Multiple suppression and wave separation

Lin Zhang

ABSTRACT

An inversion algorithm, which does multiple suppression and wave separation simultaneously, is developed to process the 3-D marine seismic data recorded in a Walk-Away survey. The key feature of the method is to use multichannel information to estimate multiple patterns and to extract seabottom multiple-free downgoing waves and upgoing waves. The inversion is performed by conjugate gradient method. The algorithm is tested on both synthetic data and field data.

INTRODUCTION

3-D Walk-Away marine seismic survey provides us a way to record the wave field in the earth's interior. The wave field is generated by shots on sea surface, and data are recorded by receivers in the borehole. Figure 1 shows the geometry of such survey. We expect that this kind of data contains more information because we make measurements at places closer to subsurfaces. However, several problems must be solved before we try to migrate the data. The first one is the higher dimensions of data set. A 3-D Walk-Away survey includes 2-D shots and 1-D receivers, plus time dimension, the data set is four dimensional, which is difficult to migrate. Since the receiver line is short, we try to combine the data from all receivers and reduce the data set to 3-D. The second problem is wave separation. We are more interested in waves reflected from deeper subsurfaces, i.e. the upgoing waves. But in the recorded data, downgoing waves and upgoing waves are mixed up. We will use the information recorded by multichannel receivers to separate these two wave types. The last one is seabottom multiple reflections which are a general problem in marine seismic survey. Two standard ways to attenuate multiples are CDP stack and predicting filter. The first one is ineffective for borehole recording because a large portion of ray paths goes through subsurfaces, so the velocity discrimination is poor. Predicting filter works on each trace independently, the multichannel information

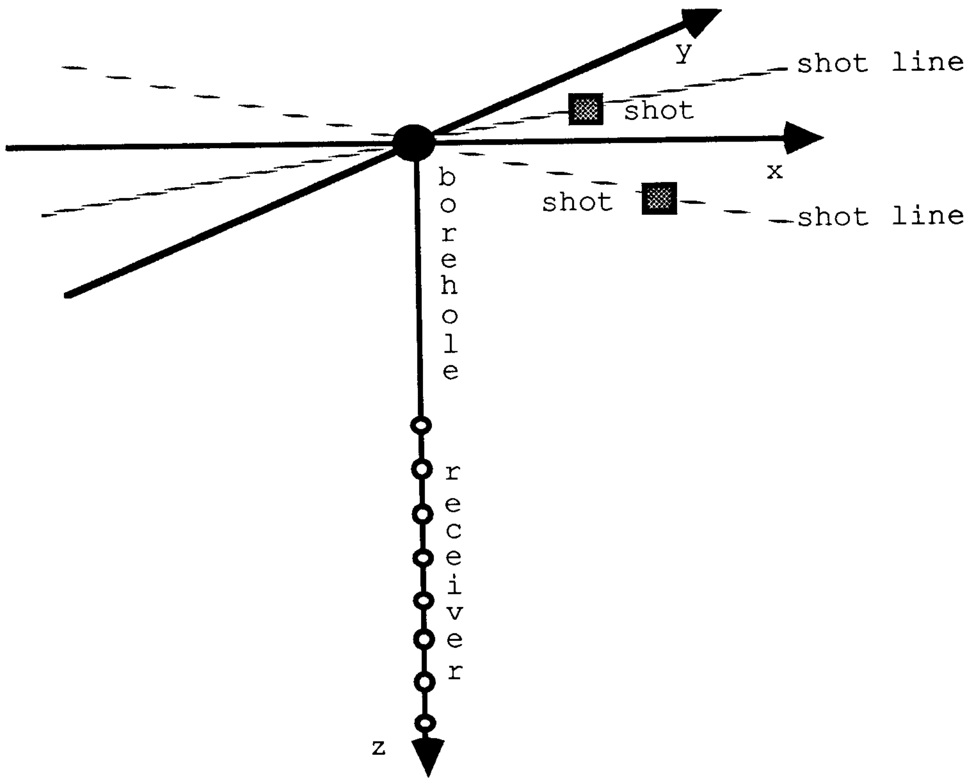

FIG. 1. The geometry of Walk-Away marine seismic survey. The 2-D shots can be partitioned into many shot lines indicated by dot lines.

is not sufficiently utilized. Also the goal of wave separation can not be reached. In this paper, I will describe a method which does all jobs simultaneously.

2-D shots can be partitioned into many shot lines which are passing over the borehole, as shown in Figure 1. Each of these shot lines can be modeled by the same concepts. Therefore, we focus to design an algorithm which works for the data generated by a single shot line. Now the data set has three axes, time, shot location and receiver level. The first step is to use slant stack to partition wave field recorded from each receiver into plane waves with various wave parameters. For each wave parameter, a common wave parameter gather is formed. Such gather can be modeled with linear convolution model. Conjugate gradient method is chosen to do inversion which results two traces, downgoing wave trace and upgoing wave trace. Finally, all traces are gathered and inverse slant stack is performed. Figure 2 shows the whole procedure. This treatment avoids the inverse operation over large multi-dimensional data set and processes information from all receivers cooperatively, so it is expected to have better performance and require moderate computation.

