Stanford Exploration Project, Report 70, December 18, 1997, pages 1–351

Short Note

Various equations for TI media

Francis Muir[1]

keywords: *velocity, approximation*

INTRODUCTION

Waves and rays are necessary components of any suite of seismic processes. In an isotropic medium the transition is seamless—not so in the simplest anisotropic material. Here, not only do velocities depend on direction, but worse, rays and waves have different velocities, and worse still, there is no local transformation from one kind of velocity to the other. In an earlier paper (Muir, 1990) I described a partial solution where approximate plane wave equations and ray equations have simple, interchangeable properties. In this note I relate the parameters of these approximants to the elastic moduli of the *TI* media that lie behind the dispersion relations that I approximate.

SPECIAL SYMBOLS

Apart from the usual ones, I use the following symbols:

W, *alacrity*, and subscripted variations thereon, is use for quantities with dimension velocity squared.

M, *sloth*, and its variations, is used for quantities with dimension inverse velocity squared.

W_{ij} is $(1/\rho)C_{ij}$, an elastic modulus divided by density.

θ is the angle from the stated axis

C is $\cos(\theta)$

S is $\sin(\theta)$

[1]**email:** not available

ELLIPTIC APPROXIMANTS FOR *TI* PLANE WAVE EQUATIONS

That is, elliptic forms that approximate the plane-wave behavior of the three wavetypes for Transverse Isotropic materials in the neighbourhoods of the Z and X axes.

SH plane waves about the Z-axis

$$W(\theta) = W_{44}C^2 + W_{66}S^2 \tag{1}$$

P & SV plane waves about the Z-axis

Begin with the exact dispersion relations for P and SV waves.

$$\begin{aligned} 2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2 \\ \pm\surd(((W_{33} - W_{44})C^2 - (W_{11} - W_{44})S^2)^2 + 4(W_{13} + W_{44})^2C^2S^2) \end{aligned} \tag{2}$$

Now convert the cosines under the surd to sines.

$$\begin{aligned} 2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2 \\ \pm\surd(((W_{33} - W_{44}) - (W_{11} + W_{33} - 2W_{44})S^2)^2 + 4(W_{13} + W_{44})^2(1 - S^2)S^2) \end{aligned}$$

Neglecting terms in S^4.

$$\begin{aligned} 2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2 \\ \pm\surd((W_{33} - W_{44})^2 - 2((W_{33} - W_{44})(W_{11} + W_{33} - 2W_{44}) - 2(W_{13} + W_{44})^2)S^2) \end{aligned}$$

Take constant out of bracket.

$$\begin{aligned} 2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2 \\ \pm(W_{33} - W_{44})\surd(1 - 2\frac{(W_{33} - W_{44})(W_{11} + W_{33} - 2W_{44}) - 2(W_{13} + W_{44})^2}{(W_{33} - W_{44})^2}S^2) \end{aligned}$$

Expand the surd in powers of S^2.

$$\begin{aligned} 2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2 \\ \pm(W_{33} - W_{44})(1 - \frac{(W_{33} - W_{44})(W_{11} + W_{33} - 2W_{44}) - 2(W_{13} + W_{44})^2}{(W_{33} - W_{44})^2}S^2) \end{aligned}$$

Return factor to original location

$$\begin{aligned} 2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2 \\ \pm((W_{33} - W_{44}) - \frac{(W_{33} - W_{44})(W_{11} + W_{33} - 2W_{44}) - 2(W_{13} + W_{44})^2}{W_{33} - W_{44}}S^2) \end{aligned}$$

Simplify.

$$2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2$$
$$\pm((W_{33} - W_{44}) - (W_{11} + W_{33} - 2W_{44})S^2 + \frac{2(W_{13} + W_{44})^2}{W_{33} - W_{44}}S^2)$$

Reintroduce cosines.

$$2W(\theta) = (W_{33} + W_{44})C^2 + (W_{11} + W_{44})S^2$$
$$\pm((W_{33} - W_{44})C^2 - (W_{11} - W_{44})S^2 + \frac{2(W_{13} + W_{44})^2}{W_{33} - W_{44}}S^2)$$

P plane waves about the Z-axis

Choose positive sign for $\pm$.

