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Kinematic residual prestack migration

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ABSTRACT

Using a kinematic approach, it is possible to derive a residual migration operator that converts a constant-offset section migrated with one slowness to a constant-offset section migrated with another slowness. Residual constant-offset migration can be decomposed into the following sequence of processes: 1) residual common-reflection-point gathering or DMO, 2) residual NMO, and 3) residual zero-offset migration. If the residual zero-offset migration part of the operator is "subtracted" from residual constant-offset migration we are left with a residual NMO+DMO operator that corrects a migrated constant-offset section for the residual NMO and residual common-reflection-point smear associated with a change in migration slowness without performing residual zero-offset migration. This ability to change the migration slowness of the data without moving events around is useful for velocity analysis.

INTRODUCTION

In a classic paper, Rothman et al., (1985) described residual zero-offset migration providing a method for changing the migration velocity of a zero-offset section without remigrating the data from scratch. Residual zero-offset migration (also known as cascaded migration) proved to be very useful. Larner et al., (1986) used residual migration to enhance the accuracy of 15 degree migrations extending them to higher dips. Beasley et al., (1988) use cascaded migration to extend Stolt (constant velocity f–k) migration to variable velocity media. Residual migration seemed so useful, that one would hope the concept of residual migration would extend to prestack migration. It might be useful for reducing sensitivity to the velocity model used for prestack migration because errors in the migration could be corrected by doing residual migration. Residual migration might also be useful for migration velocity analysis because many migrations could be obtained for little cost. It might also be possible to do prestack depth migration by first performing inexpensive constant-velocity migration, followed by a residual migration to account for $v(x, z)$.

Because of the potential rewards, Fowler and Al-Yahya (1986) tried to formulate

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residual prestack migration. They concluded that it is possible to construct an operator that converts constant-offset or shot-profile data, migrated with one velocity, to data migrated at another velocity. However, the operator is not equivalent to applying constant-offset migration or shot profile migration with some other velocity which is the case for zero-offset residual migration. Because the operator they obtained was not a "migration" in the classical sense, it seemed that residual prestack migration would not be as useful as residual zero-offset migration.

My velocity analysis project (Etgen, 1988, 1989) requires the ability to find changes in migrated images due to perturbations in the velocity model used for prestack depth migration. In the past, approximated residual migration to derive residual moveout corrections that can be applied to migrated constant-offset sections. The residual migration operators of this paper supercede the previous residual moveout operators. To obtain an inexpensive residual migration operator I make two approximations. First, I only use the stationary phase path of the full residual migration operator. Second, I build the operators only for the constant-velocity case. The residual migration is only a residual time migration; ray bending is ignored in the construction of the migration operator.

Subtracting the kinematics of residual zero-offset migration from the kinematics of residual constant-offset migration, leaves an operator that performs only residual DMO and residual NMO. This converts migrated constant-offset sections to zero-offset sections for a change in the migration velocity. Note that if the velocity doesn't change, migrated constant-offset sections are already converted to migrated zero-offset sections. Residual constant-offset NMO+DMO is useful for velocity analysis. It provides a velocity analysis method analogous to stacking velocity analysis, that works in arbitrary structure and after prestack depth migration.

RESIDUAL CONSTANT-OFFSET MIGRATION

Define residual constant-offset migration as the process that converts a constant-offset section migrated with slowness w to a constant-offset section migrated with slowness w_n. Building a kinematic residual constant-offset migration operator simply requires finding where the image of a point on a dipping reflector in a migrated constant-offset section moves as the migration velocity changes. Doing this for all points and all dips in the image defines a kinematic residual migration operator. A wave-theoretic residual migration operator can be found by cascading constant-offset modeling at the original migration slowness and constant-offset migration with a new slowness. A kinematic development will give a stationary phase approximation to this wave theoretic residual migration operator. Since my main application of this operator will be velocity analysis, the errors of the stationary phase approximation should not be important. I further approximate, by building residual constant-offset migration operators using constant velocity prestack migration. Although the kinematic arguments will be strictly valid only in the constant slowness case, the residual migration operator should still be useful, even when velocity varies. Constant-offset depth migration followed by constant-velocity constant-offset residual migration will still give an approximation to constant-offset depth migration with a new interval velocity model. The approximation gets better as the amount residual migration gets smaller.

