On-line movies of the response of beam-steered geophone arrays

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ABSTRACT

The frequency response of a two-dimensional array of geophones to an incident plane wave is easy to compute. The seismic movie program provides a useful means to view the multidimensional result of such computations. Together, these techniques yield an array analysis tool that is easy to implement and use. Beam steering, used to pass energy incident on an array from a particular direction, is formulated here as a transformation of the receiver array that can be applied prior to array response computations to determine the response of beam-steered arrays.

INTRODUCTION

A geophone array acts as a wavenumber filter. The distribution of geophones in an array is a function of one or more spatial directions. When transformed into Fourier space, this distribution is a function of one or more spatial wavenumbers. The response of the array to a signal depends on the signal's wavenumber or wavelength.

The frequency-wavenumber representation of a plane wave is well known (Claerbout, 1985). A plane wave propagating in the (x, z) plane at an angle ϕ with respect to the x axis is governed by the relations:

$$\sin\phi = \frac{v k_x}{\omega} \tag{1}$$

$$\cos\phi = \frac{v k_z}{\omega} \tag{2}$$

Thus if we want to compute the response of a one-dimensional geophone array to such a plane wave, the procedure is as follows. Fourier transform the receiver distribution to get a wavenumber filter. For a given frequency, use equation (1) to determine k_x, and examine the wavenumber filter to determine the response to this k_x.

Now suppose that the plane wave propagates in three-dimensional space. In other words, the down-dip direction of the wavefront is no longer required to be in the + or − x direction. The filtering action of the array depends on the apparent dip of the wavefront in the x direction, which we will call α:

$$k_x = \frac{\omega \sin \alpha}{v} \tag{3}$$

where α is related to the true dip of the plane wavefront by the equation:

$$\sin \alpha = \cos \theta \sin \phi \tag{4}$$

where ϕ is the angle that the down-dip direction of the plane wavefront makes with the direction along which dip is measured, measured counterclockwise. Slotnick (1959) describes the computation of apparent dip and discusses extensively the three-dimensional geometry used in the discussion of beam steering that follows.

Extending the problem to determining the response of an arbitrary two-dimensional receiver array is another straightforward step. A two-dimensional Fourier Transform of the geophone distribution is computed to give the response of the array as a function of the two horizontal wavenumbers k_x and k_y. The apparent dip of the wavefront in the x direction gives k_x, and the apparent dip in the y direction gives k_y. The values of k_x and k_y are used to find the response to the plane wave in the two-dimensional response function computed earlier.

The response of a two-dimensional receiver array to an incident plane wave is three-dimensional. It is a function of frequency, dip angle of the plane wavefront, and azimuth angle or down-dip direction of the wavefront. Thus for a given array we can construct a three-dimensional data volume that can be viewed with the seismic movie program (Ottolini *et al*, 1984).

BEAM STEERING

Array forming, or summation of the signals from the geophones in the array to produce a single output, attenuates energy that is not vertically incident. If we wish to favor energy arriving from a direction other than straight down, the geophone outputs must be properly delayed before summation, so that the wavefront from the beam-steered direction arrives at all receivers at the same time. This is the common practice known as beam steering.

The attenuation produced by a beam-steered array can be computed using the method discussed above. A geometrical interpretation of beam steering helps to see how this can be done. In beam steering, if we want to steer the array so that an arrival from a given direction is not attenuated, we need to project the array onto

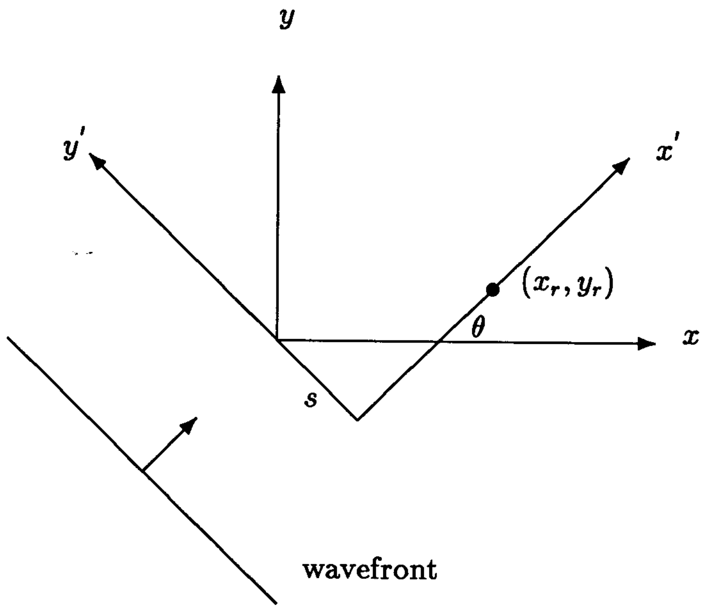

FIG. 1. Coordinate system for beam steering. The x' axis is in the down-dip direction of the arriving wavefront.

the incoming wavefront. The time shifts applied in beam steering are, in another view, the shifts in position needed to align the geophones on a plane that is parallel to the wavefront. Given this geometric view of beam steering as a transformation of the geophone array, it is easy to see that the response of a beam-steered array can be computed in the way that the response for simple receiver summation was computed. We need now to formalize the transformation that results in a beam steered array.

