Stanford Exploration Project, Report 70, December 18, 1997, pages 1–132

Anisotropic scalar imaging using the double elliptic approximation

Martin Karrenbach[1]

keywords: *approximation, dispersion, velocity, phase, anisotropy, algorithm, modeling*

ABSTRACT

The double elliptic approximation to the transverse isotropic dispersion relation can be used to image a subsurface scalar eigenfield. The rationale behind the double elliptic approximation is the use of only four parameters for 2D data. These parameters are found by conventional velocity analysis after a vector wavefield is converted to its scalar eigenfields. The dispersion relation is parameterized by horizontal and vertical direct-wave velocities and horizontal and vertical normal-moveout velocities. Estimation of those four parameters typically requires a combination of surface seismic data and cross-well or VSP data. If the algorithm is implemented in the phase domain, existing scalar imaging techniques are easily extended to anisotropy As an example a transverse isotropic medium (Greenhorn shale) is modeled and migrated using both the exact anisotropic dispersion relation and its double elliptic approximation. The approximation works very well paraxially, but it fails in areas of triplication.

INTRODUCTION

There are two fundamentally different ways to invert seismic data for subsurface properties. One is to perform nonlinear optimization and end up with an estimation of the full stiffness matrix at each medium location. Another way is to parameterize the data in a way that allows an estimation of a few parameters which describe the data reasonably well. If the problem is transformed into the frequency domain, the exact dispersion relation is not solved for, but a sparse estimate is attempted. The single elliptic paraxial approximation, developed by Dellinger and Muir (1985), was recently extended to a double elliptic paraxial form, which approximates a two-dimensional dispersion relation.

[1]**email:** not available

SCALAR ANISOTROPIC IMAGING TECHNIQUES

Vector wavefields are commonly reduced to their scalar eigenfields. This procedure reduces the data volume of multicomponent source and multicomponent receiver data into a more manageable size. After conversion to scalar fields, conventional processing techniques can be applied (Nichols, 1990). Most conventional techniques assume isotropic wave propagation. It is generally observed that NMO and vertical propagation velocities in conventional seismics do not coincide. This presents a problem when an attempt is made to properly image the subsurface: which migration velocity should be used? If the NMO velocity is used, diffractions will be focused quite well but that focus will be misplaced. If the vertical propagation velocity is used, diffractions will not collapse properly. This situation calls for the application of anisotropic imaging techniques, as shown in this paper.

Whether it is easy to include anisotropy, in the form of the exact dispersion relation, in existing conventional algorithms, depends largely upon whether the methods are implemented in the phase or group domain. In the anisotropic world, there is no easy way to go from the phase to the group domain and vice versa. However, if the double elliptic approximation is used, the two domains can be described by the same set of parameters and conversion between them is easy.

APPROXIMATING THE DISPERSION RELATION

The advantage of approximating the dispersion relation is that we do not need to use special tools to estimate the parameters, but merely the same velocity estimation and imaging techniques which have proved their usefulness in conventional scalar processing. The four parameters in two dimensions are horizontal and vertical normal moveout and the direct propagation velocities. Using conventional velocity-analysis tools paraxially, NMO velocities can be solved for or direct-wave velocities if the data coverage is sufficient. A surface seismic experiment, a cross-well experiment, and a VSP used together (each having limited aperture) would give the following information:

1. horizontal moveout velocity (surface seismic).

2. vertical direct-wave velocity (VSP).

3. horizontal direct-wave velocity (cross-well).

4. vertical moveout velocity (cross-well).

Behind each parameter is the main source of information from which it comes. These parameters we can routinely be found in data processing.

It must be kept in mind that there exist scale differences between the surface seismic, cross-well and VSP experiment types. The most obvious difference is in the source frequency content, which can vary as much as several orders of magnitude, depending on whether the source is placed at the surface or in the well. Spatial and temporal sampling varies from experiment to experiment, but typically in surface experiments frequencies are low, in cross-well experiments frequencies are high, and in VSPs frequencies are somewhere in between. Scale differences must be equalized in a "consistent manner" before the combined measurement values can be used. After scale equalization, conventional velocity analysis can be applied giving an elliptic parameter estimate. The parameters can be estimated either all together in one pass or separately. Approximation does not make it necessary to estimate the parameters either separately or together; and both ways have their advantages. The elliptic approximation does not require any particular anisotropic symmetry model. It rather serves as a basis for a least-squares estimate of elastic stiffness constants.

