The velocity transform and its preconditioning

Jon F. Claerbout

ABSTRACT

The problem is posed of finding a transformation to velocity space that is close to unitary. A preliminary attempt via Kirchhoff-type algorithms leads to moderate degradations after about 4 iterations in and out of velocity space. Results show clearly that summing only one point per trace leads severe degradation where data steps out fast whereas using more points per trace avoids such noise. A preliminary theory to make the velocity transform more unitary is based on pre and post operative diagonal scaling.

INTRODUCTION

The most important operator in reflection seismology is summation over hyperbolic trajectories. This operator is a linear operator. Knowledge of the inverse would be helpful in dealing with incomplete data coverage. The transpose operator is readily computable to machine precision Claerbout [SEP-42]. The transpose is an approximation to the inverse. The definition of the velocity transformation includes some arbitrariness in the areas of weighting functions and sampling in velocity space. So I thought it would be worthwhile to choose the arbitrariness to try to get the transpose operator as close as possible to the inverse. I haven't fully succeeded in this, but I'll tell you how far I have come.

VELOCITY TRANSFORM

Let $\mathbf{A}$ denote the linear operator that transforms the offset space (t, x) to the velocity space (τ, v). The operator $\mathbf{A}$ can be defined in a variety of ways. In this paper I will consider only two of the many possible ways of defining this operator, both are effectively "Kirchhoff" type definitions that sum along a hyperbolic trajectory.

Two Kirchhoff-style transforms

The first method sums over the offset axis taking one point per trace. This method will be called the "conventional" method. The second method will be called the "antialias" method. It sums along the same hyperbolic trajectory, but it will use more than one point