Consider a receiver located at position (x_r, y_r), with the center of the array at the origin. To beam steer plane waves arriving from some direction, we need to project all the receivers in the array onto the plane parallel to the arriving wavefronts that intersects the origin. Let θ be the azimuth angle of the wavefront to be beam steered. Define a new coordinate system (x', y') that is rotated by θ degrees relative to the original (x, y) system, so that the x' direction is the down-dip direction. Also set it up so that the y' axis intersects the origin at $y' = s$. This coordinate system is shown in Figure 1.

Transforming from the (x', y') coordinate system to the original (x, y) coordinate system is accomplished by the following equations:

$$x = x' \cos\theta - (y' - s)\sin\theta \tag{5}$$

$$y = -x' \sin\theta + (y' - s)\cos\theta \tag{6}$$

where s is given by:

$$s = x_r \sin\theta - y_r \cos\theta \tag{7}$$

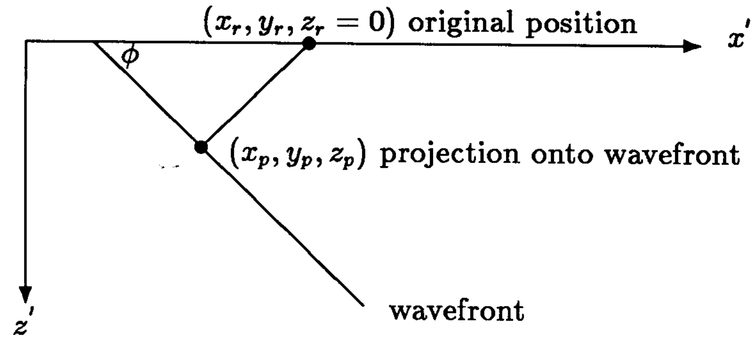

FIG. 2. Projection of receivers onto plane parallel to the wavefront arriving from the direction of beam steering

In the new coordinate system, the receiver is located at the point $(x_r \cos\theta + y_r \sin\theta, 0)$. Projecting this point onto the plane with dip angle ϕ and down-dip direction along the x' axis, we find that in the new coordinate system, the coordinates of the projection are:

$$x' = \cos^2\phi(x_r \cos\theta + y_r \sin\theta) \quad (8)$$

$$y' = 0 \quad (9)$$

$$z' = \cos\phi \sin\phi(x_r \cos\theta + y_r \sin\theta) \quad (10)$$

Figure 2 illustrates this projection onto a plane parallel to the beam steering direction.

Since the coordinate transformation did not involve depth, z_p, the depth of the projected point in the original coordinate system, equals z'. Transforming the other coordinates back to the original system, we find that the location of the projected point is:

$$x_p = x_r(\cos^2\phi \cos^2\theta + \sin^2\theta) + y_r \sin\theta \cos\theta(\cos^2\phi - 1) \quad (11)$$

$$y_p = x_r \sin\theta \cos\theta(\cos^2\phi - 1) + y_r(\cos^2\phi \sin^2\theta + \cos^2\theta) \quad (12)$$

$$z_p = \cos\phi \sin\phi(x_r \cos\theta + y_r \sin\theta) \quad (13)$$

If this transformation is applied to all receivers prior to Fourier transforming the receiver distribution, then the response of the beam-steered array will be obtained.

It is worth noting that the coordinate transformation for beam steering passes a few obvious tests. If $\phi = 0$ (zero dip), the coordinates do not change. For 90 degree dips, the z coordinates of all receivers will still be zero, and the y coordinates will be zero if $\theta = 0$ or $\theta = 180$ (inline arrivals), whereas the x coordinates will be zero when beam steering crossline arrivals.

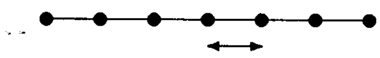

FIG. 3. Linear array geometry.

The transformation required for beam steering introduces one problem that has not yet been discussed. After the transformation, the receiver array will generally be three-dimensional. A three-dimensional Fourier transform of the receiver array is required. Thus computation of k_z is also required at each point to determine the response for a particular frequency, dip, and azimuth. But k_z is independent of the azimuth of the wavefront, depending only on the dip:

$$k_z = \frac{-\omega \cos \phi}{v} \tag{14}$$

where ϕ is the dip angle of the wavefront, and the minus sign selects upcoming waves.

Including beam steering in an array response computation, then, requires extra work mainly at the initialization stage. The receivers must be projected onto the wavefront direction, as described above, and a three-dimensional Fourier transform of the resulting receiver distribution must be taken, as opposed to a two-dimensional transform for the case of receiver summation. Once this extra work is done, though, computing the response for a beam-steered array requires only the additional computation of k_z for each frequency, dip, and azimuth combination.

RESULTS

Figure 3 depicts a linear array a geophones used to test the method. There are seven receivers, with a spacing of 20 meters between adjacent pairs.

Figure 4 is the response of the linear array of Figure 3. The front face of the cube shows the attenuation for zero azimuth. That is, for a plane wave arriving in the inline direction. On this front face, the horizontal axis is frequency (from negative Nyquist to positive), and the vertical axis is the dip angle of the wavefront, from 0 degrees at the top to 90 degrees at the bottom. The third dimension of the cube is the azimuth angle or down-dip direction of the wavefront. This ranges from 0 degrees at the front face, to 360 degrees at the back. The straight black lines show the location of the slice that is displayed on each face of the cube. The lines occur at approximately 60 Hz, 18 degrees dip, and 180 degrees azimuth. Thus the top face of the cube shows attenuation as a function of frequency and azimuth at a dip of 18 degrees, etc.

