# High-resolution velocity inversion of migrated data after structural interpretation

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## ABSTRACT

After determining a structural-velocity model, accurate velocity analysis might still be necessary in possible target zones for oil exploitation, or in areas where the velocity model is inconsistent with available geophysical or geological information. Reflections events in the constant surface location (CSL) gathers after migration are studied for this analysis. Variations in the depth of these events are inverted to find any local velocity perturbations, and the amplitude-versus-angle behavior of the events is used to verify velocity contrasts in the structural model.


## INTRODUCTION

In another article in this report (Van Trier, 1988) I describe a method to determine a global structural velocity model. The method uses ray tracing to construct a matrix that linearly relates traveltimes and velocity parameters. The rays are traced upwards from the reflectors to the surface. Although the structural velocity model is often accurate enough for interpretational purposes, further analysis of the migrated data is sometimes necessary to verify velocities at target zones, or to solve any ambiguities in the velocity model. This detailed velocity analysis is divided in two parts.

The first part can be called residual tomography. Residual variations in the depth of reflectors in the CSL gathers are picked and converted to traveltime perturbations, and the matrix mentioned above is used to find local velocity perturbations. Residual tomography can be compared with residual statics in conventional data processing: after NMO, static corrections can be applied to the traces of a CMP-gather to flatten reflection events.

The second part is concerned with the verification of velocity contrasts in the structural model. The rays calculated in the structural-velocity estimation are used to convert the offset axis in the CSL panels to an angle axis. The amplitudes of reflection events as a function of reflection angle are an indication of the velocity contrast at the reflector. This step is similar to AVO-analysis in standard seismic data processing.

Note that I have assumed that the structural model is accurate enough to trace rays accurately through the model. Also, in the residual tomography the structural model is explicitly used to determine traveltime perturbations. The assumption can be seen as a limitation to the method, but on the other hand it does not make much sense to ignore information available from previous analyses. The idea behind the method is that at the current stage of processing the global velocity model is well-known.

## RESIDUAL TOMOGRAPHY

In the determination of the global velocity model, a tomographic matrix is used to relate traveltimes, $\mathbf{t}$, to velocity parameters, $\mathbf{p}$ (Van Trier, 1988):

$$\mathbf{t} = \mathbf{A}\mathbf{p}. \tag{1}$$

If the structural model is determined in an other way and no matrix is available, the matrix can be constructed by ray tracing. The details of the parametrization of the velocity model are not important for the discussion of this paper. The traveltimes are traveltimes of rays going from the reflectors to the surface.

The matrix is now used to invert perturbations in the CSL gathers after migration. However, before this can be done, several problems have to be solved. First, the matrix only handles one-way rays (going from the reflector to the surface), instead of two-way ones (going from the source to the reflector and back to the receiver). Second, the CSL gathers are not a function of time, but a function of depth. Finally, the perturbations have to picked from the gathers.

### Constructing the two-way matrix

The matrix has to be converted to a matrix that relates traveltimes in a CSL gather to velocity parameters. $\mathbf{A}$ has been constructed by ray tracing from the reflectors to the surface, and, consequently, it can be split in two parts: $\mathbf{A}_{-\theta}$, containing all the rays from the reflector to the sources and $\mathbf{A}_{+\theta}$, containing the rays to the receivers, where $\theta$ is the angle of incidence at the reflector. So, equation (1) can be written as:

$$\begin{pmatrix} \mathbf{A}_{-\theta} \\ \mathbf{A}_{+\theta} \end{pmatrix} \mathbf{p} = \begin{pmatrix} \mathbf{t}_{-\theta} \\ \mathbf{t}_{+\theta} \end{pmatrix}, \tag{2}$$

where $\mathbf{t}_{-\theta}$ contains the traveltimes to the sources, and $\mathbf{t}_{+\theta}$ the traveltimes to the receivers. A new matrix is constructed by combining $\mathbf{A}_{-\theta}$ and $\mathbf{A}_{+\theta}$:

$$\mathbf{B} = \begin{pmatrix} \mathbf{A}_{-\theta} \, \mathbf{A}_{+\theta} \end{pmatrix}. \tag{3}$$