DOUBLE ELLIPTIC DISPERSION RELATION

The derivation of the double elliptic paraxial approximation is described by Dellinger et al. (1991). The resulting dispersion relation in two dimensions is

$$\omega^2 = \frac{(W_x k_x^2)^3 + (W_x k_x^2)^2 (2W_z + W_{znmo}) k_z^2 + (W_z k_z^2)^2 (2W_x + W_{xnmo}) k_x^2 + (W_z k_z^2)^3}{(W_x k_x^2 + W_z k_z^2)^2}, \tag{1}$$

where ω is the temporal and k_x, k_z are the spatial frequencies and W is squared velocity. The top row in Figure **??** represents the "phase domain"; the bottom row shows the corresponding "group domain". The curves are calculated from actual stiffness constants, which are used below for modeling and migration. I approximated in phase domain, since I show only implementations of phase domain algorithms in this paper.

Exact and approximated phase and group velocities are compared. In general, the approximation in the "phase domain" works very well, suggesting that it is more accurate than the approximation in the "group domain". In general, the curve produced by the double elliptic approximation lies within the true velocity curves. In the phase domain the two curves are in very good agreement, while in the group domain, the disagreement is larger. In particular, triplications can be modeled using the double elliptic approximation in the phase domain; but not in the group domain. Modeling of triplications is possible in the "phase domain", since the rational polynomial can produce concavities in the phase slowness surface. Furthermore, the elliptic parameters govern both phase and group domains; thus each domain can be easily converted into the other.

Equation (1) is a very good approximation to the exact dispersion relation, when Fourier-domain algorithms, such as the Stolt or phase-shift method are used. For modeling and its transpose operation, migration, the dispersion relation must be

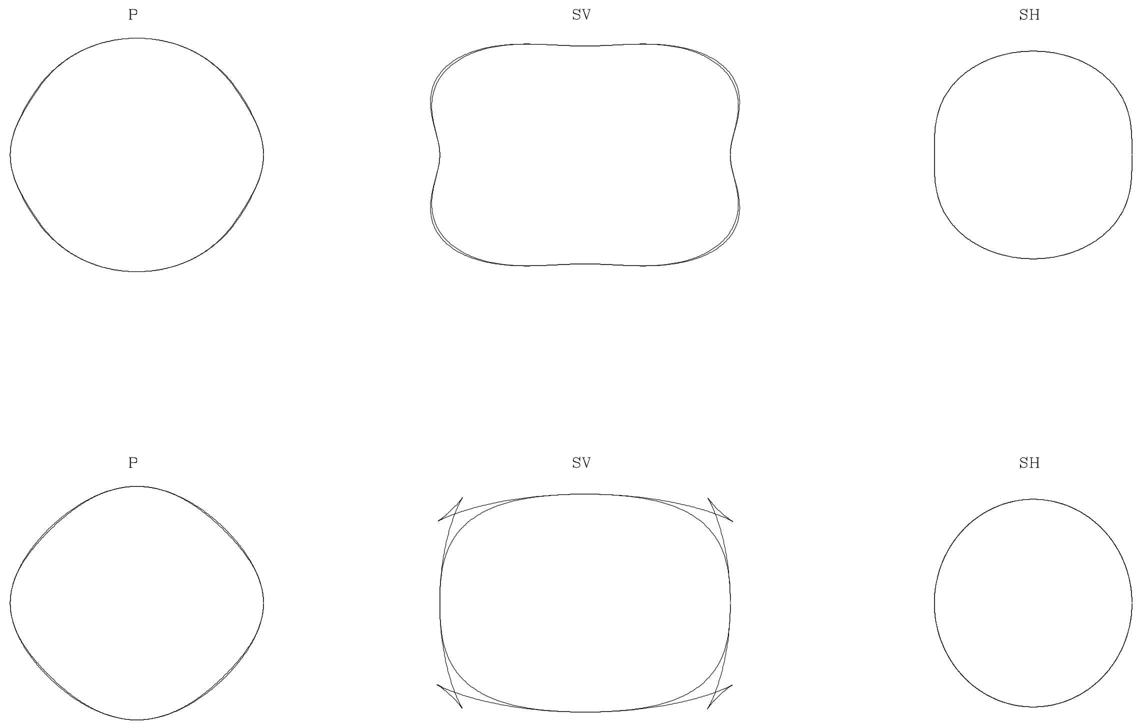

Figure 1: compvel $\boxed{\text{martin1-compvel}}$ [ER]

evaluated and solved for an unknown k_z. Both operations can be done quite efficiently using equation (1). The evaluation of the dispersion relation is straightforward. And when finding the k_z root, we must explicitly solve a third-order polynomial in k_z^2.