For this and all the remaining tests shown in the paper, the plane wave is assumed to be propagating with a velocity of 3500 meters per second. The time sampling rate is 4 milliseconds, giving a Nyquist frequency of 125 Hz.

The grey scale beneath the cube gives the attenuation level for different shades of grey. A continuous scale is used rather than picking patterns for just a few different attenuation ranges. The dark bands with attenuation levels well below nearby points are the expected "notches" in the filter response.

A slice can be extracted from the three-dimensional data volume to illustrate how the attenuation depends upon two of the three parameters while the third is held constant. Figure 5 shows the attenuation as a function of frequency and dip of the incoming wavefront for ten different azimuth angles ranging from 0 (inline) to 90 (crossline) degrees. As expected, there is no attenuation for an azimuth angle of 90° with a linear array, since the wavefront arrives at all receivers at the same time regardless of dip.

Figure 6 shows the attenuation as a function of frequency and azimuth angle for a number of dips. As expected, there is no attenuation for the crossline directions of 90 and 270 degrees, regardless of dip. Also note that the attenuation increases rapidly for small dips and then seems to increase slowly past about 30 degrees.

Figure 7 shows a single slice through the cube, attenuation as a function of dip and azimuth for a frequency of 60 Hz. The two crossline directions are evident once again, as is the rapid increase in attenuation with dip for small dips discussed earlier.

Areal arrays

If an areal array is used, the inability to attenuate energy arriving in the crossline directions is removed. Consider the array formed by adding receivers in the crossline direction as shown in Figure 8.

The three-dimensional response for this cross array is shown in Figure 9. As expected, the response has changed significantly for energy incident in the crossline direction. However, response to energy in the inline direction has changed due to the weight that has effectively been added to the center of the seven-element inline array.

Unequal weighting of receivers

The results thus far have been for arrays where all the receivers are weighted equally. The ability to weight receivers can be provided explicitly, or accomplished by putting more than one receiver at a location. For example, consider the linear

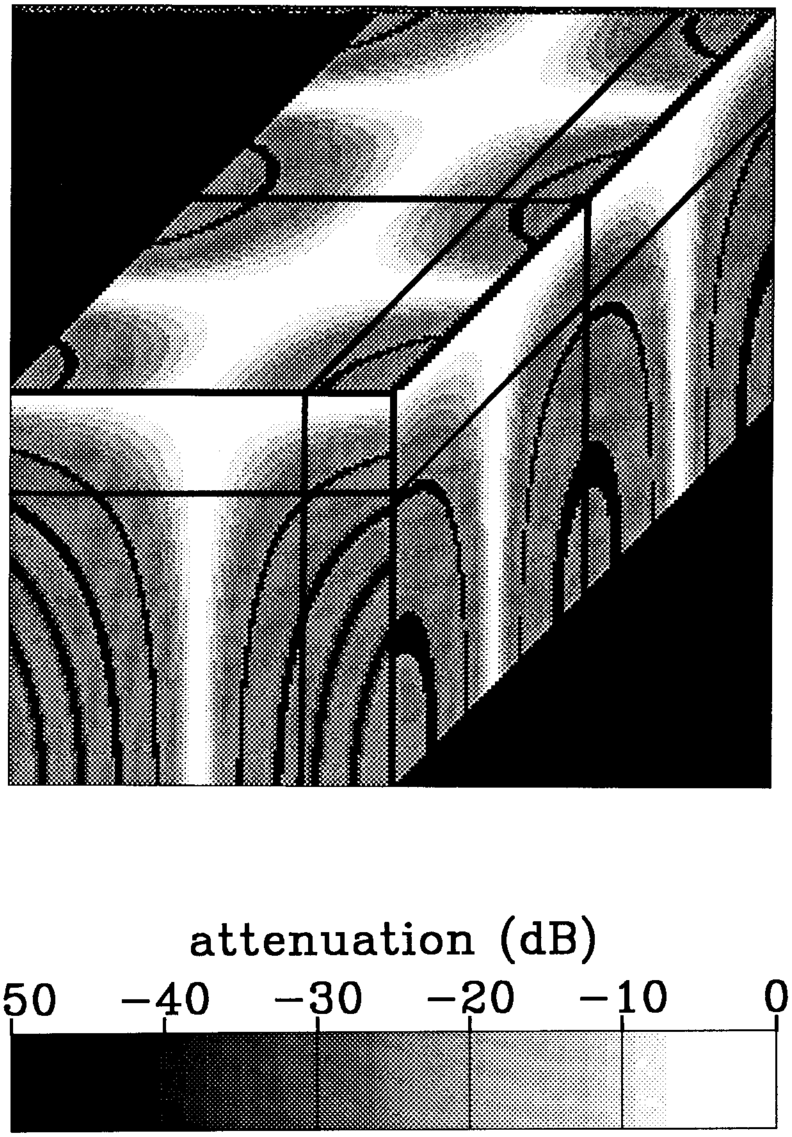

FIG. 4. Attenuation of a linear array of seven equally- weighted geophones as a function of frequency and dip and azimuth angle of the incident plane wavefront. The horizontal axis is frequency, from negative to positive Nyquist. The vertical axis is dip, from 0 (top) to 90 degrees. The third axis is azimuth, ranging from 0 degrees at the front to 360.