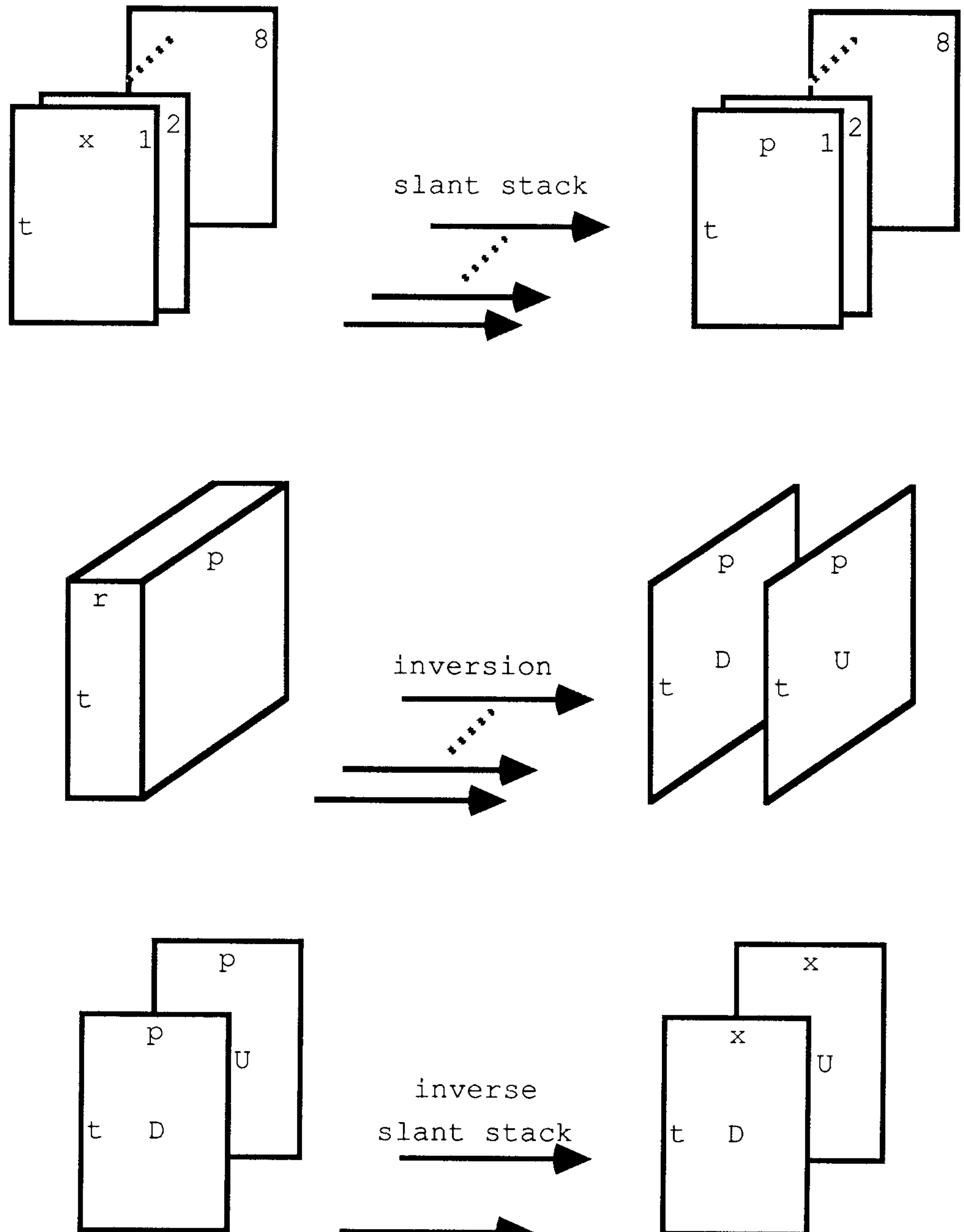

FIG. 2. The data structures in the successive steps of the algorithm. The letter x indicates the shot axis, t indicates time axis, r indicates receiver axis and p indicates wave parameter axis. D stands for downgoing wave and U stands for upgoing wave.

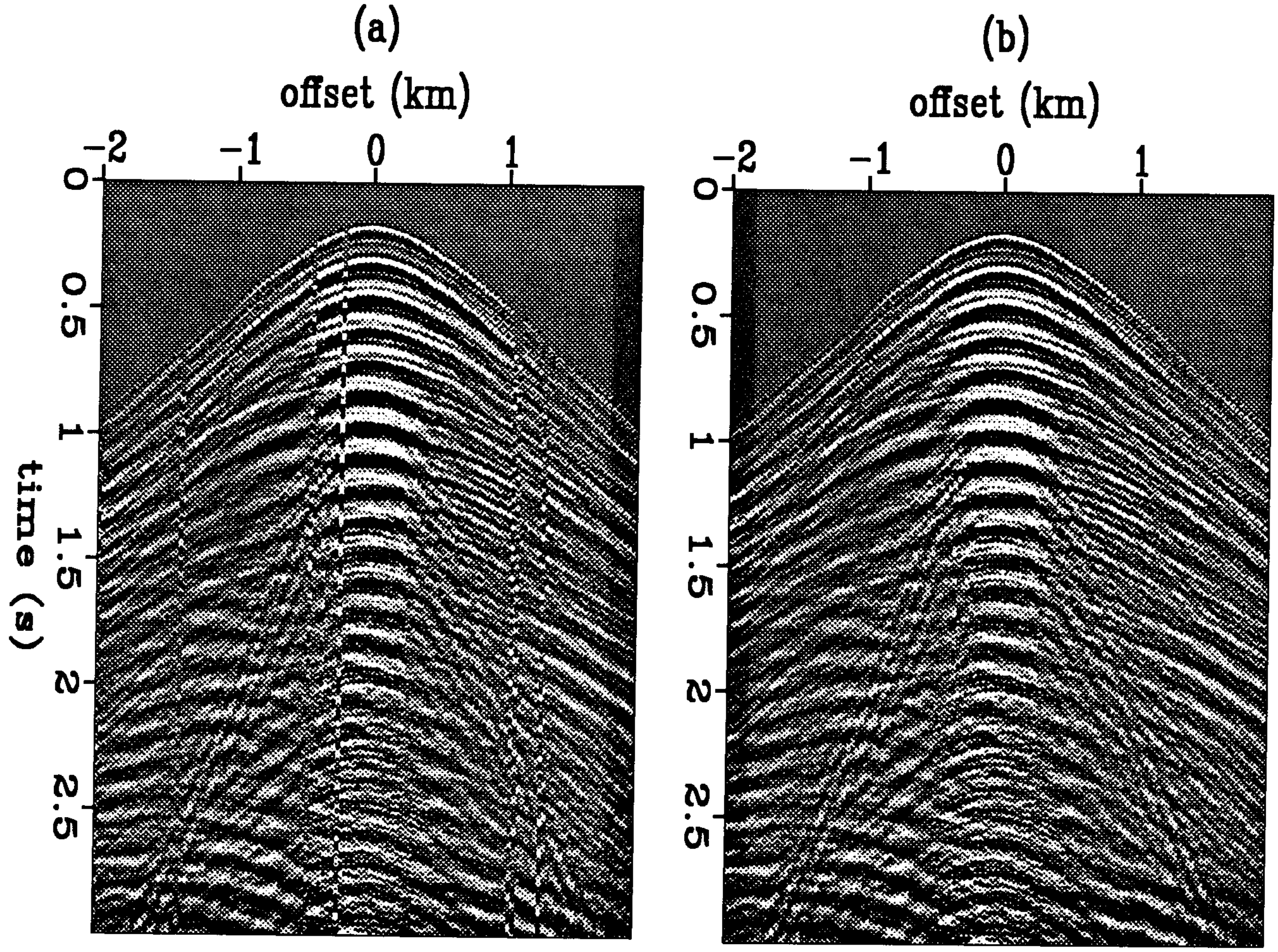

FIG. 3. The common receiver section before and after shift correction and sample reduction. (a) Before. (b) After.

DATA AND PREPROCESSING

Before trying to formulate the inversion, let us take a look at the field data we will process. Figure 3.a shows a common receiver section. The horizontal axis shows shot positions along a shot line and the vertical axis shows time. The sample rate in time domain is 0.002 and the number of samples in each trace is 1500. Obviously several isolated traces in the section are badly shifted. This turns out to be a general problem in all sections. So we need to correct these shifting errors. Also we could consider to reduce the number of samples in each trace.

Trace shift correction

Following algorithm corrects the shifting errors.

