Stanford Exploration Project, Report 70, December 18, 1997, pages 1–339

Short Note

A simplified canonical form for monoclinic systems

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keywords: *anisotropy*

INTRODUCTION

Anisotropy can be introduced into earth modeling in a useful and natural way by recognizing a correspondence between anisotropic complexity and the complexity of the fabric of an otherwise relatively undisturbed sedimentary basin. The correspondence I have in mind looks something like this:

isotropic Uniform depositional environment. Sediments are not much altered by overburden.

transverse isotropic Layering effects. Variable deposition. Alteration caused by a uniaxial (gravity) stress field. Horizontal cracks and fractures.

orthorhombic Further alteration due to a vertical crack or fracture system.

monoclinic A set of such vertical fracture systems with a variety of azimuths.

We could go on from there and describe more complex systems, but plane-layered models have a long and successful history in our discipline, and the monoclinic system with horizontal symmetry plane is a natural adjunct to these plane-layered models, leading to quite major simplifications in the algebra of modeling. With this context in mind, it is the purpose of this note to present a simplication in the description of monoclinic stiffness and compliance matrices.

CANONICAL FORMS

Conventional Wisdom

In specifying the stiffness or compliance matrix of an anisotropic material, convention dictates that the coordinate and symmetry systems are aligned to minimize the num-

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ber of non-zero coefficients; that is, we like to seperate elastic and geometric degrees of freedom. For example, the twelve (nine independent) non-zero coefficients of an orthorhombic system are so

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} \tag{1}$$

because we have aligned the coordinate system with the three symmetry axes.

Standard Monoclinic Form

In case of monoclinic systems, we recognize the one symmetry plane, the (12) plane and this leads to:

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{pmatrix} \tag{2}$$

Without loss of generality, and to simplify discussion, we now rearrange the rows and columns so that the 6th lie between the 3rd and 4th,

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{16} & 0 & 0 \\ c_{12} & c_{22} & c_{23} & c_{26} & 0 & 0 \\ c_{13} & c_{23} & c_{33} & c_{36} & 0 & 0 \\ c_{16} & c_{26} & c_{36} & c_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & c_{45} \\ 0 & 0 & 0 & 0 & c_{45} & c_{55} \end{pmatrix} \tag{3}$$

and the stiffness matrix is now in **block diagonal** form.

Form Invariance under Rotation about the 3-axis

Stiffness and compliance matrices in compact form are rotated with Bond transformations (Auld, 1990). If we look at the Bond forms for rotation about the 3-axis we see that they have the identical block diagonal structure of our stiffness matrix illustrated above. Since block matrices of the same shape form a group under matrix multiplication, this means that we can rotate monoclinic systems about the symmetry axis and leave the form unchanged. What this means physically is that the property that defines a monoclinic system, the two-fold symmetry, is not tied to azimuth in the symmetry plane.

Choosing the Rotation Angle

What angle to choose? From the point of view of parsimony we can reduce the number of variables from 13 to 12 by rotating our coordinate system around the 3-axis by an angle $= (\arctan(2c_{45}/(c_{55} - c_{44})))/2$. This will anihilate the two occurences of the c_{45} term.

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{16} & 0 & 0 \\ c_{12} & c_{22} & c_{23} & c_{26} & 0 & 0 \\ c_{13} & c_{23} & c_{33} & c_{36} & 0 & 0 \\ c_{16} & c_{26} & c_{36} & c_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{55} \end{pmatrix} \tag{4}$$

Does this make sense physically? Yes. Since there is 2-fold symmetry, the three waves propagating along the 3-axis have orthogonal directions of particle motion, and those directions corresponding to the two pure shear waves define two pseudo-symmetry axes. If the coordinate system is now aligned along these axes, the c45 term disappears,

Proposed Canonical Forms for Monoclinic Systems

Notice that the new form above has a simplified block diagonal structure and because of the group property this will carry over to the compliance matrix. We are now in a position to back out the row and column reordering, and present the new monoclinic forms for both stiffness and compliance.

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & s_{16} \\ s_{12} & s_{22} & s_{23} & 0 & 0 & s_{26} \\ s_{13} & s_{23} & s_{33} & 0 & 0 & s_{36} \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ s_{16} & s_{26} & s_{36} & 0 & 0 & s_{66} \end{pmatrix} \tag{6}$$

Practical Considerations

Since a monoclinic system "looks like" an orthorhombic system in the neighbourhood of the symmetry axis, near-vertical data such as from many surface seismic surveys and VSPs may appear to be orthorhombic, and this may lead to an over-simplified interpretation of the fracture systems. For example, double fracture systems may be mis-interpreted as single fracture systems.

ACKNOWLEDGMENTS

Joe Dellinger's Electronic Anisotropists got this result propagated, and Andy Norris of Rutgers and Klaus Helbig of Delft encouraged me to write it up.

REFERENCES

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