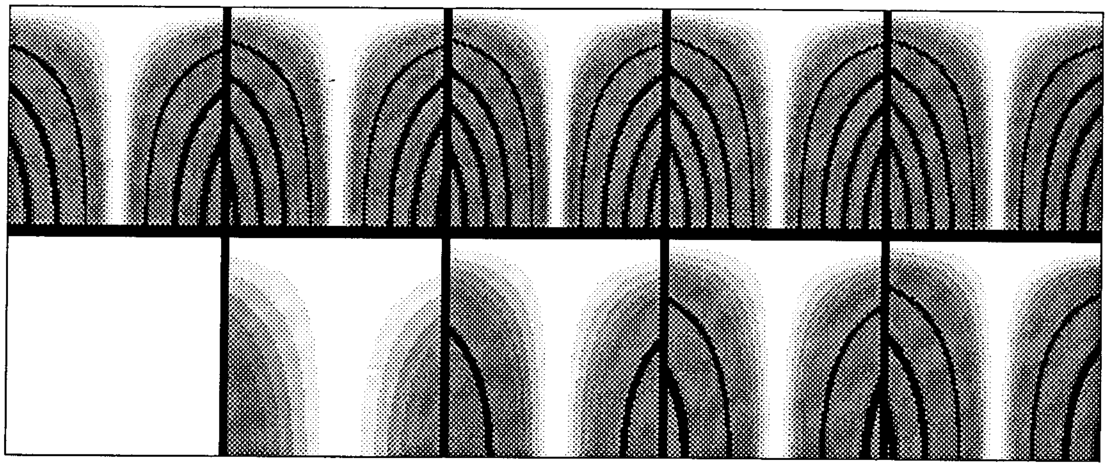

FIG. 5. Attenuation of the linear array of Figure 3 at ten different azimuth angles, ranging from 0 degrees (upper right) to 40 degrees (upper left) on the top row, and 50 to 90 degrees on the bottom row. In each plot, the horizontal axis is frequency and the vertical axis is dip angle of the wavefront, from 0 degrees at the top to 90 degrees at the bottom.

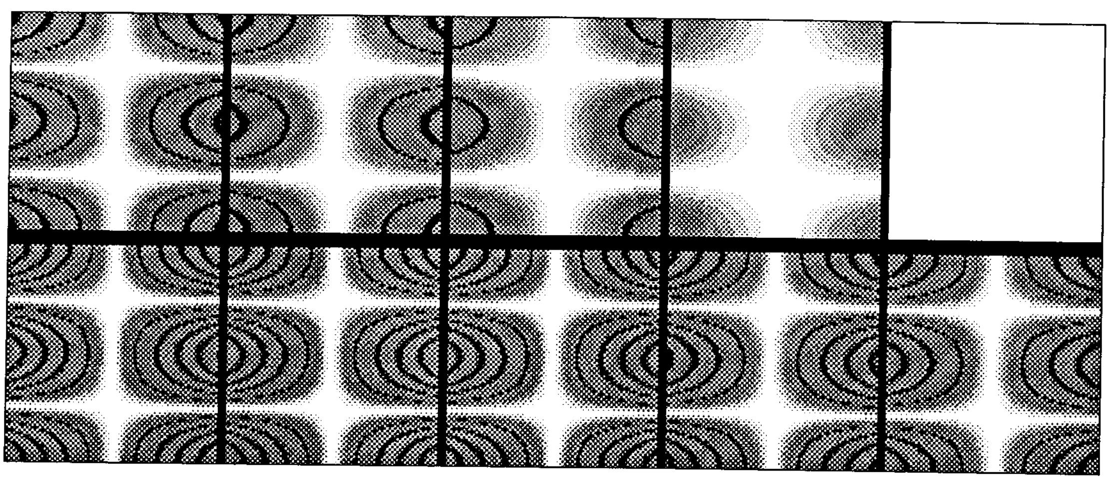

FIG. 6. Attenuation of the linear array of Figure 3 at ten different dip angles, ranging from 0 degrees (upper right) to 90 degrees (lower left). In each plot, the horizontal axis is frequency, and the vertical axis is azimuth angle, from 0 degrees at the bottom to 360 degrees at the top.