The impulse response of constant-velocity constant-offset migration is an ellipse with foci at source and receiver. Each point on the impulse response ellipse represents a different time dip in the data. A small segment of a dipping reflection event assumed to contain only a single traveltime dip, after migration becomes the tangent to the impulse response ellipse for the appropriate traveltime. The traveltime determines the size of the ellipse, and the traveltime dip determines the position of the image along the impulse response ellipse. Seen in reverse, given an image point and its depth dip, we can solve for the position of the source and receiver that have an impulse response that goes through our point and is tangent to the dipping reflector segment.

Starting at some point in depth with an assumed dip and solving for the source and receiver points that contain a specular reflection from the point, we can write an equation in polar coordinates that describes the migration ellipse that goes through the point.

$$r(\theta, w) = \sqrt{\frac{\frac{t^2}{4w^2} - h^2}{1 - \frac{4w^2}{t^2} h^2 \cos\theta}} \quad . \tag{1}$$

The origin is shifted to be the center of the ellipse. Equation (1) describes all image points that have specular reflections recorded at this shot receiver pair. Specify a point of interest as $r(\theta_0)$. Write the traveltime of the reflection from the dipping reflector as

$$t = w\left[\sqrt{(x-h)^2 + z^2} + \sqrt{(x+h)^2 + z^2}\right] \quad , \tag{2}$$

where $x = r(\theta)\cos\theta$ and $z = r(\theta)\sin\theta$. Also write the time dip of any reflection from a tangent to the ellipse as

$$\frac{\partial t}{\partial y} = w\left[\frac{r(\theta)\cos\theta - h}{\sqrt{+}\frac{r(\theta)\cos\theta + h}{\sqrt{r^2(\theta)+h^2-2r(\theta)h\cos\theta}} r^2(\theta) + h^2 + 2r(\theta)h\cos\theta}\right] \quad . \tag{3}$$

Figure 1 shows a selected point "P" with its tangent impulse response ellipse and specular reflection rays for a specified dip.

After choosing an initial depth and depth dip and solving for the shot receiver pair, equations (1)–(3) give the traveltime and traveltime dip of the specular reflection from that point. When the migration slowness changes, the migration ellipse will move to a new position. The key to finding the position of the reflector as the slowness changes is to remember that the traveltime, midpoint, and traveltime dip of the reflector segment in the constant-offset section is fixed. Only its image in depth changes as the migration slowness changes. First, find a new impulse response after the migration slowness changes for the original shot–receiver pair that has the same traveltime as the previous impulse response at the old migration velocity. Then find a point on the new impulse response that has the same traveltime dip as the original point using the original migration slowness. This procedure gives the new location and dip of the event. Keeping t fixed as the traveltime of a reflection from our original point, the equation of the new impulse response ellipse at slowness w_n can be written as

$$r(\theta, w_n) = \sqrt{\frac{\frac{t^2}{4w_n^2} - h^2}{1 - \frac{4w_n^2}{t^2} h^2 \cos\theta}} \quad . \tag{4}$$

Identify $d = t/w$ as the shot to image point to geophone distance of the reflector at the original migration slowness. Also substitute $\gamma = w/w_n$ (Al-Yahya, 1987). γ can be called the residual slowness or slowness scale factor. When $\gamma = 1$, residual migration leaves the image unchanged. With these changes, rewrite equation 4 as:

$$r(\theta, \gamma) = \sqrt{\frac{\frac{\gamma^2 d^2}{4} - h^2}{1 - \frac{4}{\gamma^2 d^2} h^2 \cos\theta}} \quad . \tag{5}$$

The time dip imaged at any point on the new impulse response ellipse is likewise given by:

$$\frac{\partial t}{\partial y} = w_n \left[\frac{r\cos\theta - h}{\sqrt{r^2 + h^2 - 2rh\cos\theta}} + \frac{r\cos\theta + h}{\sqrt{r^2 + h^2 + 2rh\cos\theta}} \right] \quad . \tag{6}$$

Equating traveltime dip of the reflector at the new and original slownesses and plugging in the new relation for $r(\theta, \gamma)$ which equates the traveltimes gives a single equation to solve for θ_n as a function of γ and the position of the original point.

$$\gamma \left[\frac{r(\theta_0, d)\cos\theta_0 - h}{\sqrt{r^2(\theta_0, d) + h^2 - 2r(\theta_0, d)h\cos\theta_0}} + \frac{r(\theta_0, d)\cos\theta_0 + h}{\sqrt{r^2(\theta_0, d) + h^2 + 2r(\theta_0, d)h\cos\theta_0}} \right] \tag{7}$$

$$= \left[\frac{r(\theta_n, \gamma)\cos\theta_n - h}{\sqrt{r^2(\theta_n, \gamma) + h^2 - 2r(\theta_n, \gamma)h\cos\theta_n}} + \frac{r(\theta_n, \gamma)\cos\theta_n + h}{\sqrt{r^2(\theta_n, \gamma) + h^2 + 2r(\theta_n, \gamma)h\cos\theta_n}} \right] \quad .$$