The foregoing procedure must be seen in contrast to solving eigenvalue problems numerically or finding roots for higher order polynomials. This is the procedure that has to be followed if the dispersion relation is evaluated exactly for anisotropic media. In this context the double elliptic approximation is an efficient way to solve the problem approximately.

MODELING AND IMAGING

The double elliptic approximation allows direct conversion from the group to the phase, and vice versa; thus making it possible to consider "$\omega - k$"or phase shift together with Kirchhoff methods. In other words movement back and forth between the wave domain and the ray domain.

Imaging procedures like Stolt's, or phase-shift incorporate anisotropic dispersion relations with ease. These algorithms explicitly require the dispersion relation in the form $\omega = f(k_x, k_z)$, where f can be any arbitrary single-valued function. In our approximation f is a rational polynomial. Some algorithms require that the dispersion relation, Equation 1, be solved explicitly for k_z. For arbitrary f this means roots must be chosen properly and in general a numerical evaluation must be performed. In the case of the double elliptic approximation at most a third degree polynomial in k_z is obtained, for which a unique solution can be found analytically.

PHASE-DOMAIN ALGORITHMS

Algorithms in the ω-k domain, such as Stolt's and phase-shift, lend themselves perfectly to anisotropic extension, since such algorithms use the dispersion relation explicitly for downward continuation and imaging. In general the agreement between the exact transverse isotropic (TI) dispersion relation and the double elliptic approximation, equation 1, is very good. The approximate algorithm is even able to model triplications. Figures **??** and **??** compare modeling and migration impulse responses. The right column plots the difference between the results in the left and middle columns using the same scale. Only the P and SV wave types show some small differences, while the SH wave is exact. The approximation fits the dispersion relation exactly on the vertical and horizontal axes. The largest discrepancy is in the area of triplication. The crucial test, however, is to model a TI media using the exact dispersion relation and then migrate it with the double elliptic approximation. Figure **??** shows that, more or less, the correct spikes are obtained. The middle column shows migration results, where both, modeling and migration, used the exact dispersion relation. The right column shows migration results where the exact form was used for modeling and the approximation was used for imaging. The algorithm, which uses the exact dispersion relation, reproduces the spike, which is convolved with a ricker wavelet, very well. The double elliptic approximation shows the spikes slightly blurred. The results suggest that phase domain algorithms can be expected to perform reasonably well, when used to image an anisotropic scalar eigenfield.

GROUP-DOMAIN ALGORITHMS

With the help of the associated ray equation, a Kirchhoff migration in x-t domain can be implemented directly or traveltimes can be calculated from the dispersion relation using anisotropic finite difference traveltimes. Triplications are caused by concavities in the phase slowness surface. The double elliptic approximation can model concavities well; thus triplications are produced. When the ray equation instead of the dispersion relation is used, the group velocity surface is approximated instead of the phase velocity surface. The approximation of the group velocity surface fails to follow the triplication. This is the fundamental difference between the $\omega - k$ and $x - t$ methods. Typical $x - t$ methods is Kirchhoff modeling and migration. In order to be accurate, such an algorithm has to handle triplications properly, which in general is not as easy in $x - t$ as in $\omega - k$. When the double elliptic form is used, the exact curve cannot be approximated well in the area of triplications. An algorithm which uses the group domain approximation (see Figure **??**, middle bottom) is not able to produce triplications.

