Improved resolution of slant stacks using beam stacks

Clement Kostov and Biondo Biondi

ABSTRACT

The separation of reflected waves by local slant stacks is adversely affected by the spatial variability and the curvature of the wavefront. The resolution of local slant stacks can be improved by replacing the straight line stacking trajectories of local slant stacks with segments of hyperbolic trajectories. In a common-midpoint (CMP) gather a unique traveltime curve is determined given a dip, an arrival time and a receiver position. The derivation of this result assumes vertically stratified velocity and planar reflectors, but makes otherwise no reference to a specific velocity function.

Knowledge of the expected traveltime curve allows an alternative stacking method, that takes into account the curvature of the wavefront. Field data examples show the improvements in resolution gained using the new stacking method in the following applications: first beamforming, that is selection of energy propagating in a specific direction, and second the design of anti-aliasing weights for a conventional global slant stack of a CMP gather.

INTRODUCTION

A beamforming transformation selects signal components propagating in a particular direction. Slant stacks are an example of beamforming — they decompose the wave field into plane waves defined by particular ray parameters. However, plane waves provide good approximations for reflected seismic signals only over limited regions of space, where both the spatial variability and the curvature of the wavefront can be neglected.

The spatial variability of the wavefront, due to the lateral changes of the Earth properties and to the finite spatial extend of the recording array, can be preserved by forming local slant stacks over windows of a few traces (Harlan 1984; Kong 1985).

The curvature of the wavefront is due to the finite spatial extend of the source and of the reflecting regions. Stacks along hyperbolic trajectories localize energy better than slant stacks. On the other hand, hyperbolic stacks sum energy scattered in all directions and thus loose the advantages of directivity.

We propose a decomposition of the wave field into beams of locally spherical waves, which approximate the curvature of the reflected waves better than plane waves. The beam is defined in the time domain for a CMP gather, under the general assumptions of a vertically stratified velocity and planar reflectors. No specific velocity function is necessary for the definition of the beams. Similarly to local slant stacks, the spatial extend of the beam is limited to a few traces, and the direction of the beam is defined by a ray parameter. Extensions of the method of beam stacking to three-dimensional data volumes are suggested.

We use the improved resolution provided by beam stacks in two applications: to enhance the energy traveling with a particular ray parameter in a CMP gather, and to reduce aliasing and truncation artifacts in a conventional slant stack. Other applications of this method for beam stacking, discussed in this report, include velocity analysis (Biondi, 1987) and signal detection (Ottolini, 1987).

BEAM STACKS AND LOCAL SLANT STACKS

Beam stacks differ from conventional local slant stacks in one respect — the data are integrated over hyperbolic curves rather than over straight lines.

Local slant stacks of a common-midpoint gather (CMP) are computed by integrating data over segments of slanted lines, defined by their slope and one point (arrival time and offset position) along the segment. The local slant stack at time t_0, offset h_0 and slope p_h of a recorded CMP gather, $Gather(t,h)$, is defined as,

$$Slant(t_0, h_0, p_h) = \int_{h_0-H}^{h_0+H} dh' \, W(h'-h_0) \, Gather\left(t_0 - p_h(h'-h_0), h'\right) , \tag{1}$$

where t is time, h is half offset, W is a windowing function and H determines the range of traces over which to integrate. The parameter p_h, that is the slope of the line over which the data is integrated, is also the ray parameter of the up-going plane wave arriving at time t_0 at the half offset h_0.

Reflected wavefronts recorded at the surface are generally curvilinear and therefore better approximated by spherical waves than by plane waves. The curvature of the reflected wavefronts reduces the resolution of both global and local conventional slant stacks.

Beam stacks decompose the data in beams of locally spherical waves instead of locally plane waves. Figure 1 shows an example of a beam of rays in a common-midpoint geometry. The rays start from a group of contiguous shots, bounce on a segment of a flat reflector and are recorded at the surface by a group of receivers. The rays in the beam have different ray paths and consequently slightly different ray parameters; still we can identify a beam by its central ray. For the beam in Figure 1, the half offset h_0, the arrival time t_0 and the ray parameter p_h of the beam are the corresponding parameters of the central ray, drawn as a wider line in the figure.

The wavefront of the beam is not planar but spherical, and the traveltime curve is not linear but curvilinear. The slope of the tangent to the traveltime curve is the ray parameter of the beam. Therefore, to select the beam with ray parameter p_h, arriving at time t_0 and half offset h_0, beam stacks integrate data over the traveltime curve with tangent of slope p_h at the point (t_0, h_0).