Unfortunately, equation (7) is very difficult to solve analytically except for the trivial cases of zero dip or zero offset. To compute the operator I solve equation (7) numerically using Newton's method for finding roots. Converting the solution of equation (7) back to Cartesian coordinates gives $x(r, \theta_n)$ and $z(r, \theta_n)$, the new position of the dipping reflector segment as shown in Figure 2.

Solving equation (7) for an initial depth and all initial dips will trace out the "spraying" operator for residual constant-offset migration. The operator traces out the new positions of events for a range of dips after the migration slowness changes for a fixed original point in the image. This curve is also the summation path if we change the role of starting and final points by redefining $\gamma = w_n/w$. It is often more convenient to write the computer code in terms of a summation operator. The equations are symmetric so the summation operator for γ is the "spraying" operator for $1/\gamma$. Figure 3 shows an example of the summation operator when the slowness increases. Depending on the change in migration slowness and the initial depth of the reflector, the operator can triplicate. This happens for image points with large offset/depth ratio. Figure 4 shows an examples of the residual migration operators for γ less than one, when slowness decreases.

To get the correct amplitudes along the summation operator, compute the points on the summation operators in equal dip angle increments in the original image. To prevent operator aliasing I resample the summation operators in equal arclength along the summation trajectory. To recover equal dip weighting of the image, the amplitude along the summation path can be taken as the jacobian of the change of variables from arclength to dip angle.

RESIDUAL NMO+DMO

Figure 5 shows the positions of a point "P_{orig}" for several different values of residual slowness γ for one initial dip and 3 different offsets. The new positions P_0, P_h, and P_H for different offsets move along different trajectories as migration slowness changes. If, for a fixed γ, the new points were above one another vertically, a residual NMO correction could convert constant-offset sections to zero-offset sections for a change in migration slowness. Residual constant-offset migration would be equivalent to residual NMO followed by residual zero-offset migration. For zero dip, as expected this is the case. Residual constant-offset migration is just residual NMO followed by residual time to depth conversion. For nonzero dip (as shown in Figure 5), the new positions do not line up vertically. Furthermore, the amount of the vertical movement as a function of offset is not the same as it is for zero dip. Using analogy to full prestack migration, call residual DMO the movement required by residual constant-offset migration and not described by residual NMO or residual zero-offset migration. Figure 6 shows how residual constant-offset migration can be built from three processes: residual DMO, residual NMO, and residual zero-offset migration.

The previous section gave the necessary equations to compute residual constant-offset migration operators; subtracting the residual zero-offset migration part of residual constant-offset migration, obtained from the same equations, gives residual NMO+DMO. Amplitudes of the residual NMO+DMO operators are obtained with the same method used to obtain the amplitudes of residual constant-offset migration operators. Figure 7 shows the residual NMO+DMO operators for a series of depth points for a large offset for $\gamma = 1.2$ and $\gamma = .833$.

Residual constant-offset migration has the desired property that it performs residual common-reflection-point gathering. For any residual slowness, any point on the output images of different constant-offset sections all correspond to the same reflection point. This is a necessary step for correct migration velocity analysis. Velocity analysis uses the traveltime of reflections from a single point in the earth; thus, when performing migration velocity analysis by comparing traveltimes versus offset, the relevant part of residual constant-offset migration is residual NMO+DMO. The residual zero-offset migration part of residual constant-offset migration confuses velocity analysis by moving the image of a fixed reflection event around the image as the migration slowness changes (Fowler, 1988; Etgen, 1989). It is preferable to ignore the residual zero-offset migration term and keep a fixed reflection event at a fixed location in the image. This is analogous to conventional velocity analysis where NMO and stacking are performed at a fixed t_0. The true-depth position of the reflector can be calculated and stored, and residual zero-offset migration can be applied later, after velocity analysis. The kinematic residual zero-offset migration is needed by tomographic velocity analysis methods.

