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Migration, tomography, seismic inversion theory, and how they relate to each other

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ABSTRACT

The Kirchhoff integral can only be used to propagate wavefields forward. Backward propagation is based on the Porter-Bojarski integral. Both integrals simplify if data is registered on a plane. Transmission and reflection data should be combined to increase spatial resolution. Angles and frequencies should vary as much as possible to cover the largest area possible of the k-space. Migration is identical to diffraction tomography for varying frequencies if the weak scatterer assumption is valid in a constant background velocity medium. Holography is backpropagation and imaging of a monofrequent wavefield. Linear and non-linear inversions improve the migration and tomography results because they consider the data as imperfect.

INTRODUCTION

Inverse scattering problems have to be solved in many scientific fields. Seismic inversion can be defined as the attempt to get a correct image of the subsurface from recorded seismograms (Claerbout, J.F., 1985). Iterative inversion tries to find the best fit between the registered data and synthetic seismograms generated by an earth model. It can be called a "statistical" approach because the best fit is defined in terms of statistics. Linear iterative inversion inverts by varying only the earth model to minimize the difference between observed and synthetic data in a Gaussian sense (Claerbout, J.F., 1989). Nonlinear iterative inversion also adapts the wave propagation matrix in each step of the iteration (Mora, P., 1987). In contrast to the iterative methods stand the non-iterative "deterministic" approach. Deterministic methods are also referred to as holography, migration, or tomography to distinguish them from the iterative, more statistical inversion methods (Mora, P., 1989). The theory of migration is based on the forward problem which can be solved exactly by the Kirchhoff integral. Migration constructs images by backpropagating only the homogeneous part of the registered wavefield. Migration algorithms have been programmed

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in different domains so that terms like Kirchhoff Migration (Schneider, W.A., 1978), the Phase-Shift Method (Gazdag, J., 1978), Stolt Migration (Stolt, R.H., 1978), and Radon Migration (Rueter, H., 1987) result. It can be shown that these migration methods are mathematically equivalent for a constant velocity medium (Tygel, M., and Hubral, P., 1990). Tomographic inversion methods gained in importance in seismology during the last decade. Traveltime and amplitude tomography describe the seismic wavefield by straight rays and, therefore, interpret the registered data as projections of the medium (Worthington, M.H., 1984). The projection and its inversion, the backprojection, can be described by the Radon transform pair (Deans, S.R., 1983). Diffraction tomography allows not only straight rays but all kinds of weak scatterer wavefields in a medium of constant background velocity (Woodward, M.J., 1989). Traditionally, reflection seismic inversion was based on rather heuristic ideas. A look over the shoulder of people working in non-destructive testing (Langenberg, 1986), on Radar techniques or medical tomography might be useful to achieve a deeper understanding of the theories and the vocabulary involved in the inversion business. A common misconception is that migration is an application of the Kirchhoff integral. Opposite starting points led to the development of tomography and migration. Many people are not aware of the relation between both. In this paper I will study the inversion problem in a general way. I introduce some terms used in the non-geophysical inversion theory and show how the different algorithms relate to each other. We will see that migration and diffraction tomography are nearly the same thing. Varying frequency and the angle of incidence for the signal and superposition of the different resulting images improve the quality of the reconstruction. Finally, we will come to the conclusion that today the nonlinear elastic inversion of seismic reflection *and* transmission data is the best algorithm that can be offered to image the earth's interior. First, the basics of the statistical and deterministic approach will be investigated.

ITERATIVE INVERSION

The statistical approach to seismic inversion considers the fact that the data is imperfect. Random noise, unwanted signals, missing data, and evanescent waves that cannot be backpropagated make a real deterministic inversion impossible. The goal of iterative inversion is to minimize the difference between the registered data vector $\underline{Y}$ and the synthetic data computed as the product of the propagator matrix $\underline{A}$ and the earth model $\underline{X}$ step by step in a Gaussian sense. To stabilize the computation, a small term $\epsilon^2 \underline{X}^2$ has to be added:

$$(\underline{Y} - \underline{A}\underline{X})^2 + \epsilon^2 \underline{X}^2 = min \tag{1}$$

Linear inversion needs a reasonable a priori velocity model $\underline{X}$ as input and to construct the propagator matrix $\underline{A}$. The choice of $\underline{A}$ is, of course, based on wave or ray theory. An iterative inversion algorithm applies migration or tomography in each step of the iteration. In so far, the "statistical approach" is also "deterministic." The iteration converges at least after N iterations against the "true" earth structure, if N is the number of unknowns to be solved for.