Following Dix's approximation for a horizontally stratified earth, the generic traveltime

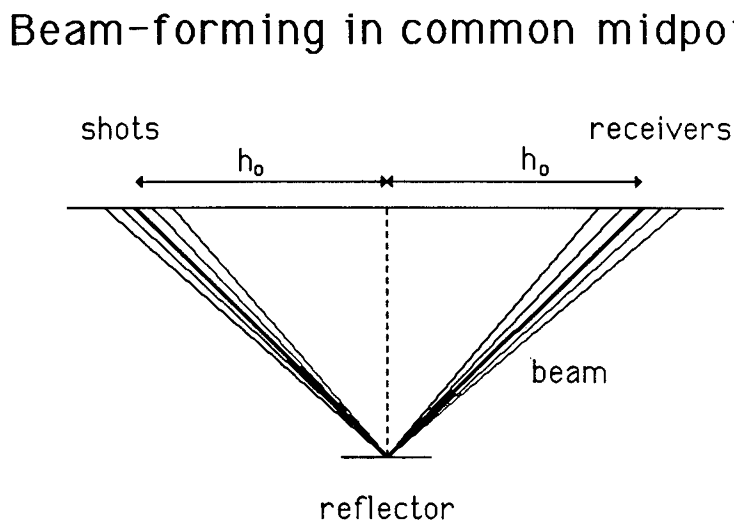

FIG. 1. A beam of rays starting from a group of shots, bouncing on a piece of reflector and recorded by a group of receivers. The recorded wave is spherical and the corresponding arrival time curve is hyperbolic. The beam is identified by the path and travel time of its central ray, drawn as a wider line in the figure.

curve in a CMP gather belongs to the following family of hyperbolas,

$$t^2 = t_a^2 + \frac{4\,h^2}{V^2}, \tag{2}$$

where the time of the apex t_a and the velocity V are free parameters. The velocity V can be a constant velocity, or it can be a RMS velocity, or even an apparent velocity in the case of dipping reflectors.

For a given arrival time t_0 and half offset h_0 related by equation (2), the direction of propagation of the energy is given by the ray parameter p_h,

$$p_h = \frac{dt}{dh} = \frac{4\,h_0}{V^2 t_0}. \tag{3}$$

Combining equations (2) and (3) we determine the velocity

$$V^2 = \frac{4\,h_0}{p_h t_0} \tag{4}$$

and the time of the apex

$$t_a^2 = t_0^2 - h_0 t_0 p_h \tag{5}$$

as a function of t_0, h_0 and p_h.

Substituting these values in the generic traveltime expression (equation 2), we find the hyperbola over which to sum the data,

$$t^2 = t_0^2 - h_0 t_0 p_h + \frac{h^2}{h_0} p_h t_0. \tag{6}$$

The beam stack at time t_0, offset h_0, and ray parameter p_h for a CMP gather $Gather(t, h)$ is thus

$$Beam_2(t_0, h_0, p_h) = \int_{h_0-H}^{h_0+H} dh' \, W(h' - h_0) \, Gather\left(\sqrt{t_0^2 - h_0 t_0 p_h + \frac{h^2}{h_0} p_h t_0} \, , \, h'\right) . \quad (7)$$

The integration curve defined in equation (6) is derived assuming hyperbolic traveltime curves. When the moveout is non-hyperbolic, beam stacks could still bring an improvement over conventional local slant stacks, because the effect of velocity variations would be of a higher order (i.e. moveout curves are closer to hyperbolas than to straight lines).

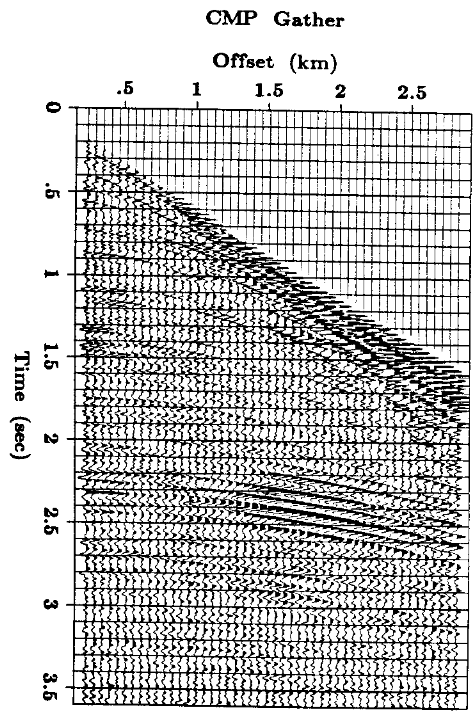

FIG. 2. CMP gather from the Gulf of Mexico. There are 48 traces in the gather and the nearest 4 offsets are missing.