EXAMPLES

To test the residual constant-offset migration and residual NMO+DMO operators, I made a synthetic constant-offset survey over a series of point diffractors. The constant-offset data were migrated with a slowness equal to .8333 times the correct slowness. I applied residual constant-offset migration at $\gamma = 1.2$ and successfully recovered the images of the point diffractors as shown in Figure 8. I also applied residual NMO+DMO to the migrated constant-offset section to obtain the lower plot in Figure 9, which should differ from the upper plot in Figure 8 by only a residual zero-offset migration. The upper plot in Figure 9 is the result of applying residual zero-offset migration to the lower plot. As hoped, the images of the point diffractors are recovered.

There are some differences between the top plots in Figures 8 and 9. The most important difference is the reduction in amplitude of the top point diffractor in the NMO+DMO+residual zero-offset migrated image. Near the cusps of the operators (see Figure 7), the stationary phase approximation breaks down because there is a discontinuity in the derivative of the slope of the summation curve. The kinematic summation operators have amplitudes that drop to zero immediately at the edge of the cusp, while in reality, energy spreads out away from the cusp. Wave-theoretic residual NMO+DMO would not have this problem, but it is more expensive to apply.

One advantage of residual constant-offset migration over residual NMO+DMO followed by residual zero-offset migration is that residual constant-offset migration has cusps only for shallow depths and large offsets. The residual NMO+DMO operator has cusps for all offsets (except zero-offset) and all depths. For large depth-offset ratios, the cusps aren't a problem however, because the operator effectively reduces to a point, namely residual NMO. This is where residual NMO+DMO has a great advantage over residual constant-offset migration. The size of the aperture of the residual constant-offset migration operator grows without bound for increasing depths, while the residual NMO+DMO operator shrinks to a point; so residual NMO+DMO is much more economical to apply.

To test the residual constant-offset migration and residual NMO+DMO methods on real data and to see if residual NMO+DMO can be used as a velocity analysis tool after migration, I migrated 180 constant-offset sections from a survey in the Gulf of Mexico. Figure 10 shows a constant-offset section from the data; this offset was at about one-half the total cable length. I chose a velocity model that varies only in depth and made the velocities increase with depth but purposefully too slow. After prestack migration, I stacked together every 12 migrated constant-offset sections to increase signal to noise, remove aliasing, and reduce the data volume. Figure 11 shows one partial stack of nearby migrated constant-offset sections. After migration and partial stacking, residual NMO+DMO was applied to the migrated constant-offset sections for the range $0.645 < \gamma < 1.15$. Figures 12, 13, and 14 show stacked sections after residual NMO+DMO. At the lower values of γ, corresponding to the largest increase in velocity, the deeper events stack well. At the higher values of γ, closer to the original velocity, the shallower events stack well. Note that events of all dips at a given depth stack well at the same value of γ. Also note that events stay fixed as their migration slowness changes. It is easier to analyze velocities using residual NMO+DMO than by scanning migration velocity with many full prestack migrations which forces events to move as the migration slowness changes.

If we wish to see the reflectors in their proper spatial locations, we can apply residual zero-offset migration to the stacks of Figures 12–14. Figures 15–17 show the result of residual zero-offset migration applied to each stack at its appropriate value of γ.

Figure 18 shows sample velocity analysis panels. Since residual NMO+DMO correctly accounts for residual common-reflection-point smear, these panels can be interpreted just as residual stacking velocity panels are interpreted for horizontal reflectors. Strictly speaking, these panels allow one to pick the best residual prestack time migration after prestack depth migration.

I plan to use residual NMO+DMO to analyze errors in interval velocity models used for prestack depth migration. My previous papers (Etgen, 1988, 1989) describe an operator that relates changes in a residual slowness measure to changes in the interval velocity model. The residual-slowness semblance panels obtained using residual NMO+DMO described in this paper are ideal for measuring residual slowness after prestack depth migration. Events don't move as residual slowness changes, the points in the images correspond to true common-reflection points and the residual zero-offset migration can be calculated and applied after velocity analysis.

CONCLUSIONS

Although the residual migration operators come from kinematic considerations, they work despite troubles near their triplications. It is possible to build the operators more closely based on the wave equation, but the resulting algorithm would resemble a phase shift migration and would be costly to apply for a broad range of residual slownesses which is needed for velocity analysis. My application for residual NMO+DMO is velocity analysis, and it is ideally suited to this task. Residual NMO+DMO can find the best residual prestack time migration slowness for fixed events which is different from finding the best migration slowness for a fixed location in the Earth. It is not confused by events moving around the image as migration slowness changes. When a high-quality final wave-equation-migration output is needed, it is probably easier to migrate the data from scratch with a new interval velocity model or use wave-theoretic residual NMO+DMO.

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