1. Compute cross-correlation functions of each trace with its two neighboring traces, find the two maximum value lags.

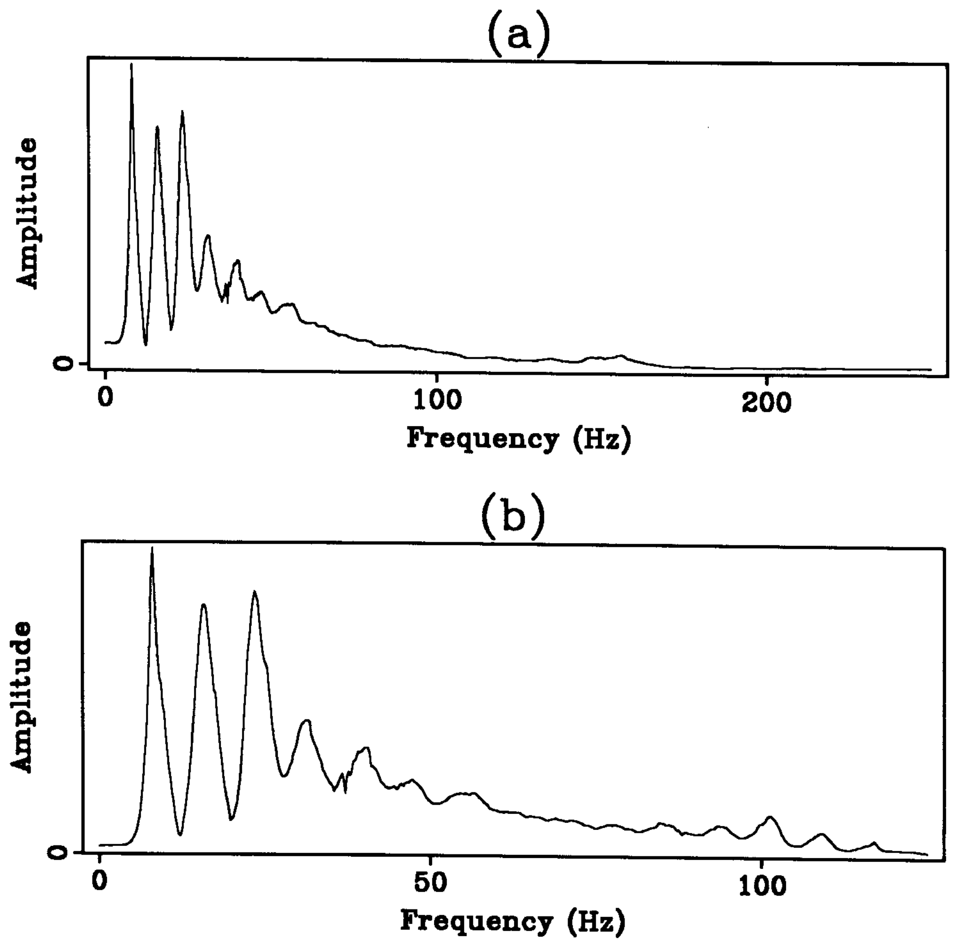

FIG. 4. Average spectrum of data section before and after samples reduction. (a) Before. (b) After.

2. If both lags are less than a threshold, nothing is done. If both larger than the threshold, shift an amount which is the interpolation of two lags.

3. If one is larger than the threshold and the other is less, compute another cross-correlation function and do extrapolation shift.

Figure 3.b shows the same section after corrections.

Sample reduction

When we look at the spectrum of the recorded data, as shown in Figure 4.a, it is noticed that signal occupies only half of the available frequency band. This indicates that we can compress the data. The redundant data can be used to increase the signal and noise ratio if we assume noise is white.

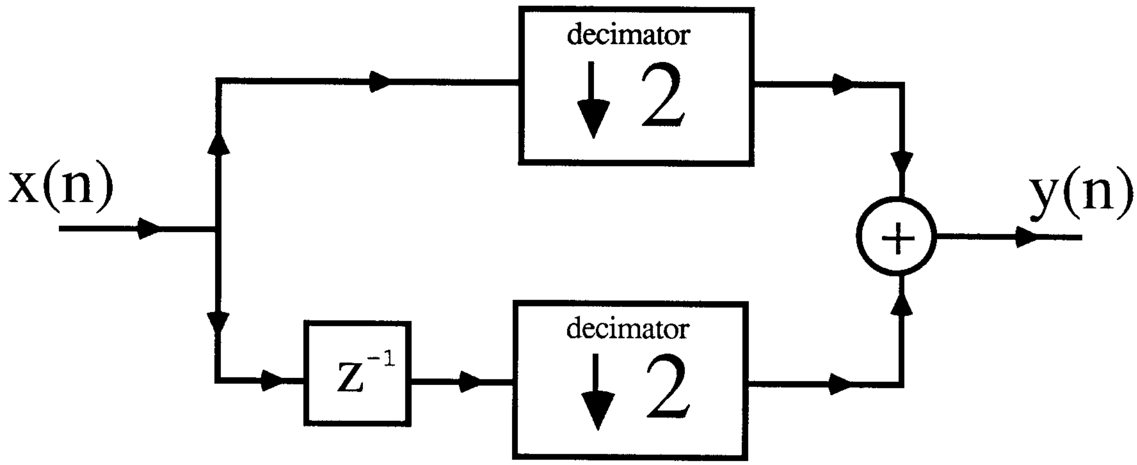

FIG. 5. The block diagram of the filter.

All these can be done by using a filter shown in Figure 5. The output signal spectrum can be expressed as

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\frac{\omega}{2}}) + H(e^{j(\omega-2\pi)})X(e^{j\frac{(\omega-2\pi)}{2}}) \tag{1}$$

where $X(e^{j\omega})$ is input signal.

$$H(e^{j\omega}) = e^{-j\frac{\omega}{4}} \cos\frac{\omega}{4}. \tag{2}$$

It has linear phase property, and $|H(e^{j\omega})| \approx 1$ in the range of $0 \leq \omega \leq \pi$. The Nyquest frequency of the output is half of the Nyquest frequency of the input. The gain of signal to noise ratio comes from the stack of two nearby samples. Figure 4.b shows the output spectrum. The data is reduced by 50%.

Result of slant stack

The standard slant stack operation is performed on each common receiver section. Figure 6.a shows an example. Now the wave field is disassemble into various plane waves with different wave parameter p. Next, common wave parameter gathers are formed. Figure 6.b shows an example of such gathers. The events in the gather are straight lines with two opposite slopes. They represent downgoing waves and upgoing waves respectively. The strong periodicity indicates seabottom multiple reflections.

THEORY

Now let us focus our attention on a common wave parameter gather. First we will model the data. Then we construct an object function which is the mean square error between real data and modeled data. Next step is to estimate the multiple