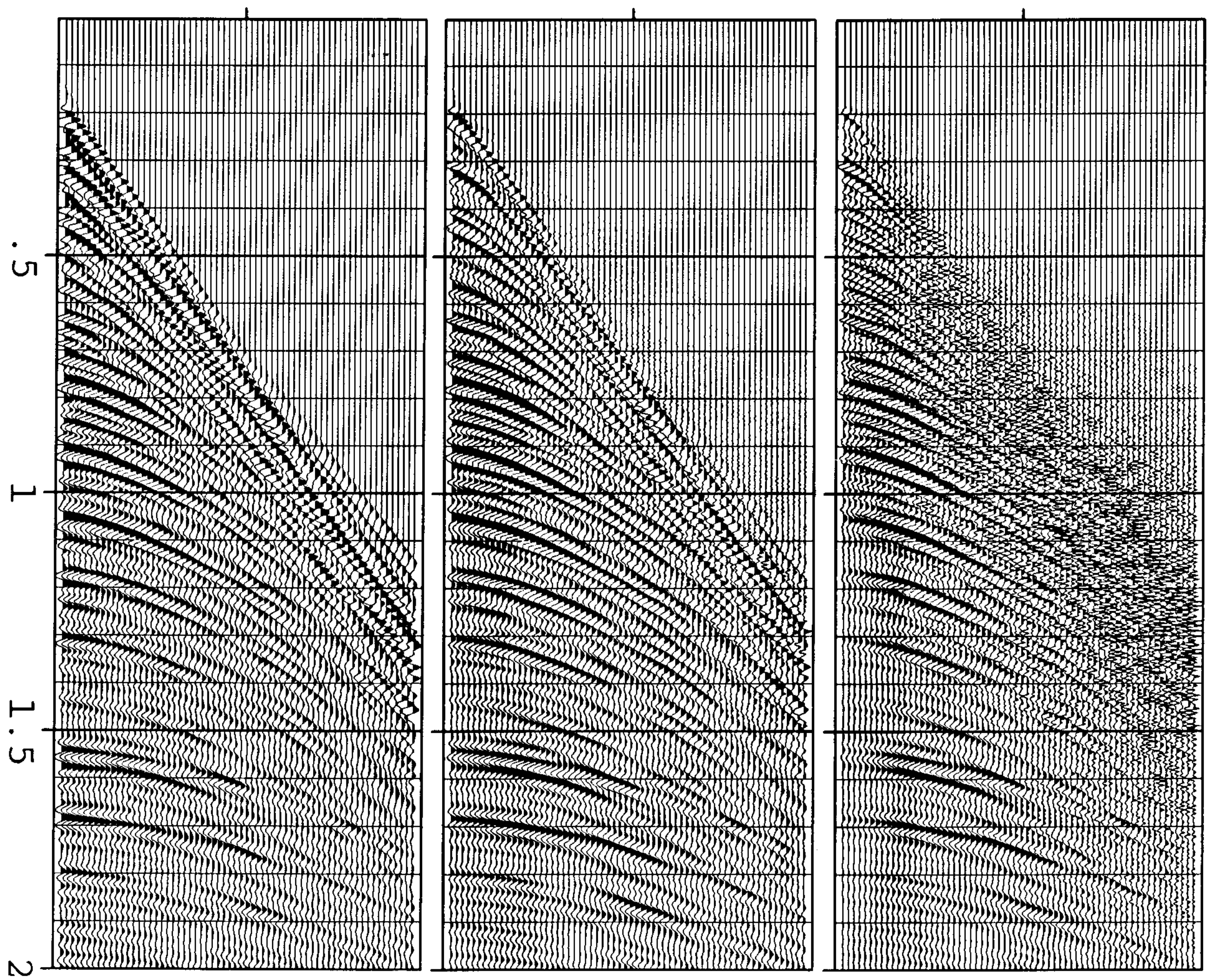

FIG. 1. Conventional summation method: Raw data (left), after one transform to velocity space and back (center), after four such transformations (right).

per trace where the hyperbolas are moving out faster than one time point per offset point. This method is depicted in IEI on page 31.

Guessed weighting functions

My goal is to be able to transform back and forth to velocity space many times with minimal degradation. After experimenting with field profile wz.27 I found least degradation when I used certain weighting functions in the Kirchoff summations. I'll describe these weighting functions.

First, I kept track of something I call $count(x, s, t)$. This is the count of times that a point located on the mesh at (x, t) will be added into slowness s. You may recall that nearest-neighbor normal moveout is exactly invertible, indeed, its conjugate equals its inverse if the moved-out points are divided by the square root of $count(x, s, t)$. By examining the degradation of iterative transformation in and out of velocity space (three times) I arrived at my guess of the best weighting functions for the transformation.

$$weight_{conventional} = \sqrt{\frac{xs/t}{\sqrt{count(x,s,t)}}} \tag{1}$$

$$weight_{antialias} = \frac{1}{\sqrt{count(x,s,t)}} \tag{2}$$

I wasn't enamored about this casual approach to operator definition, so I tried to think up a more systematic means of finding the appropriate weights. This will be described in a later section.

Rho filter

It is well known that slant stacks are invertible in principle if you incorporate a "rho" filter, that is, an $|\omega|$ filter. I guessed that something of this type would be required for the velocity stack too, so I augmented my velocity transform by a causal square root of a time derivative. Thus the operator followed by its conjugate has an $|\omega|$ behavior. I found this worked well and for the first time in my life I was able to transform in and out of velocity space 3-4 times without massive degradations.

Idempotent operator theory review

When a linear operator $\mathbf{A}$ fulfills the condition that its transpose equals its inverse, $\mathbf{A}^T\mathbf{A} = \mathbf{I}$, then the operator is said to be unitary. All the eigenvalues of a unitary operator are known to be of unit magnitude. If some of the eigenvalues are zero then the property $(\mathbf{A}^T\mathbf{A})^2 = \mathbf{A}^T\mathbf{A}$ still holds. This property is called is called *idempotence.* In practice we refer to such an operator as *pseudounitary.*

One example from filter theory is a phase shift filter, which is unitary. Another example, genuinely idempotent, is the bandpass filter, whose spectrum is generally either unity or zero, with a rapid transition.

Results without preconditioning

From the accompanying movie as a function of iteration, you conclude that the "antialias" method works all the way out to the hyperbola asymptote whereas the "conven-