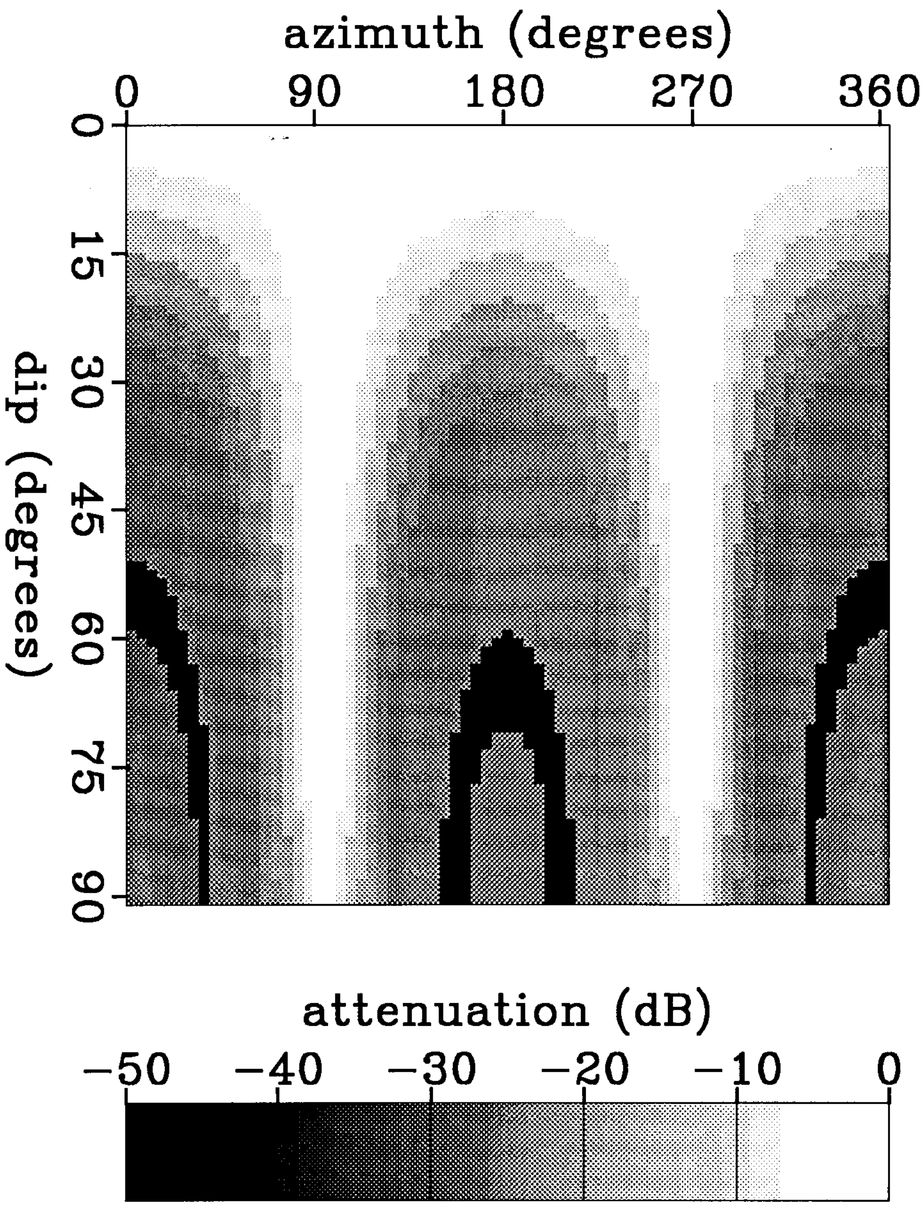

FIG. 7. Attenuation as a function of dip and azimuth angle of the wavefront for a frequency of 60 Hz.

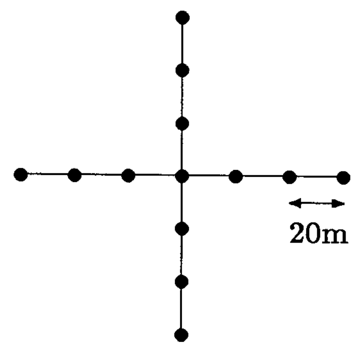

FIG. 8. Cross array geometry.

array with the triangular weighting function shown in Figure 10. The receiver spacing has been doubled as suggested by Newman and Mahoney (1973), to position the first notch at the same frequency as the uniformly-weighted linear array.

While the Fourier transform of an equally-weighted linear array is a sinc function, the Fourier transform of this triangularly-tapered array is a sinc function squared. Thus we expect the side lobes of the attenuation filter to be not as strong. Figure 11 shows the three-dimensional cube for this triangularly-weighted array.

Beam-steered arrays

Figure 12 shows the response for the linear array of Figure 3, but where the array has been beam steered to pass incident energy having a dip of 30 degrees that is incident in the negative x direction (an azimuth of 180 degrees). The white all-pass regions are centered on the top of the cube at 180 degrees azimuth and on the front of the cube at 30 degrees dip, as expected. From the side of the cube it can be seen that the shift applied to focus in the beam direction has a fairly small effect on very steep dips. For steep dips, the array does not provide much attenuation in the two "broadside" directions (azimuths 90 and 270).

Figure 13 shows the result when the cross array of Figure 7 is beam steered to pass energy incident with 30 degrees dip and an azimuth of 75 degrees. The cross array, with its areal coverage, can obviously be steered more effectively than the linear array. Note the compact pass region on the side of the cube centered at the desired dip and azimuth. Figure 13 shows constant dip slices of the response cube.