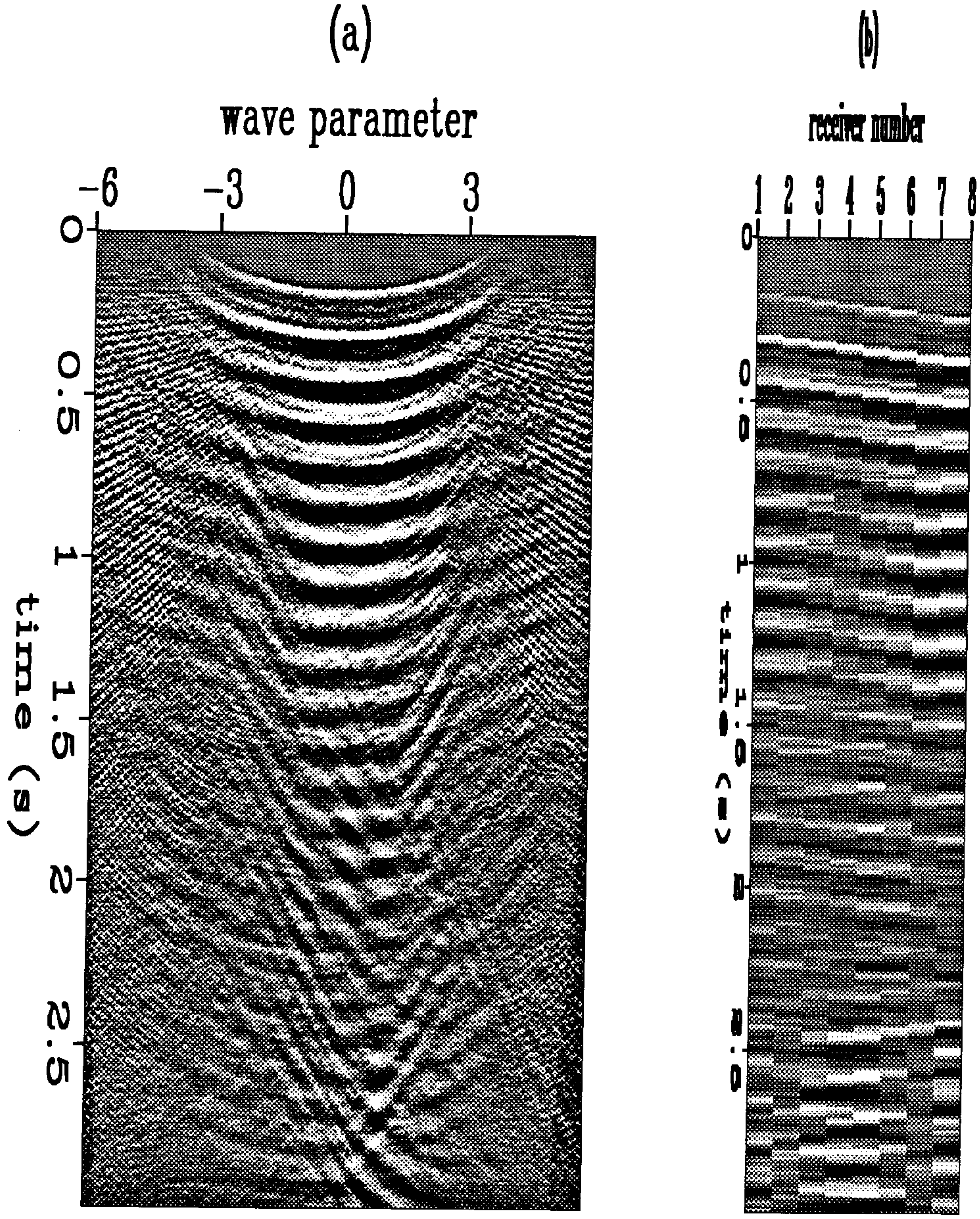

FIG. 6. Examples of (a) slant stack section, (b) common wave parameter gather.

patterns for both downgoing waves and upgoing waves. Finally we consider to use conjugate graduate method to do inversion.

Modeling

By terms of downgoing waves and upgoing waves we mean the waves propagating towards receivers from above and from below respectively. We define $D(t)$ to be seabottom multiple-free downgoing waves and $U(t)$ to be seabottom multiple-free upgoing waves. $M_d(t)$ and $M_u(t)$ are corresponding seabottom multiple patterns.

$$M_d(t) = \sum_k a_k \delta(t - kt_w) \tag{3}$$

$$M_u(t) = \sum_k b_k \delta(t - kt_w) \tag{4}$$

where t_w is the two way traveling time in water. For the plane wave with wave parameter p,

$$t_w = \frac{2d_w}{v_w\sqrt{1-(pv_w)^2}}. \tag{5}$$

d_w is the depth of sea floor. v_w is the wave traveling velocity in water.

A common wave parameter gather $\hat{P}(t,j)$ can be modeled as

$$\hat{P}(t,j) = D(t) * M_d(t) * \delta(t - t_j) + U(t) * M_u(t) * \delta(t + t_j) \tag{6}$$

where j is the coordinate of receivers. The convolutions with two δ functions model the linear moveout. t_j is the time delay of the jth receiver with respect to a reference. Now we assume the receivers are uniformly spaced with distance d_r, and all the receivers are located in a layer with interval velocity v_r. We choose the first receiver to be reference. Then

$$t_j = \frac{(j-1)d_r}{v_r}\sqrt{1-(pv_r)^2}. \tag{7}$$

Let $P(t,j)$ be real data, the inversion is performed by minimizing the following objective function

$$E = \sum_j \int [P(t,j) - \hat{P}(t,j)]dt + Damping. \tag{8}$$

which is the mean square error between modeled data and real data.

How to get $M_d(t)$ and $M_u(t)$

My original plan is to implement following algorithm.

1. Initiate $D(t)$ and $U(t)$ to be zero, and $M_d(t)$ and $M_u(t)$ to be periodic impulse function with uniform amplitude.

2. Fix $M_d(t)$ and $M_u(t)$, minimize objective function with respect to $D(t)$ and $U(t)$.

3. Fix obtained $D(t)$ and $U(t)$, minimize objective function with respect to $M_d(t)$ and $M_u(t)$.

4. Do iteration from step 2.

However, synthetic tests show failures in step 3. The reason is that if the initial multiple patterns are far away from real patterns then the solutions of downgoing waves and upgoing waves are poor. It is unlikely that these results can be used to improve multiple patterns. In fact, the bad solutions of $D(t)$ and $U(t)$ will creat a local minimum of E such that mean square error will not decrease effectly.

Our next try is to extract multiple patterns from the correlation functions of data. In order to do so, we need to make further assumptions.

Let us assume the downgoing waves $D(t)$ and the upgoing waves $U(t)$ are generated by two independent white sequences convolved with a wavelet. The covariances of two white sequences are σ_d^2 and σ_u^2 respectively. We first linearly stack the gather $P(t,j)$ along two opposite directions specified by the wave parameter p.

$$P_d(t) = Stack_{+p}P(t,j) \tag{9}$$

$$P_u(t) = Stack_{-p}P(t,j) \tag{10}$$

$P_d(t)$ and $P_u(t)$ are good estimations of multiple contaminated downgoing waves and upgoing waves. So we can model these two traces as

$$P_d(t) = D(t) * M_d(t) \tag{11}$$

$$P_u(t) = U(t) * M_u(t) \tag{12}$$

The correlation functions of these two traces are

$$B_d(t) = P_d(-t) * P_d(t) = \sigma_d^2 B_w(t) * M_d(-t) * M_d(t) \tag{13}$$

$$B_u(t) = P_u(-t) * P_u(t) = \sigma_u^2 B_w(t) * M_u(-t) * M_u(t) \tag{14}$$

where $B_w(t)$ is the correlation function of wavelet. It is reasonable to assume the width of $B_w(t)$ is less than t_w, the two way traveling time in water, if sea floor is deep enough. In this case, we can simply subsample the correlation function of $P_d(t)$ at a rate of $1/t_w$ to obtain the correlation function of $M_d(t)$. The same thing can be done for the upgoing waves.

Now the problem is how to find the multiple patterns when their correlation functions are given. Generally this can not be done because we do not have phase information. However, if we limit the multiple patterns to be exponential sequences generated by single pole transfer functions,

$$M_d(t) = \sum_k \alpha^k \delta(t - kt_w) \tag{15}$$

$$M_u(t) = \sum_k \beta^k \delta(t - kt_w) \tag{16}$$

then we can estimate α and β, so to obtain multiple patterns.