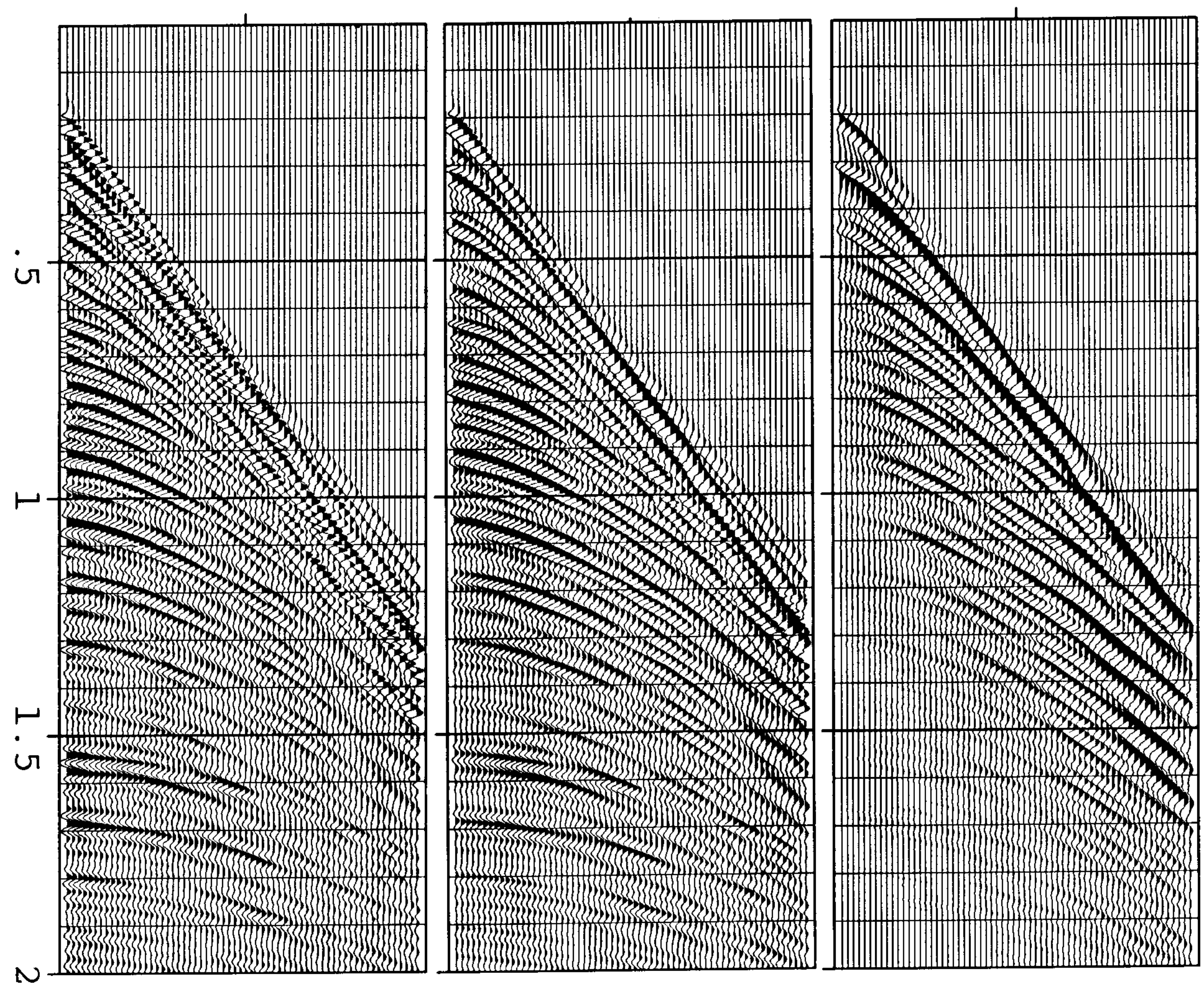

FIG. 2. Antialias summation method: Raw data (left), after one transform to velocity space and back (center), after four such transformations (right).

