-band interpolated traces.

DISCUSSION

Will it work? I do not know. What I do know is that it will fail for a well-defined class of model traces, those with the so-called *Bussgang Property*, which was defined and discussed by Rocca &al (1979). The class includes all stationary sequences (of any color) that have a Gaussian distribution of amplitudes, so it seems reasonable to expect that the method will work increasingly well as the traces depart from Gaussian.

A trace consisting of a few, well-defined compact wavelets is a good example of a far-from-Gaussian model.

How do they fail? Simply put, the Bussgang property is that if two sequences have zero correlation, then any memoryless non-linear transformation of either of them leaves them uncorrelated. In the present case, the HF and LF partitions are uncorrelated, and the amplitude cubing is a memoryless, non-linear mapping. Thus there is no prediction of the high frequencies from the low.

I have found Thomas' textbook (1969) to be a useful introduction to non-linear filtering.

EXTENSIONS

The unaliased region

Chosing to partition traces in the frequency domain, is equivalent to constraining the unaliased region to a strip in the *f-k* domain. Since it is desirable to have as large an unaliased region as possible, it might be advantageous to get away from the time domain constraint. Clearly this leads to an increase in computational complexity.

Function polynomials

By limiting the non-linear trace generation to cubing the low-frequency trace, we are also limiting our new bandwidth to three times the original. In a more general formulation, the fifth, seventh, &c. powers of the trace amplitudes can also be employed.

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