Let the subsamples of correlation functions $B_d(t)$ and $B_u(t)$ to be $R_d(k)$ and $R_u(k)$. l is the maximum number of multiples after truncating the data. We can easily derive two formulas.

$$\alpha R_d(k) = R_d(k+1) + \alpha^{(2l-k)} \tag{17}$$

$$\beta R_u(k) = R_u(k+1) + \beta^{(2l-k)} \tag{18}$$

For different k's, we can solve the equations to get solutions of α and β. The average values are used to generate multiple patterns

The error in the estimation of multiple patterns could be caused by the fact that simple linear stack may not completely separate downgoing waves and upgoing waves. We can improve the estimations once we get the intermediate results of inversion. Instead of stacking $P(t,j)$, let

$$P_d(t) = Stack_{+p}[P(t,j) - M_u(t) * U(t) * \delta(t + t_j)] \tag{19}$$

$$P_u(t) = Stack_{-p}[P(t,j) - M_d(t) * D(t) * \delta(t - t_j)] \tag{20}$$

Clearly this operation gives better results for wave separation. The overall algorithm is to do estimation and inversion repeatedly.

Inversion by conjugate gradient method

The conjugate gradient method is an effective method for geophysical inversion especially when objective function is in quadratic form. This method is ideal for our problem because it provides intermediate solutions and needs fewer iterations.

There are many versions of conjugate gradient method, some works for general cases, and some works only for quadratic functions. I use the one given by Claerbout, the one which is very easy to understand and program. The only changes I make is to always use convolution operation to avoid having large matrices. The objective function of our problem is

$$E = \sum_j \int [P(t,j) - D(t) * M_d(t) * \delta(t - t_j) - U(t) * M_u(t) * \delta(t + t_j)]dt + Damping \tag{21}$$

It is a quadratic function of the unknowns $D(t)$ and $U(t)$. The number of unknown samples is $2 \times t$. The number of data samples we have is $j \times t$ which is more than that of unknowns. It turns out that the damping term is less important. The inversion is stable even without damping.

SYNTHETIC TESTS

A synthetic gather is generated for testing the algorithm. All the parameters are chosen close to that of real data. Figure 7 is the wavelet to be used. Figure 8

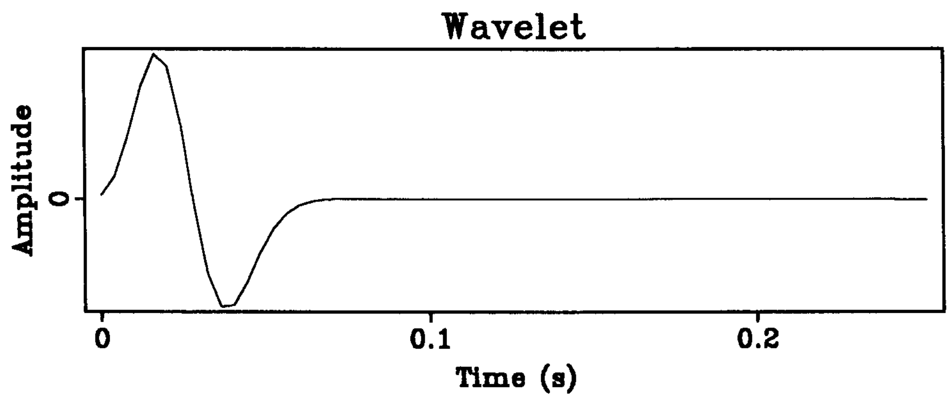

FIG. 7. Synthetic wavelet. It is Richer2 with length 64.

shows the two random sequences and their correlation functions. The sequences are considered to be white because their correlation functions are close to δ function. The multiple patterns are

$$M_d(t) = \sum_k (-0.6)^k \delta(t - t_j), \tag{22}$$

$$M_u(t) = \sum_k (-0.6)^k \delta(t - t_j). \tag{23}$$

Figure 9 shows the gather generated. The data has the dimension of 8×750. Assume the sea floor depth and water velocity are known. We first estimate the multiple patterns and then invert the data. After 10 iterations, we have the solutions shown in Figure 10. Compared with (a), (b) in Figure 8, they are the results of white sequences convolved with the wavelet.

FIELD DATA RESULTS

A common wave parameter gather from the field data is selected, as shown in Figure 11. Generally it is difficult to measure the two way travel time in water. Again we try to get this information from the correlation functions of multiple patterns. Figure 12 shows the results of linear stack. One trace is downgoing waves, the other is upgoing waves. Figure 13 shows their correlation functions, which are considered to be the approximations of the correlation functions of multiple patterns convolved with the correlation function of wavelet. If we look at the spectrum of the correlation functions, as shown in Figure 13, we can see a strong peak appears at some frequency. I consider this frequency to be the repeating frequency of multiples. So from data, I obtain all information required for estimating multiple patterns. I run the inversion algorithm for 10 steps, then use the intermediate results to improve the multiple pattern estimations. After that, I continue to run the algorithm for another 10 steps. Figure 14 shows the results of the inversion. Compared with (a), (b) in Figure 12, strong periodic multiples are suppressed. For downgoing