Nonlinear inversion goes one step further because it iterates not only for the model vector $\underline{X}$ but also for the propagator $\underline{A}$. The start propagator matrix is usually the transpose of the forward operator (Claerbout, J.F., 1989). The method of steepest descent or better conjugate gradients are in widespread use to find the minimum. To guarantee convergence a reasonable start model is a necessity.

To make sure that the true earth model can be found, the assumption that the earth behaves Gaussian has to be made. A *real* statistical approach to seismic inversion should drop this assumption and should use non-Gaussian methods like the Monte Carlo method (Tarantola, A., 1990). Only if we iterate without any a priori information (which can be wrong) can we hope to find the real inverse operator. Unfortunately, this makes the solution space infinite so theoretically we will need a computer with infinite speed. In practice it should be sufficient to combine Gaussian iterative nonlinear inversion with the Monte Carlo method to improve today's results.

KIRCHHOFF INTEGRAL AND PORTER-BOJARSKI INTEGRAL

The basis of the iterative methods is the non-iterative "deterministic" approach to seismic inversion. It ignores the random components of seismic data and is strictly speaking less deterministic than the iterative inversion. The reason it is sometimes considered as deterministic is that it is based on the forward scattering problem that can be solved exactly by the Kirchhoff integral. People took the forward solutions and simply changed the direction of the wave propagation. The resultant integrals converge only between finite integration limits that cut off evanescent energy. Therefore, this method constructs images by backpropagating only the homogeneous part of the registered wavefield. Iterative methods treat evanescent energy at least statistically so that they should lead to more accurate images.

For didactical reasons we will now consider the Kirchhoff integral in its simplest form: for an homogeneous acoustic medium. The following concepts can be extended to inhomogeneous elastic media.

The Kirchhoff integral takes a wavefield $p(R,\omega)$ and its normal derivative $\frac{\partial}{\partial \underline{n}} p(R,\omega)$ measured on an arbitrary closed surface S surrounding the sources and propagates the field forward, away from the sources to an arbitrary place R_0 outside the surface. The normal vector $\underline{n}$ points to the outside of V that is away from the sources. The volume V is enclosed by S and contains the source volume V_S. Inside of V the Kirchhoff integral is zero. A Green's function $G(R-R_0,\omega)$ extrapolates the registered wave field from R to R_0.

If R_0 is inside of V we have:

$$\oint_S [p(R,\omega)\frac{\partial}{\partial \underline{n}} G(R-R_0,\omega) - \frac{\partial}{\partial \underline{n}} p(R,\omega) G(R-R_0,\omega)] dS = 0 \qquad (2)$$

If R_0 is outside of V we have:

$$\oint_S [p(R,\omega)\frac{\partial}{\partial \underline{n}} G(R-R_0,\omega) - \frac{\partial}{\partial \underline{n}} p(R,\omega) G(R-R_0,\omega)] dS = p(R_0,\omega) \tag{3}$$

The Kirchhoff integral has to be zero inside V because we don't know where the sources are. If we knew the exact position of the source volume V_S, we could change the sign of $\underline{n}$ and propagate until close to that region. But there we had to stop because the source volume contains singularities that would make our integral diverge. Mathematically, it makes only a quantitative difference whether the sources inside are primary or secondary sources. In any case, we will run into trouble if we try to backpropagate evanescent energy, that is, if we come too close to the sources. Numerically, the whole thing could work if only weak scatterers have to be considered.

The physical interpretation of the Kirchhoff integral is that of a forward propagation and superposition of Huygen's sources on S. We assume two kinds of sources: monopoles $p(R,\omega)$ and dipoles $\frac{\partial}{\partial \underline{n}} p(R,\omega)$. In case of stiff boundary conditions (von Neumann conditions) on S, we can drop the term $\frac{\partial}{\partial \underline{n}} p(R,\omega)$ and for a weak boundary S only the gradient $\frac{\partial}{\partial \underline{n}} p(R,\omega)$ is of importance and we drop the term containing $p(R,\omega)$ (Dirichlet boundary condition).