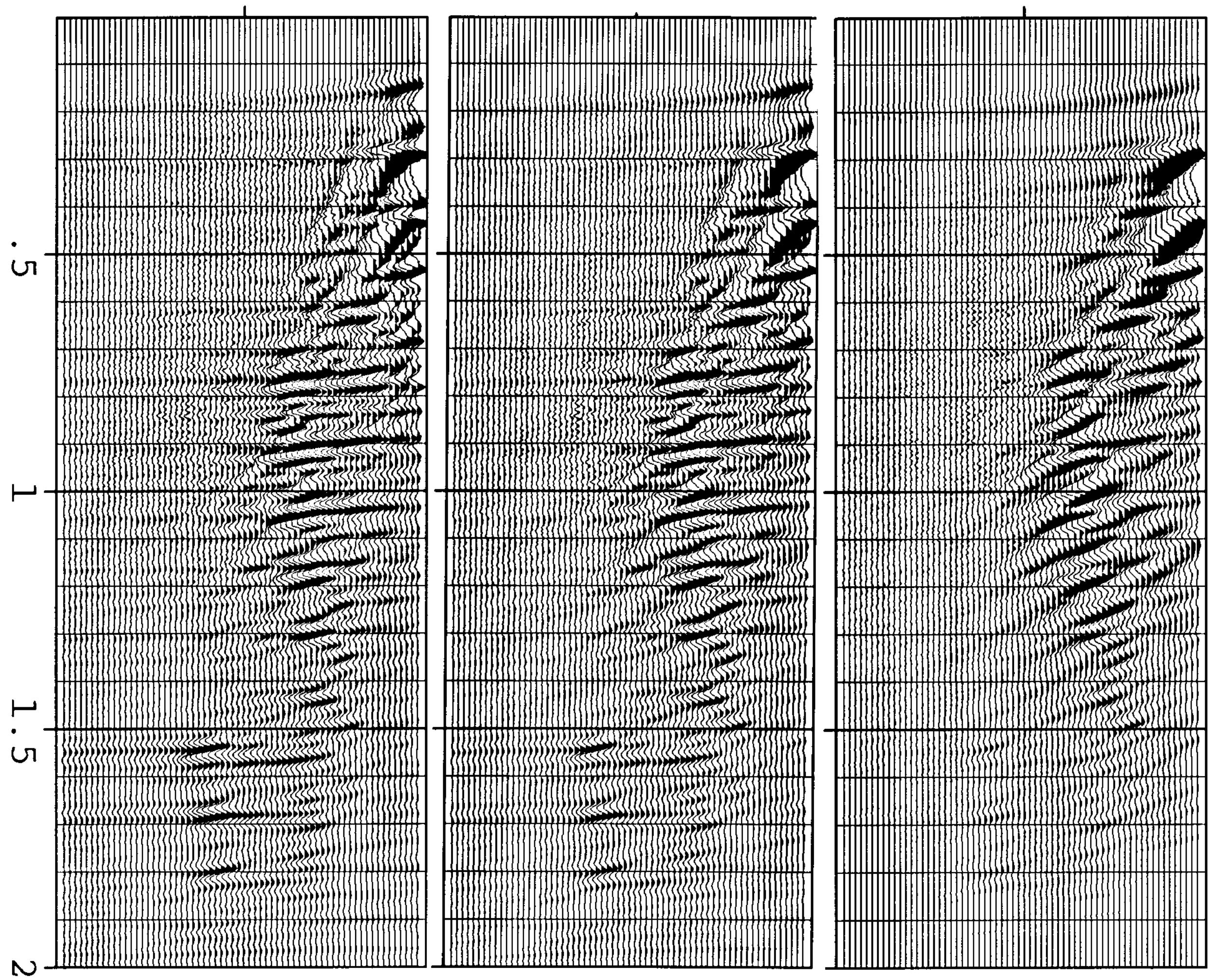

FIG. 3. Antialias summation method. Velocity space. First transform to velocity space (left), after one transform to offset space and back (center), after four such transformations (right).

tional" method gets fuzzy all around the asymptote. So far so good. But the "antialias" method deteriorates unexpectedly at the near offsets. It seems I have guessed incorrectly at the weights. Rather than continue guesswork, I tried to come up with a systematic way of choosing weights which is described next.

PRECONDITIONING

My goal here is to discuss improvement of any velocity transform $\mathbf{A}$ by diagonal pre- and post-multipliers that could be called preconditioners, i.e. multipliers that make $\mathbf{A}$ more unitary.

Define $\mathbf{e}$ to be any sized vector containing all ones. Now consider

$$\mathbf{u} \quad = \quad \mathbf{A}^{\mathrm{T}} \mathbf{A}\, \mathbf{e} \tag{3}$$

$$\mathbf{v} \quad = \quad \mathbf{A}\mathbf{A}^{\mathrm{T}}\, \mathbf{e} \tag{4}$$

Equation (3) first takes ones all over (t, x)-space and transforms them to (τ, v)-space, then presuming that $\mathbf{A}^{\mathrm{T}}$ approximates an inverse transform it transforms back to (t, x)-space. Now if $\mathbf{A}$ was already unitary, we would be so happy that we would not need to bother with preconditioning because $\mathbf{u}$ would already be all ones. Equation (4) is like equation (3) except that we start with ones in (τ, v)-space and see whether we return to all ones in $\mathbf{v}$.