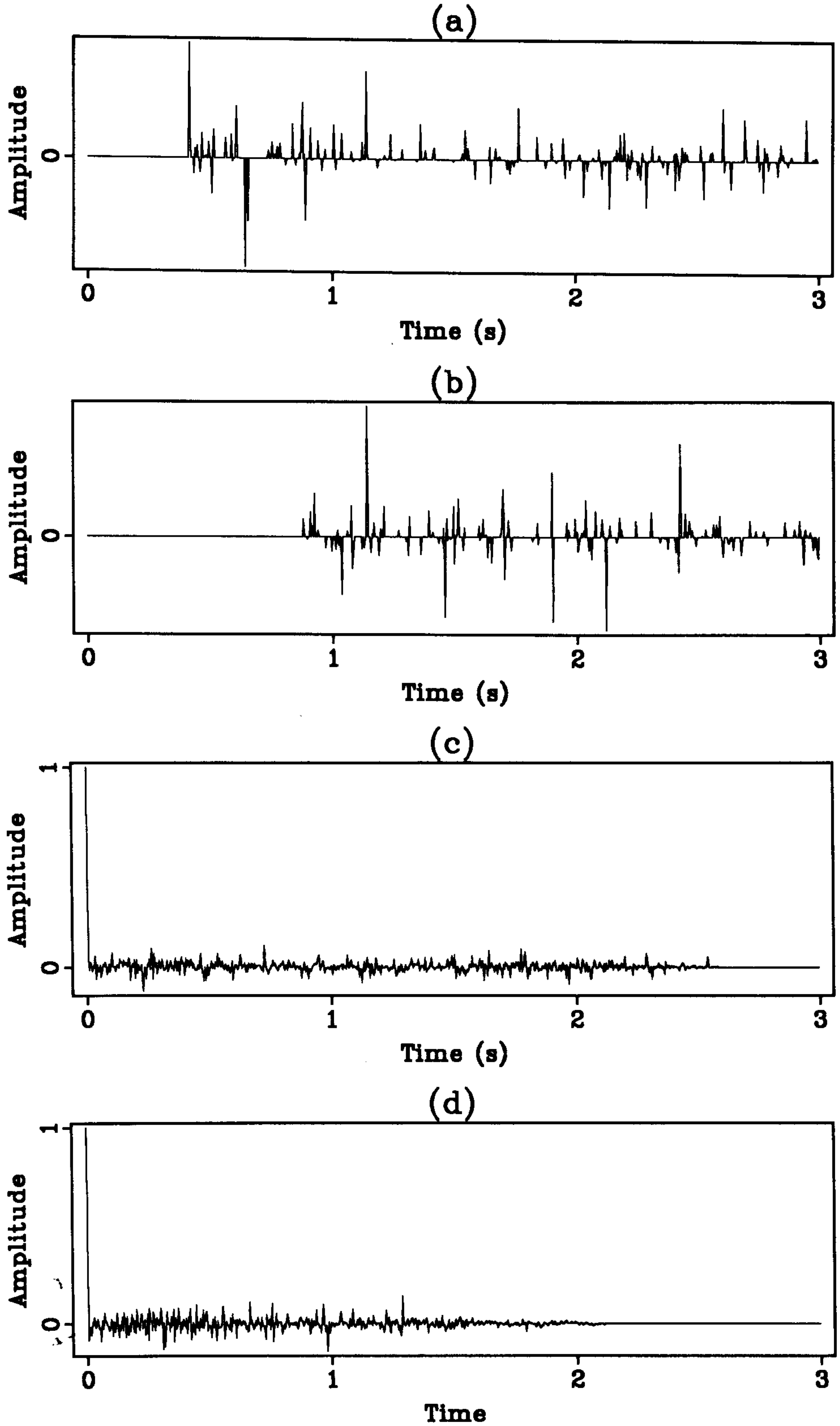

FIG. 8. (a) The white sequence used to generate downgoing waves. (b) The white sequence used to generate upgoing waves. (c) The correlation function of (a). (d) The correlation function of (b).

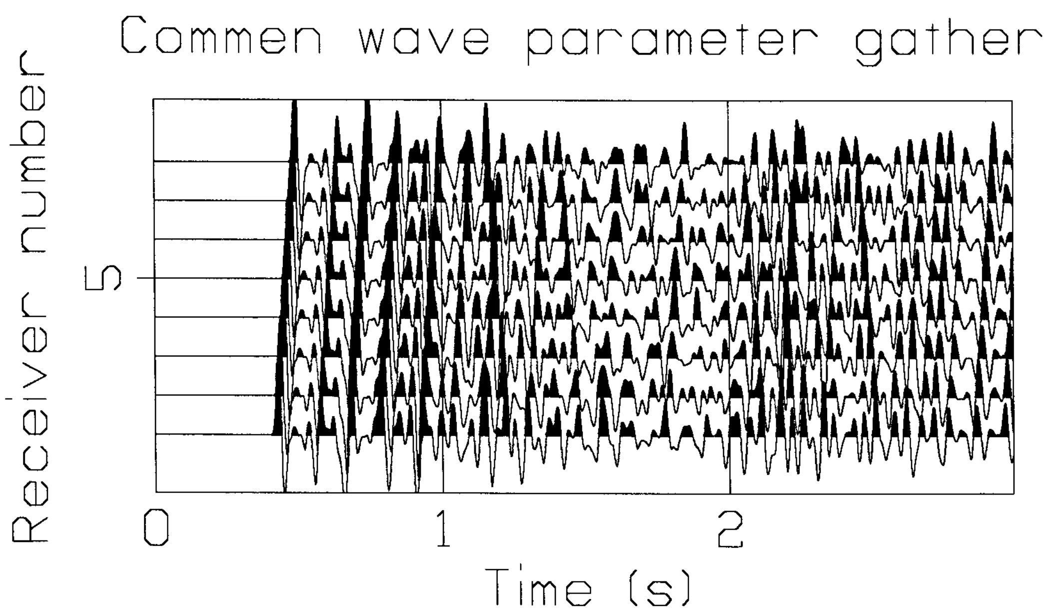

FIG. 9. The synthetic common wave parameter gather.

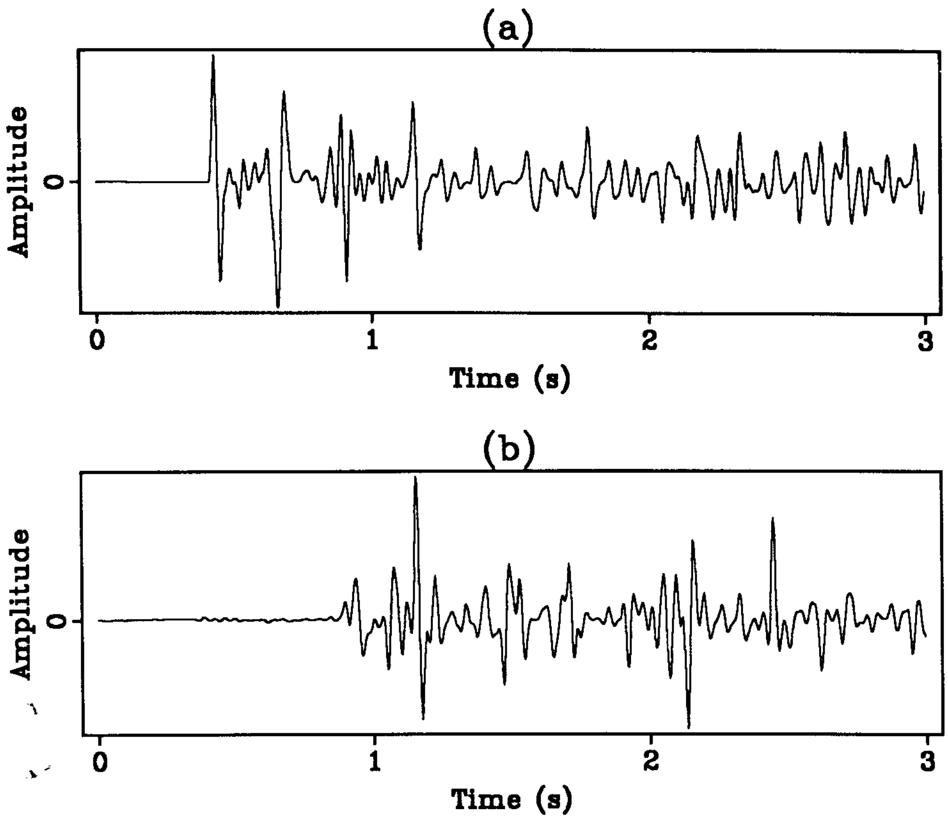

FIG. 10. Result of inversion. (a) Downgoing waves. (b) Upgoing waves.