The computation of the integral in equation (7) requires a time-variant transformation. Therefore the cost of beam stacking is higher than the cost of local slant stacking. On the other hand, the following examples show the significant improvement in resolution obtained by using beam stacks instead of slant stacks.

The CMP gather used in the comparison between local slant stacks and beam stacks, is displayed in Figure 2. There are 48 traces in the gather, and the 4 nearest offsets are missing. Figure 3a displays the result of beamforming, obtained by using local slant stacks as defined in equation (1) for a constant value of the ray parameter. Figure 3b shows the result of beam stacking as defined in equation (4). Beam stacks provide sharper results than slant stacks: not only is the amplitude of the stack higher at the correct arrival time and offset, but it also is lower where it should be zero. The two results differ most at small offset where the traveltime curves are most distinct from straight lines.

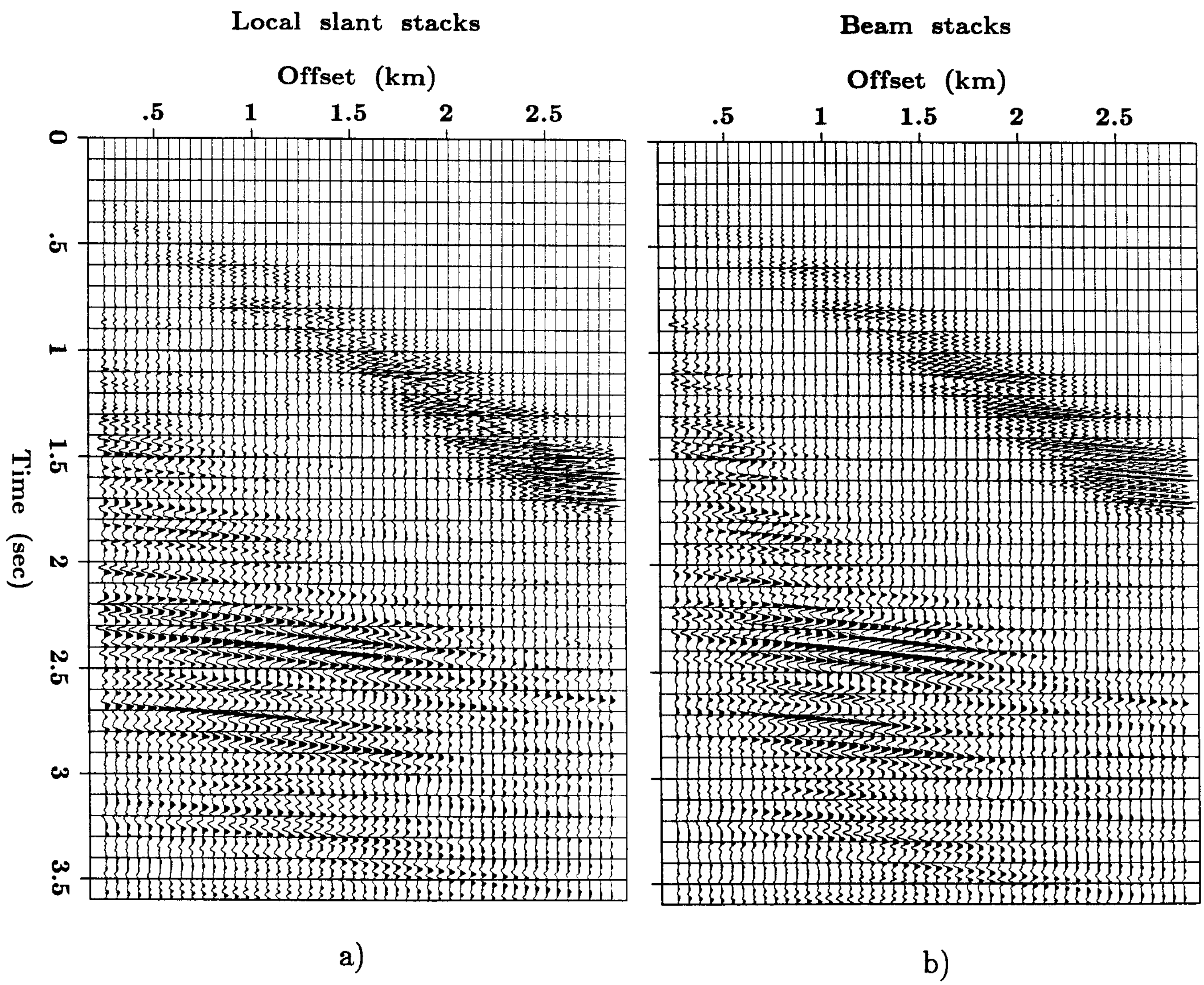

FIG. 3. a) Decomposition by local slant stacks for a fixed ray parameter p_h, of the CMP gather shown in Figure 2.
b) Decomposition using beam stacks for a fixed ray parameter p_h of the CMP gather shown in Figure 2. The result is sharper than the result obtained using local slant stacks, shown in Figure 3a. The differences are largest at small offsets where the travel time curves are most distinct from straight lines.