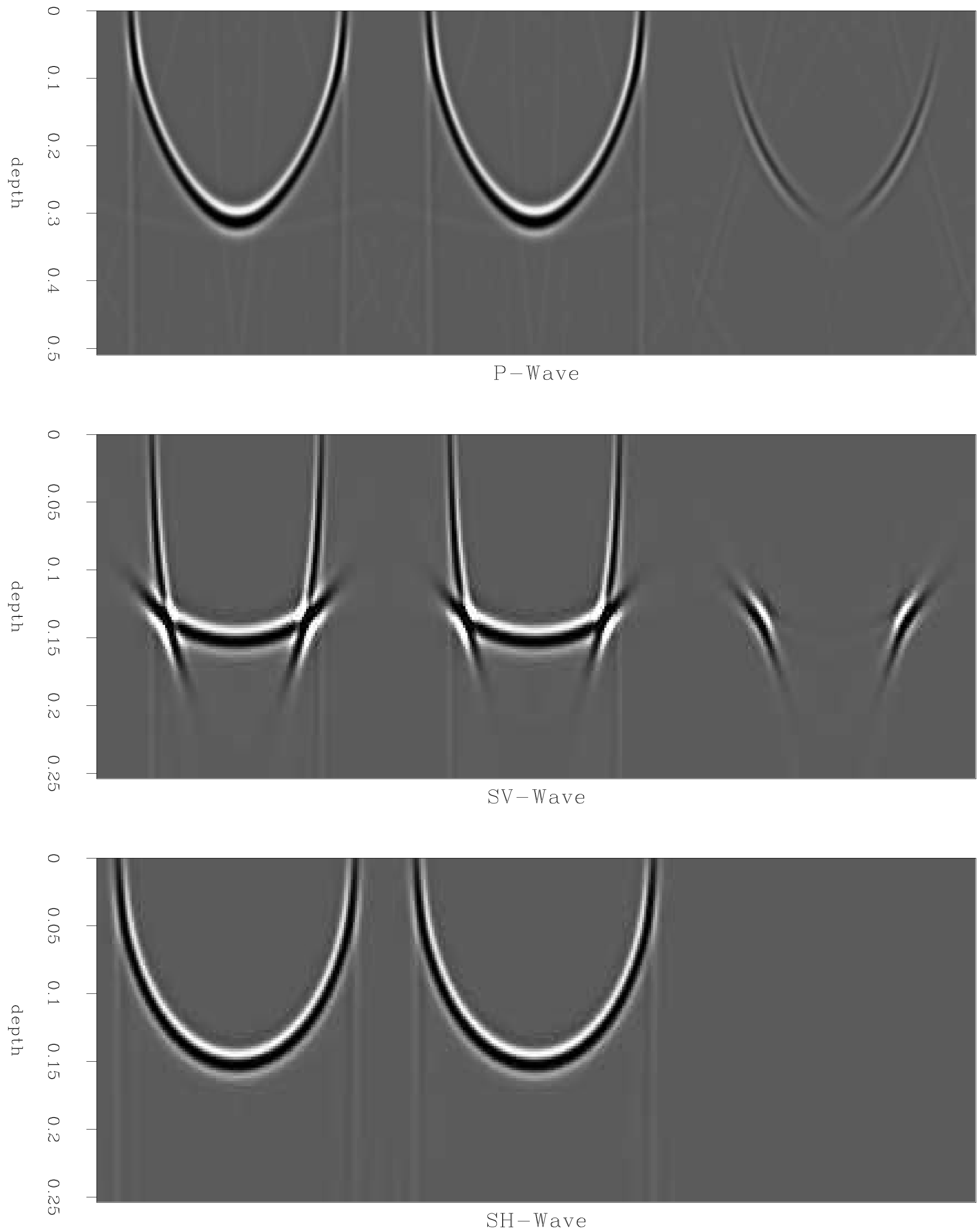

Figure 2: Scalar migration impulse response (using the Stolt method) of a TI medium is compared to the impulse response of the equivalent double elliptic approximation. The left column shows results using the exact dispersion relation, the middle column results using the double elliptic approximation, and the right column shows the difference between the two results. We are able to model even cusps to a limited extent: the approximation works very well paraxially. martin1-compimp [ER]