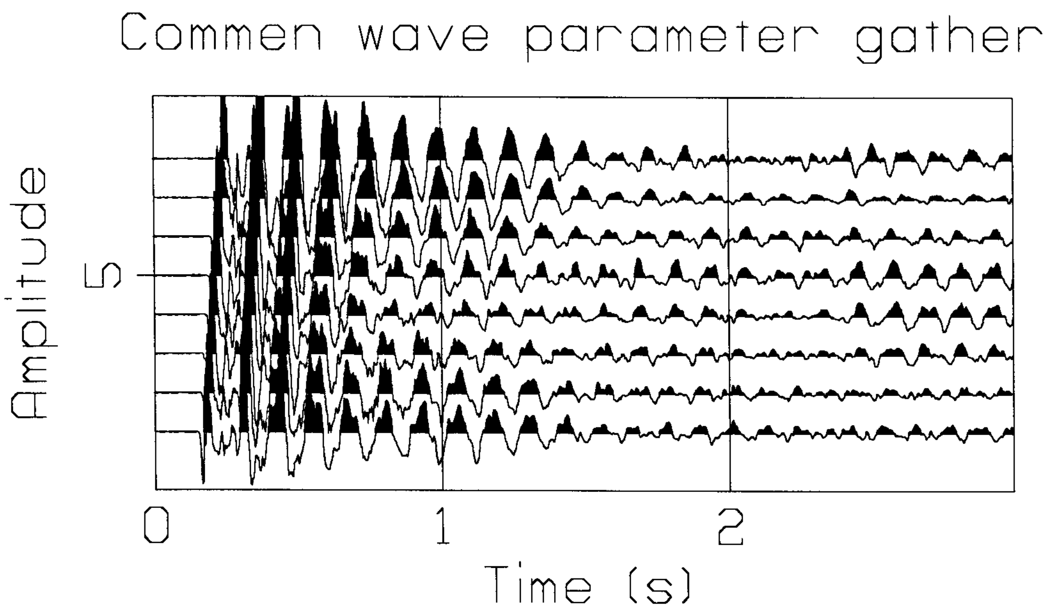

FIG. 11. A common wave parameter gather from field data.

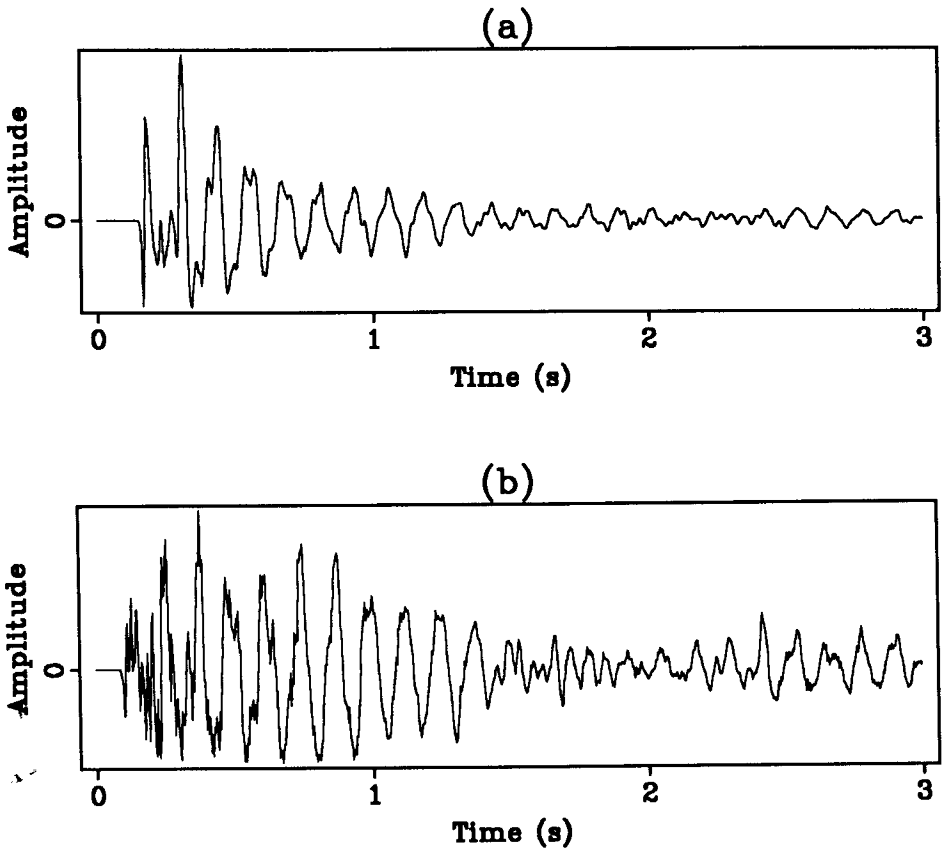

FIG. 12. Results of linear stack. (a) Downgoing waves. (b) Upgoing waves.

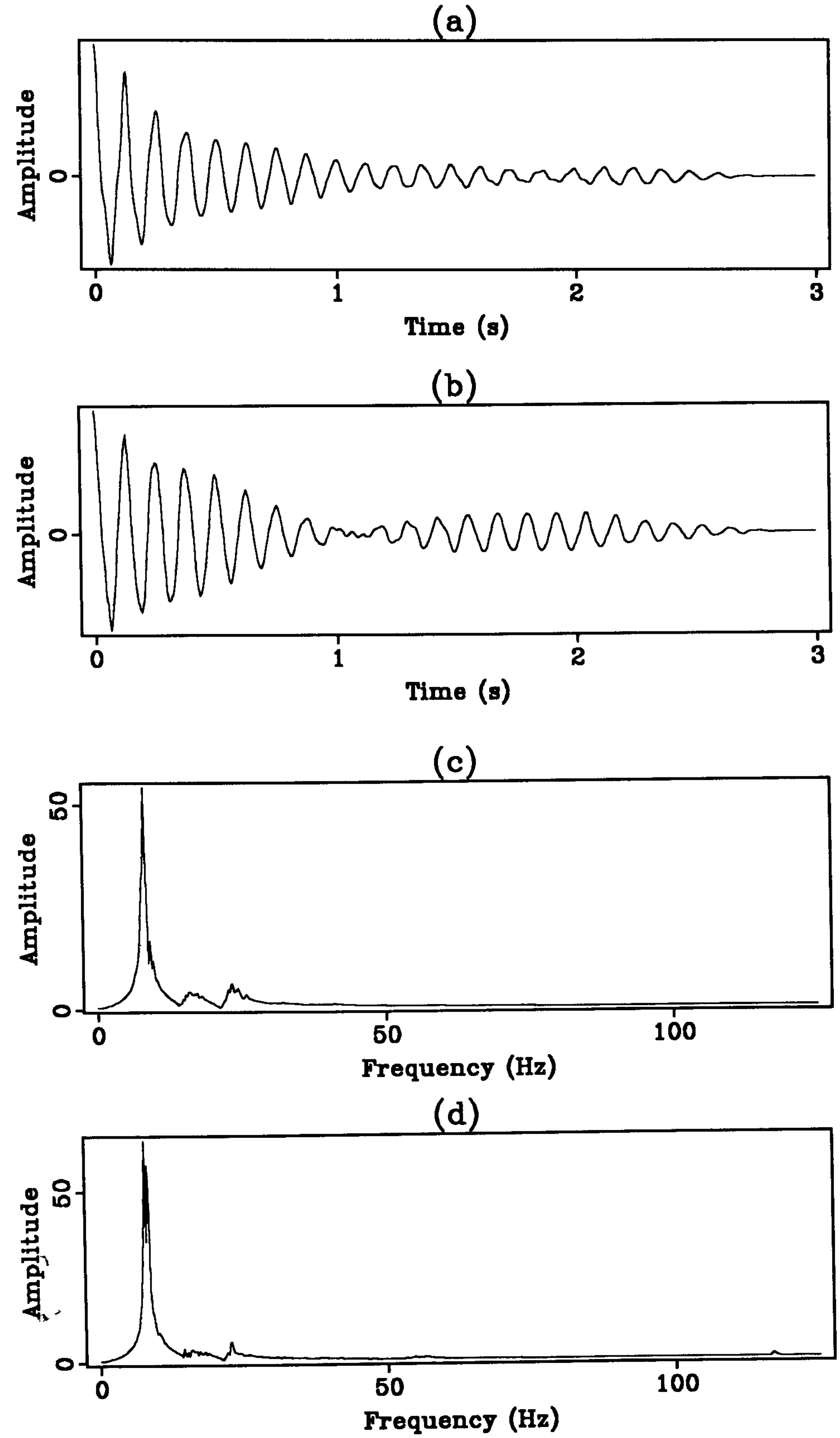

FIG. 13. Correlation functions ana their spectrum. (a) Correlation function of (a) in Figure 12. (b) Correlation function of (b) in Figure 12. (c) Spectrum of (a). (d) Spectrum of (b).