So far we learned how to forward propagation. For backpropagation, i.e. if we propagate towards the sources, the Kirchhoff integral has to be modified. Two changes have to be made:

1. change the sign of the normal vector $\underline{n}$; i.e., let $\underline{n}$ point towards the source region.
2. change the propagation direction, i.e., the Green's function G.

To reverse the wave propagation direction, we can change the sign of time in the Green's function G taken in the time domain. Because we are working in the frequency domain, we have to take the complex conjugate Green's function G^*. G^* performs the propagation of an *imploding* wavefield while G propagates an *exploding* wavefield. The two substitutions yield to the Porter-Bojarski integral (Langenberg, K.L., 1986)

$$-\oint_S [p(R,\omega)\frac{\partial}{\partial \underline{n}} G^*(R-R_0,\omega) - \frac{\partial}{\partial \underline{n}} p(R,\omega) G^*(R-R_0,\omega)] dS = p(R_0,\omega) \quad . \tag{4}$$

The implosion will take place at the source location and at the time the measured field exploded.

G and G^* solve the inhomogeneous wave equation for a point source:

$$\Delta G(R-R_0,\omega) - \frac{\partial^2}{\partial t^2} G(R-R_0,\omega) = -\delta(R-R_0) \tag{5}$$

and

$$\Delta G^*(R-R_0,\omega) - \frac{\partial^2}{\partial t^2} G^*(R-R_0,\omega) = -\delta(R-R_0) \tag{6}$$

The difference between the two inhomogeneous wave equations describes the homogeneous wavefield

$$\Delta(G^* - G) - \frac{\partial^2}{\partial t^2}(G^* - G) = 0 \quad . \tag{7}$$

In a three-dimensional medium, the new Green's function is

$$\begin{aligned} G^* - G = & \frac{\exp(-i\omega(t - \frac{|R-R_0|}{v}))}{4\pi|R-R_0|} - \frac{\exp(+i\omega(t - \frac{|R-R_0|}{v}))}{4\pi|R-R_0|} \\ = & \frac{i\sin(\omega(t - \frac{|R-R_0|}{v}))}{2\pi|R-R_0|} \quad . \end{aligned} \tag{8}$$

The Green's function of the homogeneous wavefield that is backpropagated has an interesting interpretation that is more obvious in its 3D time domain representation

$$G^*(R - R_0, t) - G(R - R_0, t) = \frac{\delta(t - \frac{|R-R_0|}{v})}{4\pi|R - R_0|} - \frac{\delta(t + \frac{|R-R_0|}{v})}{4\pi|R - R_0|} \quad . \tag{9}$$

Our Green's function propagates the wavefield of *time symmetric* point sources on the surface S. The time symmetric waves implode for negative times, run through a focus at $t = 0$, and explode for $t > 0$.

The homogeneous wavefield is used in standard seismic imaging where we have to superpose the forward extrapolated wavefield and the backward extrapolated waves to get a representation of the subsurface by homogeneous waves at the time they were scattered from a discontinuity. Usually we propagate with the *same* sign of $\underline{n}$ for both forward and backward propagating Green's functions. This explains why we subtracted the forward and backward extrapolated waves, whereas in prestack migration, they have to be added.

The surface integral of Porter and Bojarski also has a volume integral representation

$$\int\int_V\int q(R,\omega)[G^*(R - R_0, \omega) - G(R - R_0, \omega)]dV = p(R_0, \omega) \tag{10}$$

where $q(R, \omega)$ is a source at R inside of V. This tells us that the scattered wavefield at R_0 can be obtained by the superposition of volume sources located in the volume enclosed by S. $G - G^*$ extrapolates only an *homogeneous* wavefield.