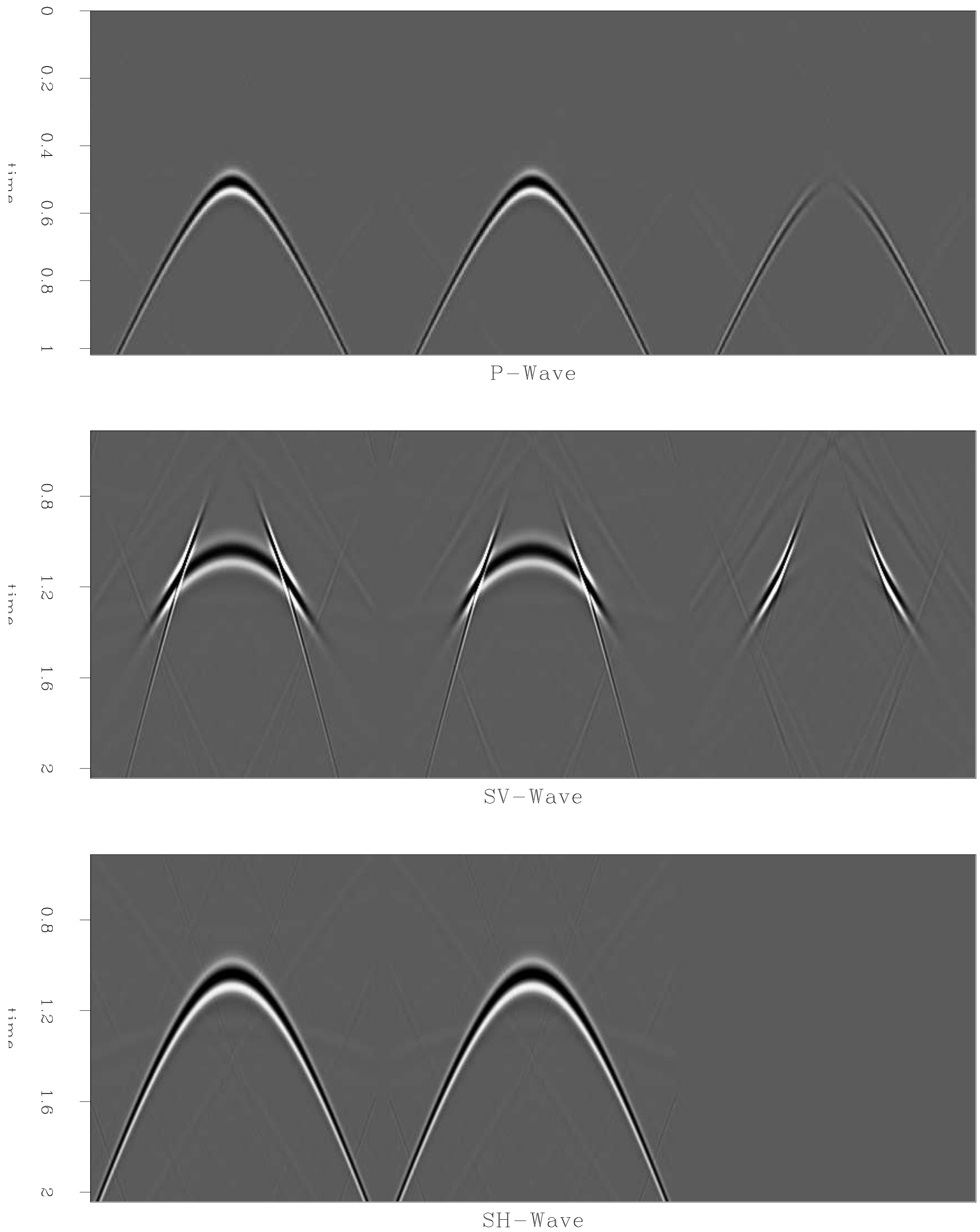

Figure 3: Scalar modeling impulse response (using the Stolt method) of a TI medium is compared to the impulse response of the equivalent double elliptic approximation. The left column shows results using the exact dispersion relation, the middle column results using the double elliptic approximation, and the right column the difference between the two. As in Figure 19,, the approximation works very well paraxially. martin1-compmod [ER]