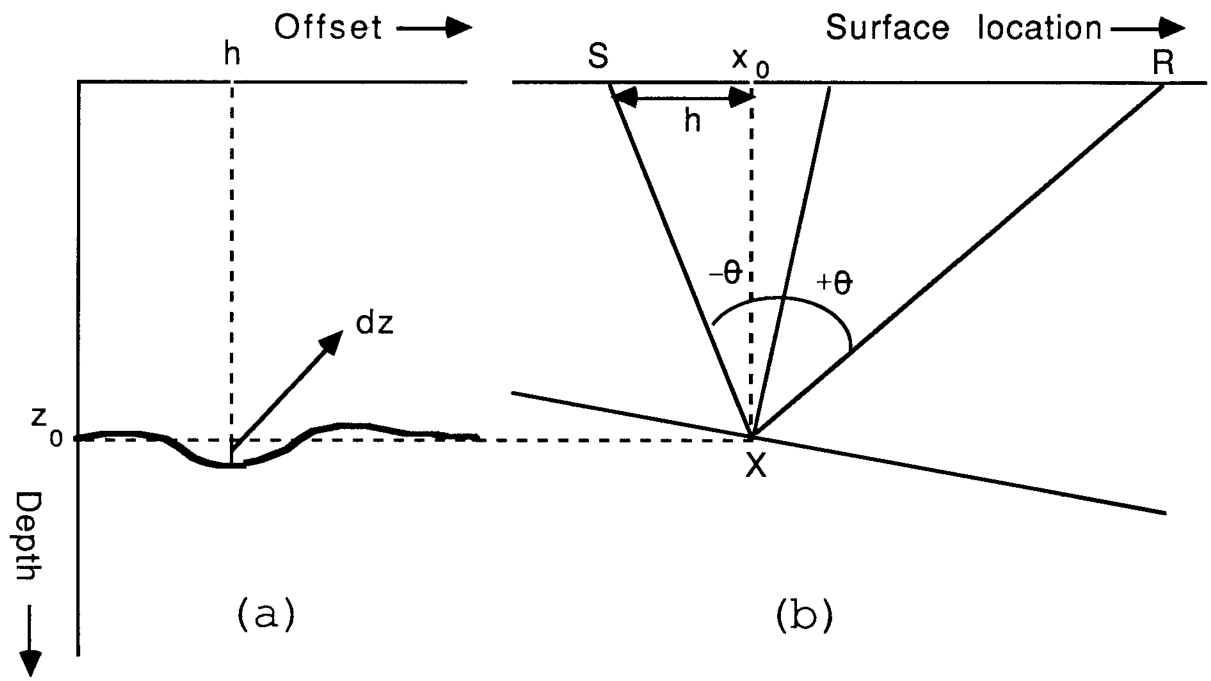


FIG. 1. CSL gather after migration with the structural model (a), and corresponding ray picture (b). The small variations $\delta z$ in the event are caused by local velocity anomalies.

This matrix models traveltimes from sources to receivers for selected depth points:

$$\mathbf{B}\,\mathbf{p} \;=\; \mathbf{t}_{-\theta} \;+\; \mathbf{t}_{+\theta} \;=\; \mathbf{t}_{h}, \tag{4}$$

with $\mathbf{t}_h$ the traveltimes as a function of the offset $h$ in the CSL gather. If rays do not arrive at surface locations corresponding to source and receiver positions, traveltimes can be found be interpolation. As for the matrix, selecting the nearest ray is accurate enough if the ray fans are sufficiently dense.

## Conversion of depth anomalies to time anomalies and inversion

After migration with the structural model, the gathers contain more or less flat events (Figure 1). The events show the behavior of reflections from certain depth points as a function of offset between shot and surface location, $h$.

Ignoring any wave effects, a spike at $(h, x_0, z_0)$ in the CSL gather at surface location $x_0$, can be found in the original seismic data at

$$(s, g, t_{SX} + t_{XR}) = (s, g, t_{-\theta} + t_{+\theta}), \tag{5}$$

with $s$ the source position, $g$ the geophone position, $t_{SX}$ the traveltime of the ray traveling from $S$ to $X$, and $t_{XR}$, the traveltime of the ray traveling from $X$ to $R$ The position of the source $S$ is related to the offset in the CSL ($h = x_0 - s$), the position of the receiver $R$ has to be determined from ray tracing (see figure 1). Note that for dipping beds the normal incidence ray does not arrive at zero offset in the CSL gather.

An average velocity for the point $(h, x_0, z_0)$ in the CSL gather can now be defined as:

$$\overline{v}(h, x_0, z_0) = \frac{s_{-\theta} + s_{+\theta}}{t_{-\theta} + t_{+\theta}}, \tag{6}$$

where $s_{-\theta}$ is the length of the ray traveling from source to reflector, and $s_{+\theta}$ the length of the ray traveling from reflector to receiver. Both the traveltimes and the ray lengths in equation (6) are known from ray tracing.