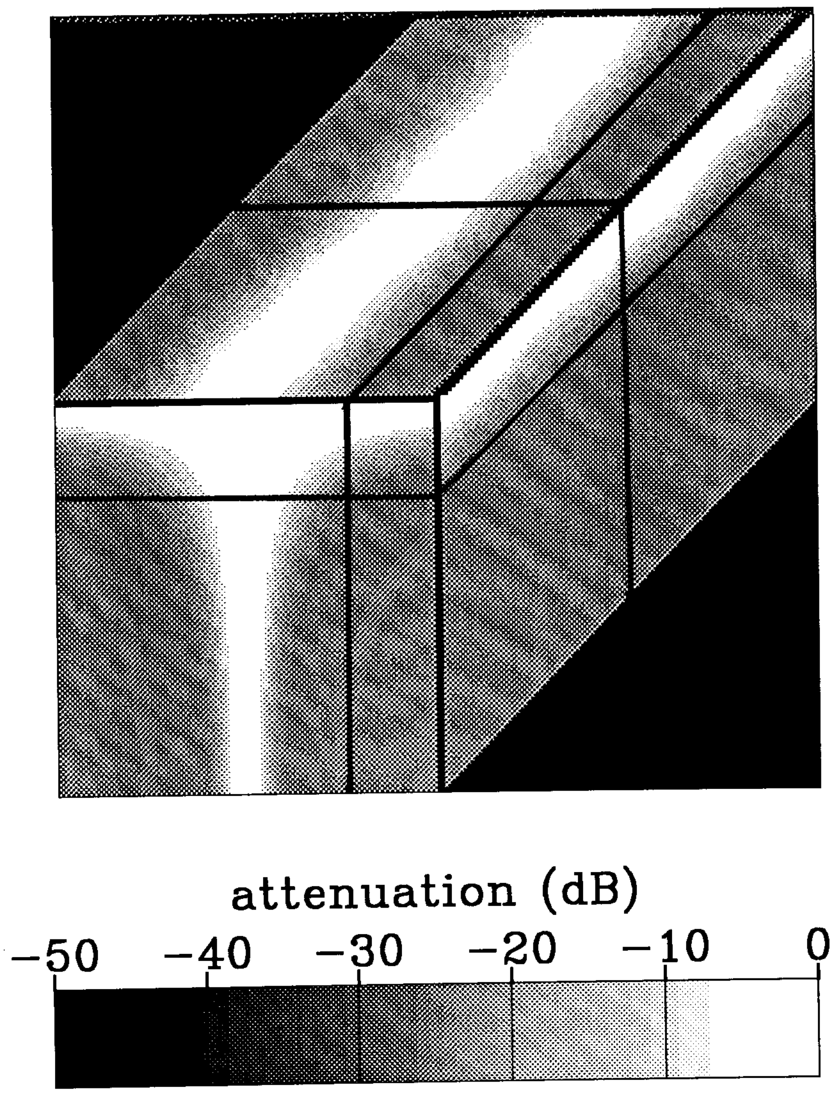

FIG. 9. Attenuation of the areal array shown in Figure 8. Note the attenuation now provided in the crossline direction.

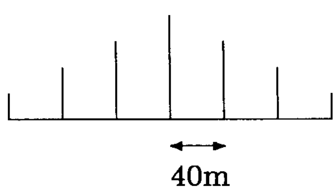

FIG. 10. Triangular taper used to weight receivers in the linear array.

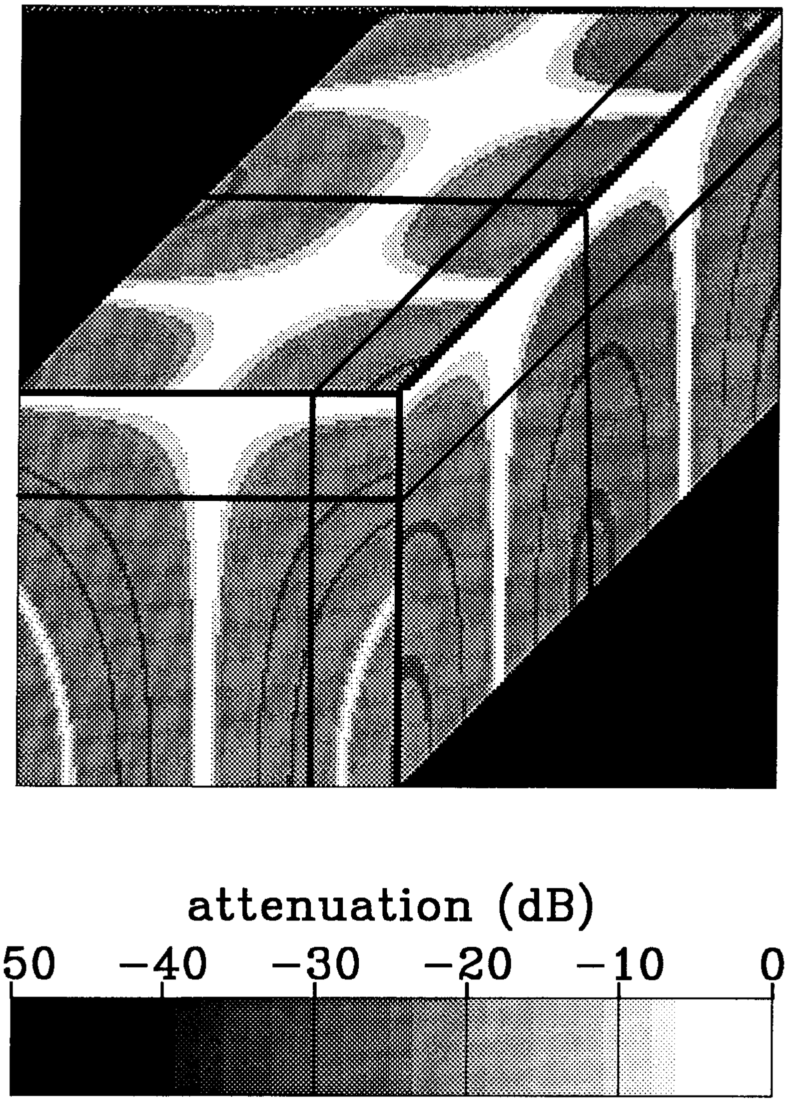

FIG. 11. Attenuation of a the linear seven-element array of Figure 3 with a triangular weighting function applied.