Notice that the operator $\mathbf{A}$ is effectively squared in (3) and (4). This suggests that the square roots of $\mathbf{u}$ and $\mathbf{v}$ be incorporated into the operator $\mathbf{A}$ as scaling multipliers. Applying each in its own space gives

$$\mathbf{A}_{new} \quad = \quad \mathbf{diag}(1/\sqrt{u})\ \mathbf{A}\ \mathbf{diag}(1/\sqrt{v}) \tag{5}$$

as scaling multipliers.

Dilemma

The matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is positive definite. But that does not mean that each element of $\mathbf{u}$ will be positive. I found that for the simplest most obvious definition of $\mathbf{A}$ with summation on hyperbolic trajectories, that the elements of $\mathbf{u}$ were indeed all positive, but when I went to the improvement of including a *rho* filter, a causal filter that in the Fourier domain is $\sqrt{-i\omega}$, that then components of u were often negative, so the divisions in (5) were inappropriate because the square root divisors were imaginary and could be zero.

THREE-DIMENSION CHARACTER OF THE KERNEL

With nearest-neighbor NMO for a single velocity and a single offset x, the matrix $\mathbf{A}$ contains ones along the hyperbolic trajectory $t^2 = \tau^2 + (x/v)^2$. The matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is a diagonal matrix with *count* on the diagonal. In the continuous domain, *count* may be some Jacobian of the transformation. Thus *count* can be expected to be appropriate for the "conventional" transformation to velocity space.

For the "antialias" method the matrix $\mathbf{A}$ contains ones between two hyperbolic trajectories. I think of this as roughly a rectangle function in the time domain. The matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is not diagonal. I think of it as being roughly a triangle time function centered along the main diagonal of a Toeplitz matrix. Raising this matrix to higher powers certainly leads to broadening. It would be nice to imagine sincs instead of triangles, or, better yet, the Dellinger and Muir functions [SEP-48, p. 261] Although "count" may relate to some Jacobian in the continuous domain, it hardly seems the appropriate weighting function for the antialias velocity transform.

CONCLUSION

Results of the preconditioning were poor, worse than without preconditioning. I have not yet determined if this is an error in the theory or in the program.

Also, I feel I have not been able to bring adequate theory to bear on the problem. In particular, even should I get the preconditioning to work, notice that the original guessed weighting functions do not factor into a (t, x)-space part and a (τ, v)-space part. The weights are intrinsically fully three-dimensional in (t, x, s)-space.

Stew just gave me a possibly-relevant paper by Gene Golub. What I need is a Siberian winter to think this through.

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IEI is an abbreviation for: Imaging the earth's interior, by Jon F. Claerbout, Blackwell Scientific Publications, 1985.

DOSECC Continental Scientific Drilling Program

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DOSECC Investigators and Staff
DOSECC, Inc., Washington, D.C.

Introduction

Deep Observation and Sampling of the Earth's Continental Crust (DOSECC, for short) is a nonprofit corporation, currently composed of 39 member universities, that was founded to manage Continental Scientific Drilling Programs somewhat as Joint Oceanographic Institutions (JOI), Inc., manages the Ocean Drilling Program. Funding is provided by the National Science Foundation, with additional support from the U.S. Geological Survey (USGS) and the Department of Energy (DOE). DOSECC currently has two projects in operation and several under development.

The long-term DOSECC program may be separated into categories based either on drilling depth or on objectives. The first category consists of shallow to intermediate depth drilling (up to about 5 km) designed to attain targets related to a better understanding of active processes in the continental crust. The second category of targets push the limit of drilling technology in terms of depth and sometimes with respect to temperature, pressure, and/or corrosive fluid environments. Ultimately, DOSECC drilling projects are expected to achieve depths exceeding 15 km. Such ultradeep holes will not only examine dynamic processes in the crust but will also explore crustal history, structures, and conditions at depth. Current budget constraints allow drilling of projects in the first category, and planning for eventual deeper drilling at a number of locations is in progress.

Project Upper Crust

"Project Upper Crust" is directed by Randy Van Schmus and Pat Bickford (University of Kansas, Lawrence). This project focuses on the midcontinent region of the United States, where the Precambrian basement is hidden beneath a relatively thin veneer of Phanerozoic sediments. Three major activities of Project Upper Crust are

- acquisition, curation, and distribution of samples from commercial drillsites;
- add-on or "piggy back" drilling to core basement rocks wherever feasible; and
- geochronologic and geochemical analysis of samples acquired, with interpretation of the data in the context of continental evolution.