HOLOGRAPHY

The volume integral representation of the Porter-Bojarski integral allows the backpropagation of the homogeneous portion in the measured data. We can *not* backpropagate the inhomogeneous portion. Nevertheless, if the inhomogeneous waves are weak in amplitude compared to the homogeneous waves, we can get a good subsurface image. Although we didn't use any weak scatterer approximation for the derivation of the Porter-Bojarski integral, the image obtained from the integral will not be complete if there are strong scatterers near our geophones. The seismic exploration industry solves this problem by cutting off evanescent energy and being content with

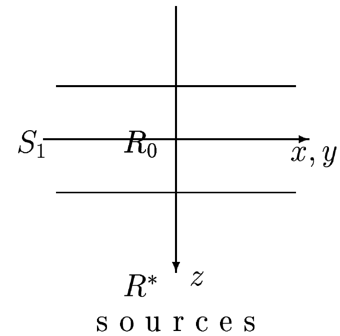

Figure 1: Derivation of the Raleigh integrals.

the homogeneous wave image. It is more clever to continue downward the homogeneous waves to a certain reflector, let the reflector explode, and use the difference of synthetic and registered data to guess the evanescent energy and to update the reflection coefficients. This can be done iteratively.

Programming the Porter-Bojarski integral for the backpropagation from an arbitrary surface S leads to an algorithm called *generalized holography*. It performs the backpropagation of homogeneous waves with a constant frequency.

Let us for a moment consider the Kirchhoff integral for the special case of a planar registration surface S_1. We assume Cartesian coordinates and some scatterers only in the lower halfspace. We construct a halfsphere $S = S_1 + S_2$ in the upper halfspace with the x-y-plane being the registration plane. If the radius approaches infinity, we can ignore the energy scattered through the part of the halfsphere with $z \neq 0$ but *not* through the registration plane. Remember that the Kirchhoff integral is zero if we march from S_1 towards the sources. Now subtract the wavefields from a plane slightly above and parallel to S_1 in a distance R_0 and a plane very close but below S_1 in a distance R^* and also parallel to the observation plane

$$p(R,\omega) = \oint_{S_1} [p(R_0,\omega)(\frac{\partial}{\partial \underline{n}} G(R-R_0,\omega) - \frac{\partial}{\partial \underline{n}} G(R-R^*,\omega)) + \frac{\partial}{\partial \underline{n}} p(R_0,\omega)(G(R-R^*,\omega) - G(R-R_0,\omega))] dS \quad . \tag{11}$$

Close to S_1 both Green's functions are identical. The normal derivatives differ only in their sign. Therefore, we simplify the Kirchhoff integral to obtain the *Raleigh II* integral

$$2 \oint_{S_1} [p(R_0,\omega) \frac{\partial}{\partial z} G(R-R_0,\omega)] dS = p(R,\omega) \quad . \tag{12}$$

The *Raleigh II* integral allows forward propagation by the superposition of *monopoles* $p(R_0,\omega)$ along the plane $z = 0$.

If we add the integrals for above and below the observation plane, we get the *Raleigh I* integral

$$2 \oint_{S_1} [\frac{\partial}{\partial z} p(R_0,\omega) G(R-R_0,\omega)] dS = p(R,\omega) \quad . \tag{13}$$

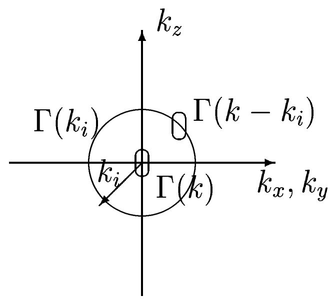

Figure 2: Cut through the Ewald sphere $\Gamma(k_i)$. The object function $\Gamma(k)$ is shifted by the incident wave k_i and intersects with the Ewald sphere.

It says: superpose all *dipoles* $\frac{\partial}{\partial z}p(R_0, \omega)$ on S_1 and obtain the wave field $p(R, \omega)$ via forward propagation.

Equivalent Raleigh formulations for the Porter-Bojarski integral are obtained by inserting the conjugate complex Green's functions G^*. If we consider the registration surface of reflection seismic surveys to be a plane, it will be sufficient to backpropagate only monopoles with a Raleigh II type representation of the Porter-Bojarski integral. We don't have to know the gradient of the wavefield. This shouldn't be news for you, but the explanations given for this fact by Schneider (1978) and others are rather heuristical.

Programming the Raleigh I or Raleigh II integrals leads to the so-called *Raleigh Sommerfeld Holography* (Langenberg, K.L., 1986). It performs the backpropagation of homogeneous waves emitted by dipoles or monopoles for constant frequencies.