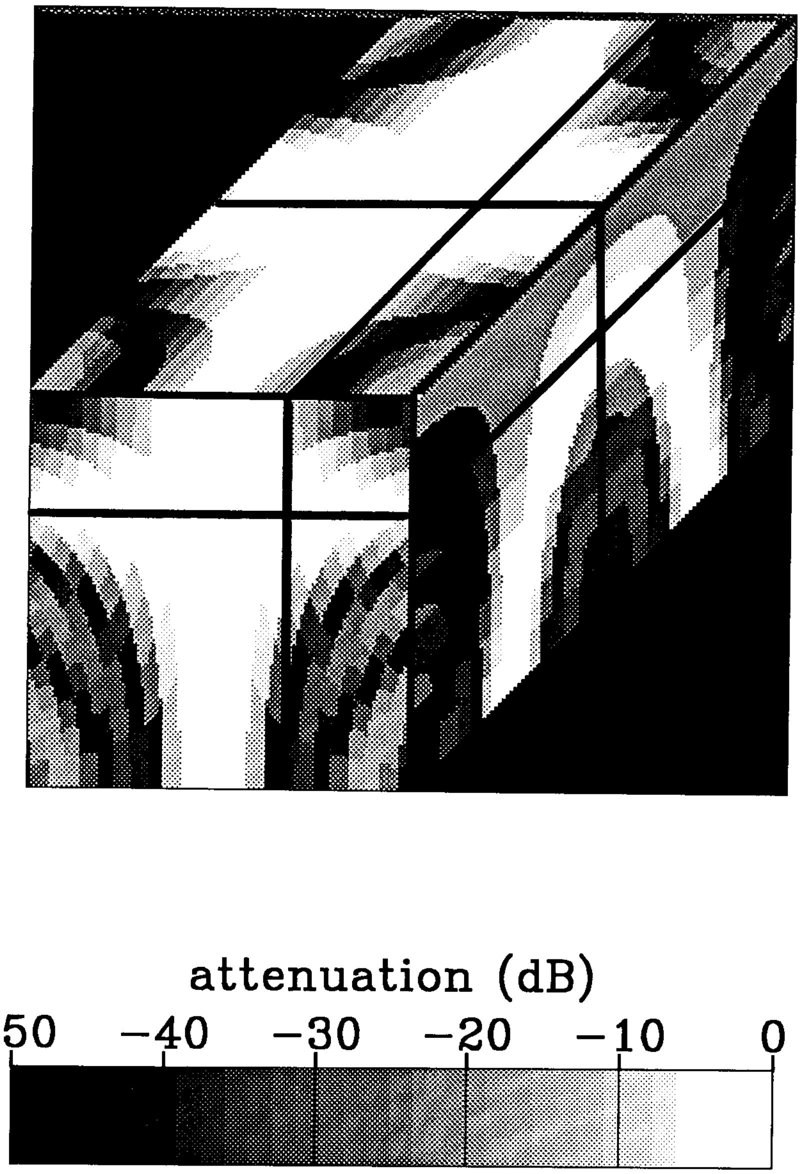

FIG. 12. Response of the linear array of Figure 3 beam steered to pass plane waves incident at 30° dip and 180° azimuth.

Dip ranges from 0 to 90 degrees in 10 degree steps. Note how the pass zone opens up at 30 degrees dip and then closes again.