The picture emerging from this work is one of a Precambrian basement in the central and western United States that consists mainly of Proterozoic crust, about 1800 to 2000 Ma old. This crust was accreted to Archean cratons of the Canadian Shield that were welded together and locally bordered by early Proterozoic orogenic complexes, some 1830–1900 Ma ago. South of the exposed shield, Proterozoic crust decreases in age through several distinct orogenic and petrogenetic provinces. The oldest of these is a belt of crustal rocks, 1700–1800 Ma old, that extends from the southwestern United States to the Great Lakes region and perhaps farther east. This belt is bounded on the south by a crustal belt, 1600–1700 Ma old, that is also trending northeast. The combined width of these two belts is about 1000 km. Sm-Nd data indicate that the belts consist of juvenile crust that is derived directly from the mantle, with little contamination by older continental material. Thus these belts record about 1000 km of lateral growth of the North American continent within a 200-Ma interval. This apparently has resulted from lateral accretion or orogenic arc complexes and related terranes and represents an episode of rapid genesis of continental crust, known in very few other places.

The early Proterozoic crust underwent extensive partial melting during two distinct periods of the middle Proterozoic. The older episode occurred 1450–1500 Ma ago and affected most, if not all, of the 1600–1800-Ma-old terrane. It is locally represented by felsic volcanics, but anorogenic granitic plutons are the most common remaining products of this event. A similar but more restricted episode occurred between 1340 and 1400 Ma and is assumed to have affected the south-central portion of the earlier Proterozoic orogenic belt. The products of this orogeny consist of an extensive veneer of rhyolite and epizonal granite that presumably overlies older Proterozoic crust. The last major Proterozoic event occurred about 1200–1000 Ma ago and includes formation of the Grenville Province, the Llano Province of southern Texas, and the Midcontinent Rift System of the central United States.

Project Upper Crust is acquiring data to provide further details of this complex geologic environment. Upper Crust investigators plan eventually to drill dedicated holes in regions where lack of immediate economic potential has restricted basement penetration. Proponents have also sought funds to drill into major untested gravity and aeromagnetic anomalies in the region to determine the features of the Precambrian basement and to use the data to refine models for Proterozoic crustal evolution. A group of geoscientists (Continental Interior Crustal Studies Consortium, or CICSCO; for more information, contact DOSECC, 1755 Massachusetts Avenue, N.W., Washington DC 20036) is requesting support for traverses of shallow, dedicated wells to provide knowledge of basement petrology, structure, and history where no data now exist. Project Upper Crust could gradually evolve into a program of dedicated shallow holes in the areas of greatest interest.

Cajon Pass Project

The first dedicated scientific hole in the DOSECC program is presently being drilled at Cajon Pass near San Bernardino, Calif. Mark Zoback of Stanford University (Stanford, Calif.) is chief scientist for the project, representing a group of principal investigators that includes Tom Henyey (University of Southern California, Los Angeles), Lee Silver (California Institute of Technology, Pasadena, Calif.), and Wayne Thatcher (USGS, Menlo Park, Calif.). More than 30 other scientists from nine universities, USGS, Lawrence Berkeley Laboratory (Berkeley, Calif.), Los Alamos National Laboratory (Los Alamos, N.Mex.), and Sandia National Laboratories (Albuquerque, N.Mex.) are currently involved in drilling-related experiments. Proposals for additional studies either in the hole or on samples and data from it are being accepted by DOSECC's Science Advisory Committee.

One of the primary experiments scheduled for the Cajon Pass hole is a test of the fundamental relationship between the geothermal regime and the state of stress along the nearby San Andreas Fault. A better understanding of earthquakes and the tectonic forces that drive them will result from measurements of the magnitude of existing stresses along earthquake-producing faults. This involves measurement of the frictional resistance of the faults, which is overcome during earthquakes and which should be accompanied by the generation of significant heat. Calculation of conditions along the San Andreas fault near Cajon Pass imply that if average fault friction were 50–100 MPa, frictional heat would generate a local heat flow anomaly along the fault with a magnitude of approximately 1 HFU (~40 mW/m^2). Heat flow experiments suggest that an anomaly of 0.1 HFU should be detectable, but none has been found. This suggests that the average dynamic friction is less than 10 MPa and, allowing for the energy dissipated by seismic radiation, the total fault strength appears to be limited to about 20 MPa.