Now consider an event in the CSL gather at depth $z_0$. Perturbations $\delta z$ in the depth of the event as a function of offset can be converted to traveltime perturbations $\delta t$ using the average velocity $\overline{v}$:

$$\delta t(h, x_0, z_0) = \frac{\delta z(h, x_0, z_0)}{\overline{v}(h, x_0, z_0)} \tag{7}$$

In the above equation I make the assumption that $z_0$ is known; the residual depth perturbations are caused by small velocity perturbations in the model, not by errors in the depth of the reflector.

Assuming the velocity perturbations to be small, Fermat's principle can be applied, and the matrix $\mathbf{B}$ can be used to relate the traveltime perturbations to velocity perturbations, $\delta\mathbf{p}$:

$$\delta \mathbf{t}_h = \mathbf{B}\delta\mathbf{p}. \tag{8}$$

The velocity perturbations are found by inverting the matrix using standard techniques (for example, see Van Trier, 1988).

### Picking

The picking of the depth perturbations is guided by the structural model. At a given surface location in model, structural boundaries are determined. After selecting a certain event in the CSL gather, the depth of the corresponding boundary in the model is used as a reference level to determine depth variations in the event. A possible problem in the picking is the problem of what to pick: after migration the phase of the source wavelet might be distorted, and simply picking peaks or throughs might lead to errors. Also, the CSL gathers are the result of summing seismic data along the geophone axis. Consequently, we are not dealing with spiky events as assumed in the above section, but with events that have a certain width (the Fresnel zone) in the original data. Ignoring this width might give erroneous results. Tests have to show how wave effects influence the picking.

## AMPLITUDE VERSUS ANGLE

Reflection amplitudes carry information about velocity and density contrast at the reflector. Shuey (1985) shows that the Zoeppritz equations that give the expression for the compressional reflection coefficient $R(\theta)$ can be simplified to:

$$R(\theta) \;=\; R_0 \;+\; a\sin^2\theta \;+\; b(\tan^2\theta - \sin^2\theta), \tag{9}$$

with $\theta$ the angle of incidence. The first term gives the amplitude at normal incidence, the second term characterizes $R(\theta)$ at intermediate angles, and the third term describes the approach to critical angle. Neglecting density contrasts, $R_0$ is proportional to the contrast in compressional velocity at the reflector, and $a$ to the contrast in Poisson's ratio.

Classical amplitude-versus-offset (AVO) analysis is based on Shuey's observations. After NMO, the traces in the common midpoint gather are converted to angle traces by an approximate partial stacking method. Then, for precritical angles, the observed amplitude behavior as a function of $\sin^2\theta$ is fitted by a line. The intercept of the line with $\theta = 0$ gives $R_0$; the slope of the line gives $a$. The AVO analysis assumes flat-bed geometry and has proven useful in the identification of gas-bearing sands.

Here the analysis is applied to surface location gathers instead of common midpoint gathers. After migration with the (almost) correct velocity, a CSL gather contains reflections from true common depth points, whereas a CMP does not. As illustrated in Figure 1, the information in the CSL gathers is still angle-dependent. However, for general geology the conversion to angle traces is not as simple as it is for flat-bed geology: ray tracing is necessary to unravel the reflection angles present in the CSL gather. Luckily, the ray tracing has already been carried out, and conversion of the offset axis in the CSL gather to an angle axis can be done without much effort.

Once the CSL gather is converted to an angle gather, one can do an amplitude-versus-angle analysis as in classical AVO. This gives $R_0$ and $a$ along the reflector. $R_0$ can be translated into P-wave velocity contrasts that can be used to verify velocity contrasts in the structural model. $a$ may be used in an S-wave velocity analysis.

Note that I don't try to model the absolute amplitudes in the data, but only relative amplitudes at a given depth, and that the amplitude behavior is modeled by only two, or at most three parameters. In other words, I only want to incorporate zero-order amplitude effects in the inversion. I believe that any higher-order effects are not likely to be found, because of gain applied to data, absorption effects, and distortion of amplitude by migration. Also, the amplitude analysis is only meant to be used as a verification, preferably in an interactive environment, giving the user information about where errors might have been made in the geological interpretation. Amplitude estimation is generally not robust enough to justify an automatic velocity inversion scheme.

## CONCLUSIONS

A structural model can be determined after extensive processing and interpreting of the seismic data. Parts of the model often require further investigation. The described method refines and verifies the structural model. It uses selected parts of the seismic data after migration. In this paper I have discussed some of the issues involved in the selection of the data and their inversion. The goal is to integrate the method into an interactive scheme that facilitates detailed structural interpretation, where the emphasis is on verifying possible structural models with the seismic data, rather than on a full-blown inversion of the data.

## REFERENCES

Shuey, R. T., 1985, A simplification of the Zoeppritz equations: Geophysics, **50**, 609–615.

Van Trier, J., 1988, Geological constraints in velocity inversion: SEP–**57**.