Until now we always considered holography in the $\omega - R-$space. Let us study our problem in the frequency wavenumber domain. A homogeneous plane wave is in the k-space located on the *Ewald sphere* (Ashcroft, N.W., Mermin, N.D., 1981).The Ewald sphere is a sphere in the Cartesian k-space with the radius

$$|k_i| = \frac{\omega}{v} = \sqrt{{k_x}^2 + {k_y}^2 + {k_z}^2} \quad . \tag{14}$$

where $k_i = (k_x, k_y, k_z)$ determines the direction $\frac{k_i}{|k_i|}$ and frequency $\omega = |k_i|v$ of the incident wave in a medium of constant velocity v. Suppose we want to image an object function $\Gamma(k)$ with an incident wave defined by k_i. The object function may describe a velocity heterogenity of the medium. On the observation plane we measure an object function shifted in space $\Gamma(k - k_i)$. To be more precise: for a given angle of incidence and a given frequency, we measure the intersection of the shifted object function and the Ewald sphere. Backpropagation means shifting the intersection back, i.e., to undo the space shift due to the wavepath.

To image we have to superpose as many intersections of Ewald spheres and object functions as possible to get a high resolution. There are basically two possibilities to get different intersections

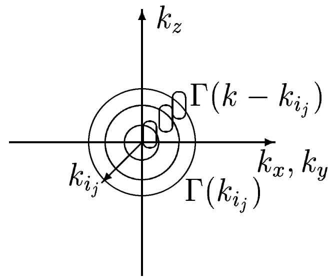

Figure 3: Frequency diversity. The shifted object function $\Gamma(k - k_{i_j})$ intersects with different Ewald spheres $\Gamma(k_{i_j})$.

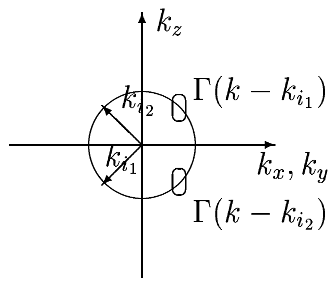

Figure 4: Angular diversity. The shifted object function $\Gamma(k - k_{i_j})$ intersects with the same Ewald sphere at different angles.

1. frequency diversity
2. angular diversity .

Frequency diversity varies the frequency ω and doesn't change the angle of incidence. Each frequency defines a new Ewald sphere so that different intersections with the object function will have different curvatures. *Angular diversity* changes the angle of incidence but holds the frequency constant. If the angular coverage is large enough, we can cover one whole Ewald sphere. Bandpassed records are located within a circular ring and limit the spatial resolution as a poor angular coverage does, i.e., if only parts of the Ewald sphere are covered.

Superposition of all possible backshifted intersections, i.e., a complete coverage of the k-space, yields to an "optimal" image with homogeneous waves. But we still miss the information of inhomogeneous waves due to strong scatterers or sources near the geophones.

Doing generalized holography or Raleigh Sommerfeld Holography for different frequencies leads to the *frequency diversity filtered backpropagation* algorithm:

- measure the intersection of the Ewald sphere with the shifted object function for different frequencies but constant incidence angle

- make a temporal deconvolution
- shift the Ewald sphere intersections back to the origin (backpropagation)
- superpose the images for different frequencies
- superpose the backpropagated and incident wavefields

The angular diversity generalized filtered backpropagation algorithm is similar:

- measure the cut of the Ewald sphere with $\Gamma(k - k_i)$ for different angles of incidence but for the same frequency. (We apply a lowpass because the intersections are within the sphere $2|k_i|$.)
- shift the Ewald sphere intersections back to the origin (backpropagation)
- superpose the images for different angles
- superpose the backpropagated and incident wavefields

Either frequency diversity or angular diversity can image the object if we have a complete coverage of the k-space. In practice, our data is bandpassed and registration on a plane limits the angular resolution. The seismic processing will be more sucessful if we can combine all the information we have. Prestack migration yields to better results than Zero-offset migration because the bandpassed frequency diversity of Zero-offset migration is completed by a limited angular diversity.

If our data is recorded on a plane, we should speak of the Raleigh Sommerfeld frequency diversity or angular diversity algorithms. For a constant velocity medium, Raleigh Sommerfeld angular diversity holography is nearly the same as *diffraction tomography* that will be studied in the next section.