Three-dimensional beam stacks

A seismic dataset is usually composed of several common-midpoint gathers therefore it is desirable to generalize beam stacks of two-dimensional datasets to beam stacks of three-dimensional datasets (three-dimensional beam stacks). Three-dimensional beam stacks integrate data over hyperbolic cylinders instead over hyperbolic curves and can be computed by local integration, along the midpoint direction, of two-dimensional beam stacks. Assuming some continuity in the reflectors, it is appropriate to use straight lines as integration curves along the midpoint axis. Therefore the three-dimensional beam stack at time t_0, midpoint y_0, half offset h_0, ray parameters p_h and p_y, is given by the integral

$$Beam_3(t_0, h_0, y_0, p_h, p_y) = \int_{y_0-Y}^{y_0+Y} dy' \, W(y' - y_0) \, Beam_2 \left(t_0 - p_y(y' - y_0), h_0, y', p_h\right), \quad (8)$$

where Y determines the range of midpoints over which to integrate and $Beam_2$ is defined in equation (7).

For some applications, such as velocity analysis (Biondi, 1987), it can be convenient to have a beam stacks decomposition of the dataset in shot and receiver coordinates instead of midpoint and half-offset coordinates. The transformations of spatial variables from midpoint y and half-offset h coordinates to shot s and receiver r coordinates are

$$s = y - h \qquad \text{and} \qquad r = y + h. \quad (9)$$

The ray parameters are equal to the partial derivatives of travel time with respect to the spatial coordinates; therefore the following relations between ray parameters can be directly derived from the transformations in equation (9)

$$p_s = \frac{(p_y - p_h)}{2} \qquad \text{and} \qquad p_r = \frac{(p_y + p_h)}{2}. \quad (10)$$

It is thus possible to transform beam stacks computed in midpoint and half offset coordinates to beam stacks defined in shot and receiver coordinates using the transformations expressed in equations (9) and (10).

REDUCTION OF ALIASING AND TRUNCATION ARTIFACTS

Stacking along a slanted line is a filtering operation designed to select plane waves arriving at the surface with a given ray parameter p_h and to attenuate signals traveling in different directions. Aliasing occurs when signals propagating in directions significantly different from p_h are not sufficiently attenuated.

Several different methods for reducing aliasing and truncation artifacts in slant stacks (Schultz and Claerbout 1978, Stoffa and al. 1982, Noponen and Keeney 1986, Brysk and McCowan 1986) use knowledge about the expected signal and design window functions applied either before or after stacking of the data.

To enhance signal in the slant stacks, we determine weights depending on the ray parameter, the offset and the arrival time, and apply these weights to the data before stacking. The weights are obtained by computing the semblance along a portion of the hyperbolic traveltime curve defined in equation 6. Our method is closely related to that of Schultz and