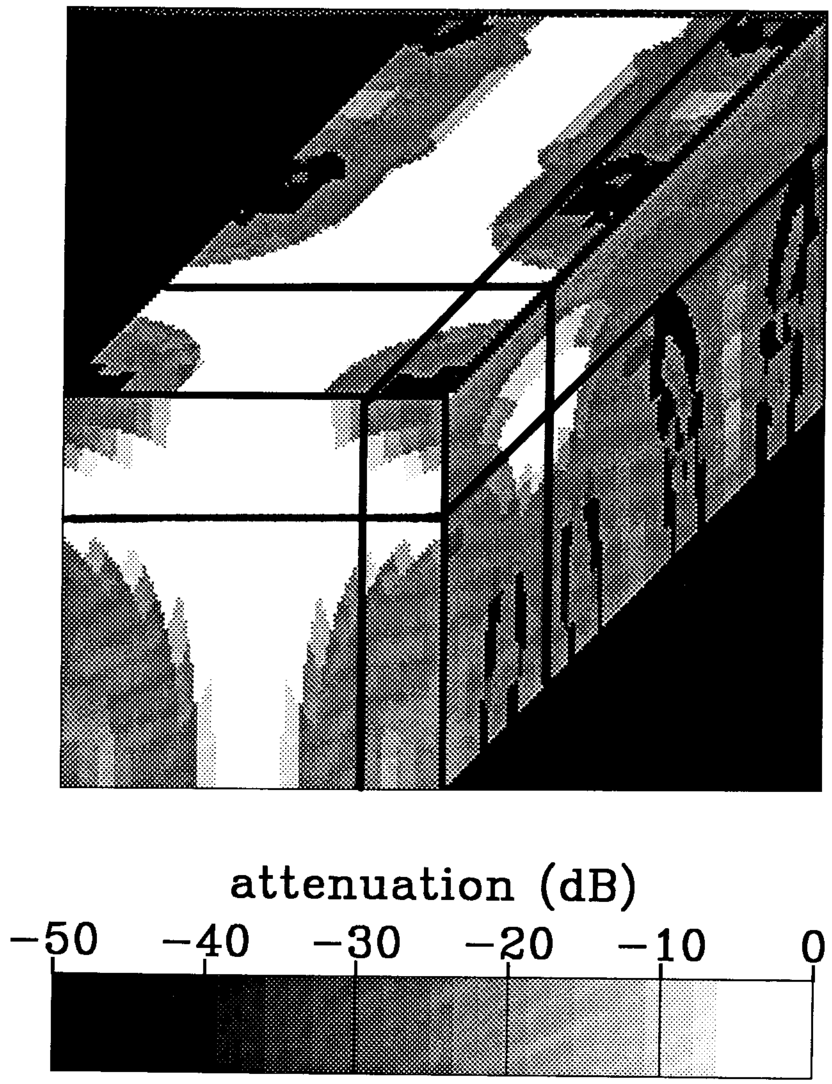

FIG. 13. Response of the cross array of Figure 7 beam steered to pass plane waves incident at 30° dip and 75° azimuth. The side panel shows attenuation as a function of dip and azimuth for a frequency of 80 Hz. Note the pass region centered at the correct dip and azimuth.

CONCLUSIONS

In this paper, a simple method for computing the frequency-dependent response of a two-dimensional array of receivers to incident plane waves has been presented. Beam steering the array, or delaying the individual channels before summation to enhance the energy arriving in a particular direction, has been formulated here

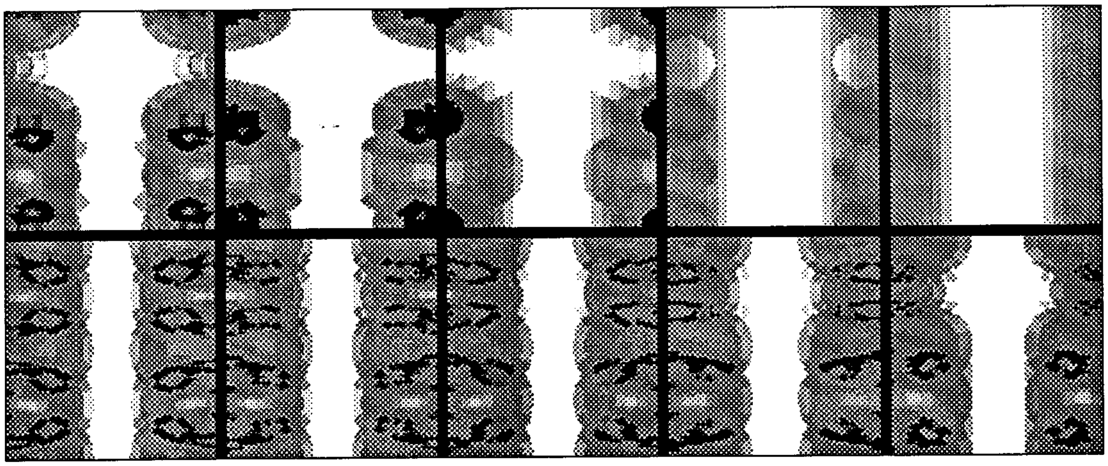

FIG. 14. Constant-dip planes of the response shown in Figure 13. Dip ranges from 0° in the upper right to 90° in the lower left. In each frame, the horizontal axis is frequency and the vertical axis is azimuth. Note the pass region that opens at 30° dip and closes again for steeper dips.

as a spatial transformation of the receiver array that generally makes it three-dimensional. This spatial transformation allows the inclusion of beam steering in the array response analysis for a small additional effort.

Software capable of displaying a three-dimensional cube of data such as the seismic movie program is an ideal means to view an array response computed using the methods described here. However, the theory presented here can be used to compute the response as a function of one or two of the three variables for presentation on conventional displays.

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RESPONSIBILITIES - September 1987

Problem	Primary Responsibility	Backup Responsibility
air conditioning	Paul	Niki
backups	Marta	Kostov
Convex	John	Stew/Nichols
*cleaning - room 467&471	COCORP	COCORP
Disco	Craig	Jill/Paul
Gigis	Pat	
Envision	Joe	Francis
ethernet	Rick/Jos	Chuck/Phil
*Imagen	Kostov	Steve
merlin	Paul	Craig
MicroVax	Rick	
modems/serial ports	Biondo	Steve
plot program maintenance	Joe/Rick	Steve
*printers	Marta	Zhang
RasterTech	Rick	
Sun	Jos/Rick	Jean Luc
SI disks	Paul	John
*tape drives	Marta	Craig
tape archives	Rick	COCORP
TeX maintenance	Niki	Jos/Joe
Seplib	Joe/Steve	Jon

* Items requiring paper, ribbons, tapes, soap suds, etc. The person responsible for this item is also responsible for notifying Kellie when supplies get low.