TOMOGRAPHY

Diffraction tomography is based on the *"Diffraction Slice Theorem"*:

> *The Fourier transform of the field scattered by a weak scatterer which was registered on a plane perpendicular to the normal vector of the wavefronts leads to values of the 2-D Fourier transform of the object function along halfcircles with the radius k.*

The theorem can be derived as follows (Langenberg, K.L., 1986): Write the Porter-Bojarski integral as a volume integral and perform a two-dimensional spatial Fourier transform to the $k_x - k_y$–space. Now assume weak scattering and take the wavefield on a registration *plane* $z = const.$ The Born and Rytov approximations are most

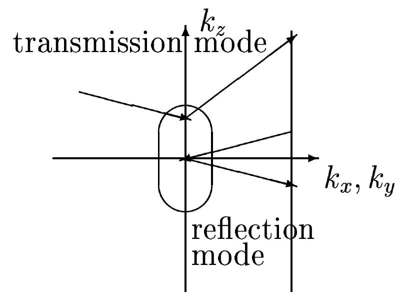

Figure 5: Transmission mode and reflection mode are defined by the angle between the incident and outgoing wave vector.

common to introduce the idea of weak scatterers. Both linearize the integral equations to be solved while assuming the displacement (Born), respectively the gradient of the displacement (Rytov) due to scattering, to be small. Small-scale heterogeneties can be best imaged by the Born approximation whereas to image large-scale heterogeneties it is better to use the Rytov approximation (Pratt, R.G., 1989).

With the above derivation in mind, it is obvious that the diffraction slice theorem is the weak scatterer equivalent to the Raleigh integrals. Diffraction tomography is the weak scatterer form of Raleigh Sommerfeld Holography with angular diversity. Although an extension of diffraction tomography to the elastic case is possible, it has not yet been done.

The old-Greek word "tomé" means "slice", and therefore we can translate "tomography" with "imaging by slices (through the object function)." The slices to be superposed are essentially the intersections of the object function and the Ewald halfcircles. Because we allow weak inhomogeneties, small areas around the intersections can be included in the superposition (Woodward, M.J., and Rocca, F., 1989). In practice, there is no significant difference. This has been shown by Menges and Wenzel (1990) for migration.

It is important to distinguish between transmission and reflection mode. *Transmission mode* means that the difference between the angle of incidence and the reflection angle, respectively diffraction angle, is small for the recorded scattered wavefield. The absolute value of this angle should be between 0 and $\frac{\pi}{2}$. We speak of *reflection mode* if the absolute value of the difference between both angles is between $\frac{\pi}{2}$ and π. Zero offset migration performs the backpropagation of reflection mode data measured under a difference angle of π and uses the one-way wave equation, or a transmission mode theory. In case of transmission mode or reflection mode, only the halfcircle of the Ewald sphere closed towards the registration plane and its closest surroundings can be reconstructed because we register only the energy scattered towards the receivers. One conclusion is that an optimal image can only be obtained if the scatterer is completely surrounded by receivers. In addition, the signal must be broad-banded.

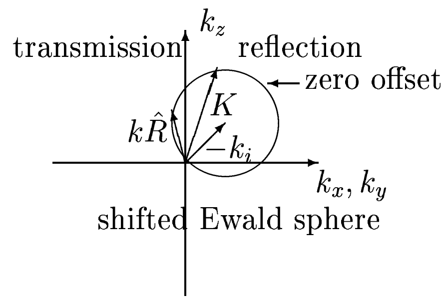

Figure 6: The transmission mode has lower resolution than the reflection mode. The unitvector $\hat{R}$ points towards the receiver and $-k_i$ towards the source. The maximum resolution is where $|K| = |k\hat{R} + (-k_i)|$ has its maximum value. This is at zero offset.

If we superpose reflection mode diffraction tomography images for different frequencies, we perform a prestack weak scatterer migration, i.e., a weak scatterer frequency and angular diversity Raleigh Sommerfeld Holography. Constant offset migration is the reflection mode of diffraction tomography with frequency but not angular diversity, if we assume weak scatterers.