This thermal indication of low frictional stress is generally viewed as inconsistent with in situ stress measurements made near the fault to maximum depths of approximately 1 km. These measurements have indicated that shear stress increases with depth at a rate of approximately 10 MPa/km, a value consistent with laboratory measurements of rock friction. Extrapolation of this trend to depths of only 5 km yields average stresses that are about twice the upper limit imposed by heat flow, and extrapolation to the base of the seismogenic zone yields stresses almost an order of magnitude larger. This contradiction is referred to as the heat flow–stress paradox.

Although there are other viable models, the two most generally accepted models resolving the heat flow–stress paradox are the following:

- The average frictional stress is high in the upper crust (~100 MPa), and a thermal anomaly is not detected because heat is carried off by circulating groundwater; in this case, both high shear stress and high heat flow should be observed in the bottom of the hole; or
- The average frictional stress is low (~10 MPa), and little frictional heat is generated, possibly because the fluid pressure in the fault zone is anomalously high at depth.

To resolve the heat flow–stress paradox it will be necessary to obtain the best possible heat flow and stress measurements in the hole, as well as to garner information regarding pore pressures, permeabilities, and regional hydrology.

Several methods are available for making in situ stress measurements. Hydraulic fracturing will be used in the Cajon Pass hole, supervised by Mark Zoback, Jack Healy (USGS, Menlo Park) and others. This method has been widely used, and both its advantages and limitations are fairly well known. It involves sealing off an unfractured section of the drill hole with inflatable packers and pressurizing the isolated interval until a tensile fracture is generated in the drill hole wall. The fracture propagates into the rock along a plane perpendicular to the least principal compressive stress (see Figure 1). To accomplish hydrofracturing under the conditions expected is technologically difficult. For example, it is possible that extremely high breakdown pressures will be encountered at depth in the hole. In order to accommodate such pressures and other unpredictable conditions in the hole, the inflatable straddle packers must have three basic characteristics. First, they must be able to withstand high breakdown and pumping pressures. Second, they should be able to work at elevated temperatures, possibly exceeding 200°C. Finally, an efficient packer system should be easily reset for multiple tests on each trip into the hole. As a backup to this system, the occurrence of wellbore breakouts will be noted to determine the orientation of principal stresses. Breakouts are stress-induced wellbore elongations that are measured with four-arm, oriented calipers. The full cross-sectional shape of a hole containing breakouts can be obtained through the use of a borehole televiewer, and the relative size and shape can be determined. It is then possible to study the breakouts to determine the relative magnitudes of principal horizontal stresses, if the minimum principal stress is known from hydraulic fracturing.

An experimental wireline stress measurement method that analyzes stress relief and utilizes downhole holography will be employed in Cajon Pass by Tom Ahrens (California Institute of Technology). Although this method is still evolving, it could provide independent information on stress magnitude and provide support data for the required stress measurements. Time-dependent stress release in core is being examined by Larry Teufel (Sandia National Laboratories).

Preliminary geologic studies at Cajon Pass suggest that the thermal regime probably has been effected by a history of extremely rapid depositional and erosional changes over the last million years. It has been further complicated by structural activity involving formations with contrasting thermal properties, at least in the upper 1 km of the crust. Heat flow measurements will be made by Art Lachenbruch and John Sass (USGS, Menlo Park) during the drilling and for a period of time following the drilling to ensure that the hole has reached thermal equilibrium.

Pore pressure, permeability, and fluid flow are parameters of great interest in understanding both the state of stress and the heat flow data. Measurements of these properties will be made by John Bredehoeft and Healy (USGS, Menlo Park) and by Zoback. The state of effective stress at any point within the crust is partially controlled by fluid pressure at that point. The fluid pressure at depth depends on the permeability of the rock mass. The permeability of intact rock can be measured in the laboratory; however, the actual permeability of a crystalline rock unit is also controlled by the nature and density of fractures, which must be measured in the hole.