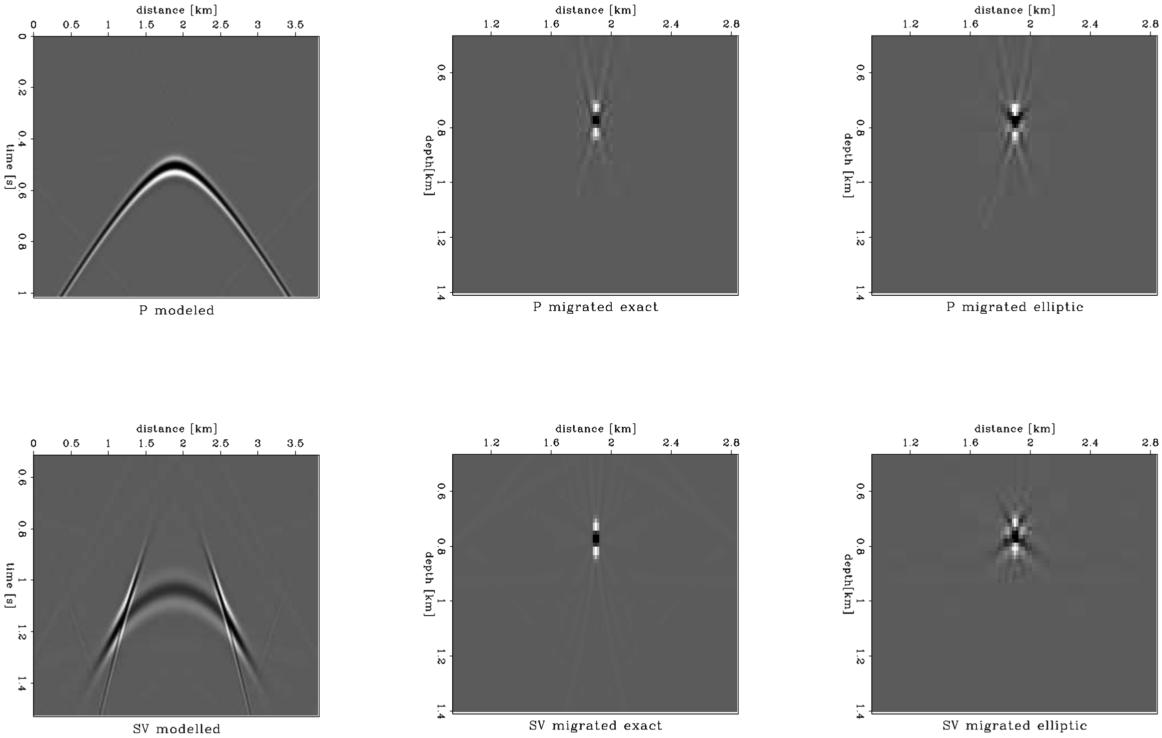

Figure 4: The exact scalar modeling (using the Stolt method)) of an point scatterer (left) in a TI medium is migrated once using the exact (middle) and the approximated (right) dispersion relation. The approximation works better for the P wave than for the SV wave, where it fails to collapse some energy from the cusps. martin1-cmodmig [ER]

Prestack or poststack?

I have been very careful not to make a distinction between prestack and poststack algorithms: all of the above applies to both domains. The prestack version use the dispersion relation in a form that solves for $k_z = F(\omega, k_x)$. Sources and receivers are downward continued in depth by calculating

$$k_z = k_{z,s} + k_{z,g} = F(\omega, k_{x,s}) + F(\omega, k_{x,g}), \tag{2}$$

where $k_{x,s}$ and $k_{x,g}$ are spatial frequencies in shot-geophone coordinates. Again F depends on the exact anisotropic dispersion relation or its double elliptic approximation.

CONCLUSIONS

Existing one-way imaging and modeling algorithms like Stolt's and phase-shift methods can be extended to the anisotropic case and are computationally attractive when the exact anisotropic dispersion relation is approximated by a rational polynomial. This double elliptic approximation works very well in the phase domain and, like the exact dispersion relation, produces triplication in the group domain. The same parameter which describes the dispersion relation can be used to approximate the group velocity curve however, then triplications are not reproduced. This suggests that phase velocity curves should be approximated, and implies that the processing should be done in the $\omega - k$ or $\tau - p$ domain.

REFERENCES

Dellinger, J., and Muir, F., 1985, Two domains of anisotropy: SEP–**44**, 59-62.

Dellinger, J. and Muir, F., 1991, The double elliptic approximation in the group and phase domains: SEP–**70**.

Muir F., and Dellinger J., 1985, A practical anisotropic system: SEP–**44**, 55-58.

Nichols, D.E., 1989, Converting surface motions to subsurface wavefields: SEP–**61**, 179-200.