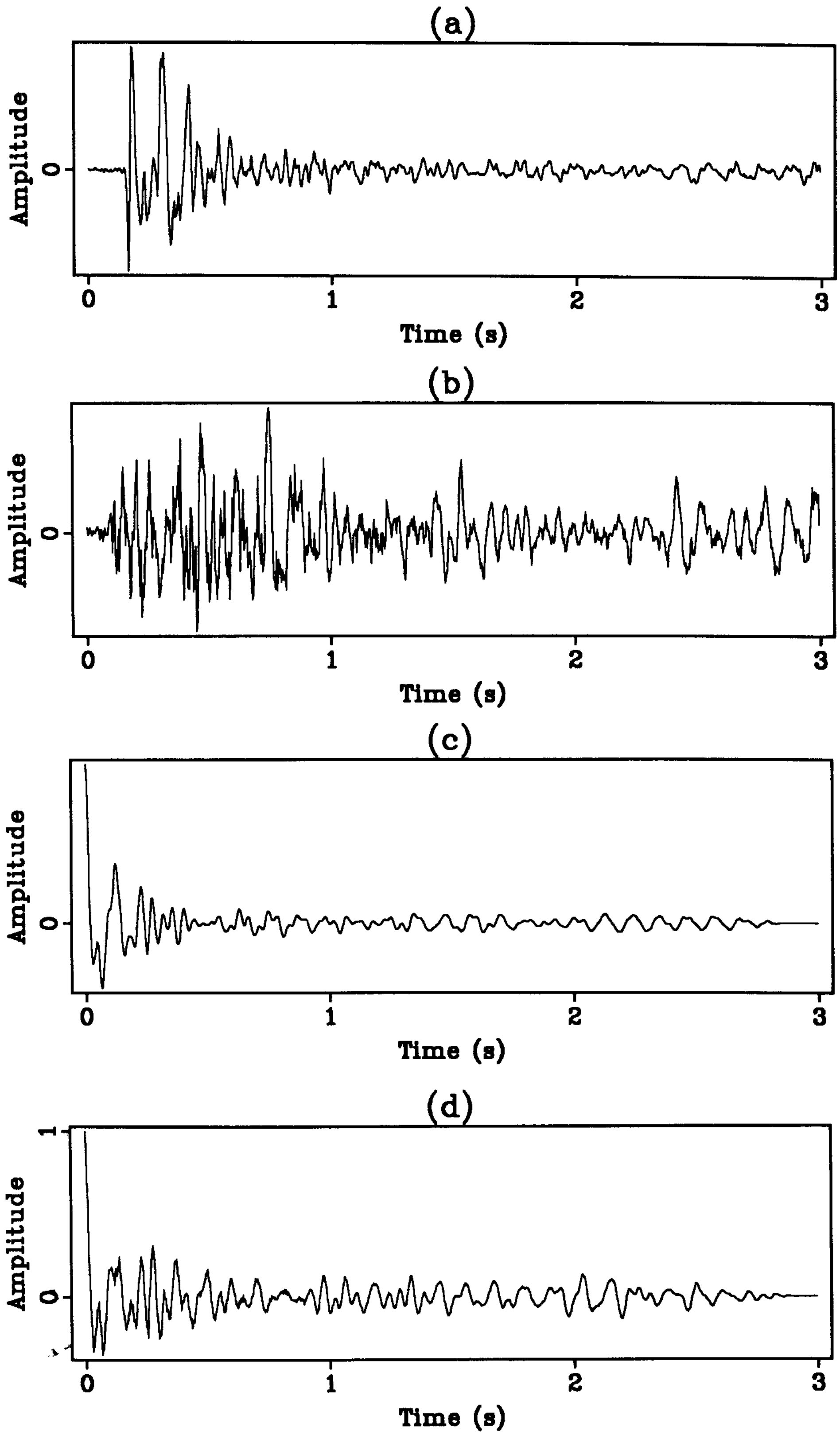

FIG. 14. Results of inversion and their correlation function (a) Downgoing waves. (b) Upgoing waves. (c) Correlation function of (a). (d) Correlation function of (b).

waves, the first multiple is not completely suppressed because the multiple pattern is computed by statistically averaging, so it may not be optimum for an individual multiple. There is no obvious event in late time. For upgoing waves, several events can be identified. Figure 14 also gives the correlation function of downgoing waves and upgoing waves. Clearly to some extend, the signals are whitened.

CONCLUSION

The development of this inversion algorithm shows that for Walk-Away seismic data, multiple suppression and wave separation can be done simultaneously. The algorithm successfully extracts the downgoing wave trace and upgoing wave trace from a common wave parameter gather which contains strong seabottom multiples. To see completely how well the algorithm works, the whole data section should be processed. The future work includes the improvement of multiple pattern estimations and the development of more complex model which puts noise into consideration.

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A man, a plan, a caret, a ban, a myriad, a sum, a lac, a liar, a hoop, a
pint, a catalpa, a gas, an oil, a bird, a yell, a vat, a caw, a pax, a wag,
a tax, a nay, a ram, a cap, a yam, a gay, a tsar, a wall, a car, a luger, a
ward, a bin, a woman, a vassal, a wolf, a tuna, a nit, a pall, a fret, a
watt, a bay, a daub, a tan, a cab, a datum, a gall, a hat, a fag, a zap, a
say, a jaw, a lay, a wet, a gallop, a tug, a trot, a trap, a tram, a torr, a
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total, a cedar, a tap, a gag, a rat, a manor, a bar, a gal, a cola, a pap, a
yaw, a tab, a raj, a gab, a nag, a Pagan, a bag, a jar, a bat, a way, a
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leer, a lever, a hair, a pad, a tapir, a door, a moor, an aid, a raid, a
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anus, a gnat, a lab, a cadet, an em, a natural, a tip, a caress, a pass, a
baronet, a minimax, a sari, a fall, a ballot, a knot, a pot, a rep, a
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sap, a zag, a fat, a hall, a gamut, a dab, a can, a tabu, a day, a batt, a
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regular, a call, a war, a stay, a gam, a yap, a cam, a ray, an ax, a tag, a
wax, a paw, a cat, a valley, a drib, a lion, a saga, a plat, a catnip, a
pooh, a rail, a calamus, a dairyman, a bater, a canal, Panama.