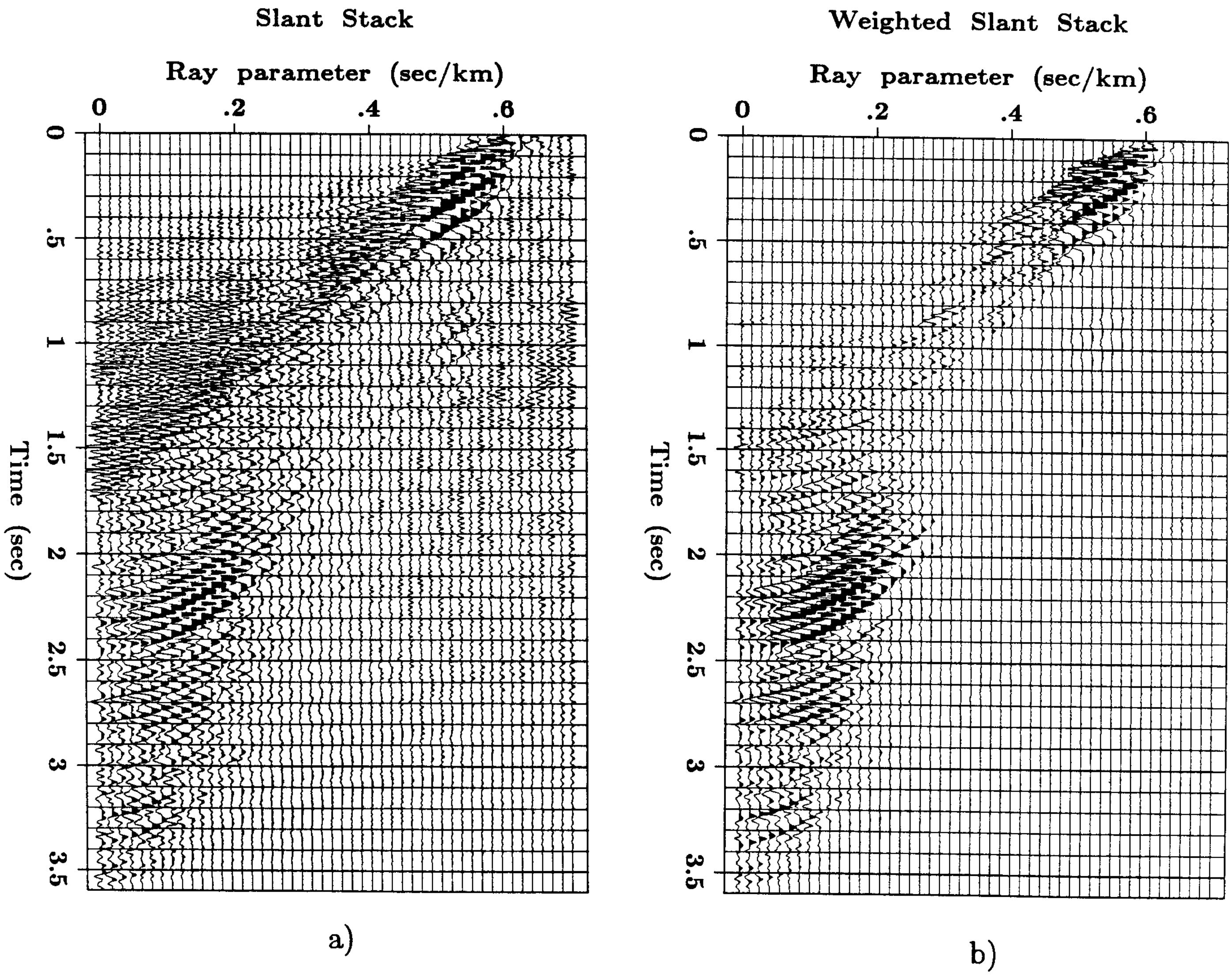

FIG. 4. a) Conventional slant stack of the CMP gather shown in Figure 2. Aliased energy from the strong events with ray parameters about .58 sec/km has obscured signals in a wedge shaped region. Artifacts from the truncation of the offset axis appear as straight lines across all offsets.
b) Weighted slant stack of the CMP gather shown in Figure 2. For each ray parameter, the data are multiplied by semblance weights before slant stacking. Both aliasing and truncation artifacts are reduced. The amplitudes for small ray parameters are lower than the corresponding amplitudes in the conventional slant stack (Figure 4b), indicating an increased resolution, since events with horizontal moveout are missing from the original data.

Claerbout (1978), with one significant difference however — we don't need a velocity model to compute the weights.

Figure 4a shows the conventional slant stack of the data displayed in Figure 2. Aliasing from the strong event at ray parameter about .58 obscures the signal in a wedge shaped region, up to times of 1.7 seconds. Artifacts from the truncation at near and at far offset appear as straight lines extending across all ray parameters.

Figure 4b shows the slant stack obtained using weight functions derived by computing semblance along hyperbolic trajectories. The aliasing is eliminated and signal at early times and low ray parameters appears clearly. Truncation artifacts are also strongly reduced, since the response of each sample in the transform domain is localized to some ray parameters determined by the semblance measure. The amplitude for low values of the ray parameter is lower in the weighted slant stack (Figure 4b) than in the conventional slant stack (Figure 4a). This effect shows the increased resolution of the weighted slant stack, since there is no energy with horizontal stepout in the original gather (Figure 2).

CONCLUSION

Given a direction of wave propagation, we have determined an expected traveltime trajectory for the reflected signals which is independent from the knowledge of velocity. The two applications presented — first a method for beam stacking in the time domain, and second the design of velocity independent weights to be applied before conventional slant stacks — suggest ways of improving both local and global conventional slant stacks.

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