If the signal frequency, i.e., the wavenumber k, approaches infinity, the Ewald halfcircles have infinite radius and can locally be approximated by straight lines. The diffraction slice theorem simplifies to the *Fourier slice theorem*:

> *Straight lines through the object function (i.e., plane waves registered under a certain angle θ_i) correspond to straight lines through the two-dimensional Fourier transform of the object function with the same angle θ_i.*

The larger the object to be mapped, the sooner the approximation of halfcircles by lines is valid. For $ka \gg 1$ (with the scatterer length a) bended rays are getting straight rays and diffraction tomography simplifies to travel-time tomography. Instead of a *backpropagation based on the wave equation*, it is sufficient to perform a *backprojection based on the inverse Radon transform.* Straight cuts through the object function are the slices to be superposed.

Langenberg (1986) and Mora (1989) showed that the spatial resolution for the transmission and reflection modes are different. For an incident wave of constant frequency, the wave vector $\underline{k}$ is smaller for transmitted waves than for reflected waves. The smaller the offset between source and receiver the more the wave vectors of the backscattered wavefield and the negative of the incident wave vector to point in the same direction. Their sum has its maximum at zero offset.

Since reflection mode data covers better the higher spatial frequencies than the transmission data, it has higher spatial resolution. The resolution maximum is at zero offset. (Transmission) tomography is the better tool to image big structures because it covers better the low wavenumber areas. Of course these comparisons are only valid if the reflected and transmitted data have the same frequencies. In practice, this will rarely be the case. Cross-hole tomography registrations with frequencies of some thousand Hertz usually lead to a much better spatial resolution than surface data migration with frequencies of two orders lower.

CONCLUSIONS

Holography, migration, and tomography can be derived from the Porter-Bojarski integral. The different algorithms simply deal with data differing in angular and/or frequency diversity and whether inhomogeneous waves are completely ignored or assumed to be weak. If we apply these methods to image the subsurface, we should cover the biggest portion of the k-space possible: We should combine transmission mode *and* reflection mode data. Angles *and* frequencies should be as diverse as possible to allow superposition of many halfcircles. If we remember that the data and a priori knowledge is imperfect, a non-linear inversion should be applied on the highly diverse data. Mora (1989) summarized these ideas as he wrote “inversion = migration + tomography.”

REFERENCES

Ashcroft, N.W., and Mermin, N.D., 1981, Solid State Physics: Saunders College, Philadelphia.

Claerbout, J.F., 1985, Imaging the Earth’s Interior: Blackwell, Oxford.

Claerbout, J.F., 1989, Model fitting by least squares: SEP–**61**.

Deans, S.R., 1983, The Radon Transform and Some of its Applications: J. Wiley and Sons, New York.

Gazdag, J., 1978, Wave equation migration with the phase shift method: Geophysics, **43**, 1342-1351.

Langenberg, K.L., 1986, Applied Inverse Problems: Fraunhofer-Instut fuer zerstoerungsfreie Pruefverfahren, Kassel, F.R. Germany.

Mora, P., 1987, Elastic wavefield inversion: SEP–**52**.

Mora, P., 1989, Inversion = migration + tomography: Geophysics, **54**, 1575-1586.

Pratt, R.G., 1989, Wave equation methods in cross-hole seismic imaging: Ph.D. thesis, Imperial College, London.

Rueter, H., 1987, Migration and tomography: radon migration: First Break **5**, 399-402.

Schneider, W.A., 1978, Integral formulation for migration in two and three dimensions: Geophysics, **43**, 49-76.

Stolt, R.H., 1978, Migration by fourier transform: Geophysics, **43**, 23-48.

Tarantola, A., 1988, Theoretical background for the inversion of seismic waveforms, including elasticity and attenuation: in Scattering and Attenuation of Seismic Waves, Birkhaeuser, Basel.

Tygel, M., and Hubral, P., 1989, Constant velocity migration in the guises of plane-wave theory: Surveys in Geophysics **10**, 331-348.

Wenzel, F., and Menges, D., 1990, A comparison between born inversion and frequency-wavenumber migration: Geophysics, **54**, 1006-1011.

Woodward, M.J., 1989, Wave-equation tomography: Ph.D. thesis, SEP–**62**.

Woodward, M.J., and Rocca, F., 1988, Wave-equation tomography II: SEP–**57**.

Worthington, M.H., 1984, An introduction to geophysical tomography: First Break **2**, 20-26.