$$W(\theta) = W_{33}C^2 + (W_{44} + \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}})S^2 \quad (3)$$

SV plane waves about the Z-axis

Choose negative sign for $\pm$.

$$W(\theta) = W_{44}C^2 + (W_{11} - \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}})S^2 \quad (4)$$

P plane waves about the X-axis

Interchangeing W_{11} and W_{33} wherever they occur in Eqn. (3)

$$W(\theta) = W_{11}C^2 + (W_{44} + \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}})S^2 \quad (5)$$

SV plane waves about the X-axis

Interchangeing W_{11} and W_{33} wherever they occur in Eqn. (4)

$$W(\theta) = W_{44}C^2 + (W_{33} - \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}})S^2 \quad (6)$$

DOUBLE ELLIPTIC FORMS

These apply to all three wave types, and are the equations that are the heart of an earlier paper (Muir, 1990).

Plane wave equations referenced to the Z axis

$$W(\theta) = \frac{W_z^3 C^6 + W_z^2 (2W_x + W_{xnmo}) C^4 S^2 + (2W_z + W_{znmo}) W_x^2 C^2 S^4 + W_x^3 S^6}{(W_z C^2 + W_x S^2)^2} \tag{7}$$

Ray equations referenced to the Z axis

$$M(\theta) = \frac{M_z^3 C^6 + M_z^2 (2M_x + M_{xnmo}) C^4 S^2 + (2M_z + M_{znmo}) M_x^2 C^2 S^4 + M_x^3 S^6}{(M_z C^2 + M_x S^2)^2} \tag{8}$$

ELLIPTIC COEFFICIENTS

P waves

$$W_{P,x} = W_{11} \tag{9}$$

$$W_{P,xnmo} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}} \tag{10}$$

$$W_{P,z} = W_{33} \tag{11}$$

$$W_{P,znmo} = W_{44} + \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}} \tag{12}$$

SV waves

$$W_{SV,x} = W_{44} \tag{13}$$

$$W_{SV,xnmo} = W_{11} - \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}} \tag{14}$$

$$W_{SV,z} = W_{44} \tag{15}$$

$$W_{SV,znmo} = W_{33} - \frac{(W_{13} + W_{44})^2}{W_{11} - W_{44}} \tag{16}$$

SH waves

$$W_{SH,x} = W_{66} \tag{17}$$

$$W_{SH,xnmo} = W_{66} \tag{18}$$

$$W_{SH,z} = W_{44} \tag{19}$$

$$W_{SH,znmo} = W_{44} \tag{20}$$

Converting wave and ray coefficients

$$M_* = \frac{1}{W_*} \tag{21}$$

PSEUDO ISOTROPIC CONDITIONS

If the paraxial move-out velocity equals the axial velocity, then the medium will be isotropic in kinematic approximation for the wavetype concerned.

Pseudo-isotropy about the Z axis

The condition is that

$$W_z \quad = \quad W_{xnmo} \tag{22}$$

For P waves this means

$$W_{33} \quad = \quad W_{44} + \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}} \tag{23}$$

which is

$$(W_{13} + W_{44})^2 \quad = \quad (W_{33} - W_{44})^2 \tag{24}$$

SV waves

Similarly.

$$W_{44} = W_{11} - \frac{(W_{13} + W_{44})^2}{W_{33} - W_{44}} \tag{25}$$

which is to say

$$(W_{13} + W_{44})^2 = (W_{11} - W_{44})(W_{33} - W_{44}) \tag{26}$$

and for SH waves

$$W_{44} = W_{66} \tag{27}$$

which is an exact result

Pseudo-isotropy about the X axis

Completely equivalent results are obtained by switching W_{11} and W_{33} wherever they occur. Thus for P waves

$$(W_{13} + W_{44})^2 = (W_{11} - W_{44})^2 \tag{28}$$

and for SV waves

$$(W_{13} + W_{44})^2 = (W_{11} - W_{44})(W_{33} - W_{44}) \tag{29}$$

and for SH waves

$$W_{44} = W_{66} \tag{30}$$

Note that shear waves, SV or SH, the same condition holds in both the X and Z paraxial approximations.

REFERENCES

Muir, F., 1990, A modified anisotropic system: **SEP–67**, 41–42.