In addition to heat flow–stress paradox experiments, numerous other related studies are planned at Cajon Pass. Silver will coordinate studies of the regional geologic and tectonic setting of the drill site, lithologic logging of drill cores and cuttings, basement rock chemistry, and geochronology and isotope studies of the U-Th-Pb and Rb-Sr systems. Downhole logs will be run at specific depths by Roger Anderson (Lamont-Doherty Geological Observatory, Palisades, N.Y.) and Dan Moos (Stanford University) and will include standard as well as highly sophisticated geochemical logs, electrical imaging, multichannel sonics, televiewer, borehole radar, magnetic susceptibility, complex resistivity, borehole gravity, and temperature logs. A vertical seismic profiling (VSP) program proposed by Henyey and Tom McEvilly (Lawrence Berkeley Laboratory) will explore physical properties, including fracture anisotropy, *P* and *S* wave velocity, attenuation, and heterogeneity associated with the San Andreas Fault. A second objective is to follow geologic structures encountered in the drill hole into the surrounding and underlying crust to provide a structural tie to adjacent surface geology. These VSP data will be supplemented with regional common depth point (CDP) seismic profiles run by CALCRUST (the California Consortium for Crustal Studies, headed by Tom Henyey) for interpretation of the regional tectonic setting of the Cajon Pass area. Other proposed work includes investigation of natural fracture systems, deformation fabrics, and rock magnetics.

While many interesting studies are already proposed for the Cajon Pass hole, there are still possibilities for additional work if imaginative and feasible proposals are brought forward. Scientists proposing experiments to be conducted in the hole, on samples from it, or on completion of the drill hole itself may submit a proposal to DOSECC's Science Advisory Committee, 7711 SW 103rd Avenue, Gainesville, FL 32608. It is suggested that interested parties consult with Mark Zoback (Department of Geophysics, Stanford University, Stanford, CA 94305) to ascertain whether the proposed experiments are in the existing science plan or if they are compatible with experiments presently scheduled. The plan called for the 16,000-ft (~4900-m) hole to be drilled by rotary methods with 10% coring at specified intervals and the option of an additional core to be taken on the basis of lithology and structure encountered in the hole. At this writing the hole is at a depth of 6938 ft (~2115 m), and hydraulic fracturing experiments are in progress in open hole below the 6000-ft (~1830-m) casing string.

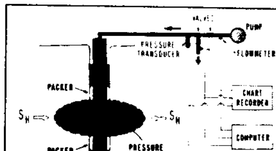

Fig. 1. Diagram of the hydraulic fracturing method, showing the position of structures and stresses mentioned in the text.

When the drilling is concluded, the hole will become the responsibility of USGS, which will establish a permanent seismic observatory at its bottom. This should ensure that the hole will produce valuable data for many years in this region, which has a high probability for a large earthquake in the next few decades. Core and cutting samples from the hole will be available for distribution once initial experiments have been completed.

Katmai Project

A plan is being developed for coring multiple holes up to 1 km in depth into the vent and eruption products of the Valley of Ten Thousand Smokes. The purpose is to understand the thermal, mechanical, and chemical behavior of silicic magma as it intrudes the shallow crust by probing a large, relatively simple, and very young igneous system. In June 1912, the Katmai region of Alaska produced the largest volcanic eruption of this century and the largest rhyolitic eruption in recorded history. The volcano produced 15 km^3 of magma, including 7 km^3 of high-silica rhyolite, in 60 h through Mesozoic basement. The vent is well defined by a 2.6-km-diameter system of ring and radial fractures, with central structural and extrusive domes. Geophysical anomalies and the formation of a second 3-km-diameter caldera, 10 km away, simultaneously with the event provide evidence of a laterally extensive reservoir beneath the region. Because this was the only large-volume eruption in the area and the only rhyolitic event, results from drilling and related geophysical investigations should be more readily interpretable in terms of the origin and behavior of magma than is the case in more common, mature silicic systems. The drilling, which might reach magmatic conditions, would establish for the first time the early postemplacement conditions of a major historic igneous intrusion. The proposed program would attempt to determine

- the structure of the 1912 vent and intrusion,
- the relative roles of syneruptive erosion, collapse, and forcible injection in the development of the structure,
- the physical conditions under which explosive and nonexplosive degassing of magma occur,
- the chemical consequences of vapor flow, including metal transport, and
- the present thermal and chemical state of the intrusion and the relative roles of conduction and convection in reaching that state.

Creede Project

The Creede Project is designed to investigate a multiphase silicic caldera complex and associated hydrothermal mineralization at Creede, Colo. The isotopic character of the

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