Water bottom multiple deconvolution

Kamal Al-Yahya and *Peter Mora*

INTRODUCTION

Multiples from a hard flat sea floor can be suppressed in a deterministic manner provided two parameters are know, namely the travel time through the water layer (filter lag) and the reflectivity of the water bottom (filter coefficient). The filter lag can be obtained by measuring the time between two consecutive water-bottom reflections while the reflectivity can be measured by taking the relative amplitudes of these reflections. Single measurements of this type are not very reliable so it is better to use an automated procedure that can utilize the redundancy in the data set to obtain these parameters.

One way to approach the problem is to determine the optimal (in the least squares sense) set of parameters in the two-dimensional space of filter lag and filter coefficient (see Figure 1). One (very inefficient) way to achieve this is by using the Monte Carlo method to search the entire space for the filter parameters that yield the best deconvolution (i.e. least reverberation). The problem is greatly simplified if the data is gained to remove the transmission and reflection losses so the filter coefficient can (at least approximately) be ignored. Subsequently, a one dimensional search for the best filter lag can be performed thereby reaching point A in Figure 2(a). The search is made easier by starting with an approximate lag picked from the gather. This method does not give good results and significant reverberations remain after the deconvolution.

However, the addition of an automated step to determine the best possible reflection coefficient removes almost all the remaining reverberations.

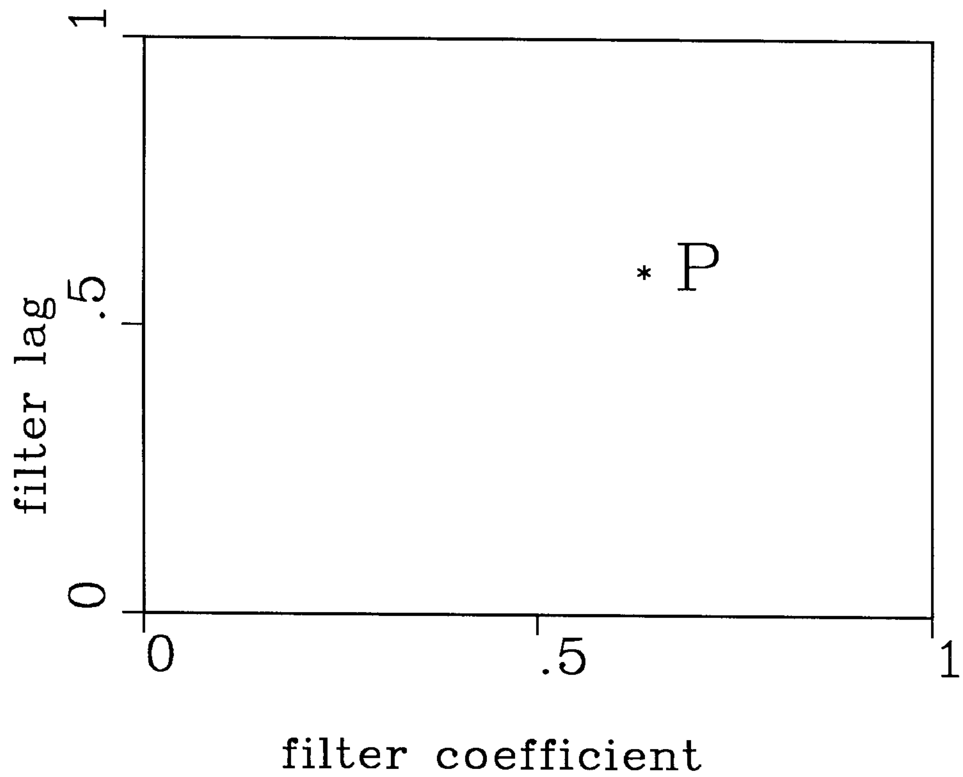


FIG. 1. A representation of the problem as a search in a two-dimensional space; the solution is point P. Scale is for illustration only and is not significant.

OUTLINE OF SCHEME

In this paper we present an efficient method for the automatic determination of the filter coefficient by using an iterative least squares descent method. The descent method is fast because the problem is close to being linear and so convergence is achieved in two or three iterations. Prior to this step, the filter lag must be determined using the technique outlined above. Specifically, the data is gained to equalize the strength of the water bottom multiple reflections so a filter coefficient of about 1 may be used in some preliminary deconvolutions. Several lags centered about the approximate reverberation period picked from the data are used and the optimal value is chosen that minimizes the energy in the deconvolved data (and hence the reverberations). We have reached point A in Figure 2(a) at this stage.

Although it would be possible to stop here, it has been found that the deconvolution is not optimal because of the inaccurate approximation that after gaining the data the filter coefficient is about 1. Therefore, the method we present here goes one step further by fixing the lag and determining the optimal reflection coefficient stopping at point B in Figure 2(b). Starting at a reflection coefficient of zero is not necessary.

THE DECONVOLUTION FILTER

The water-bottom is assumed to be flat and horizontal with reflection coefficient r. The source and receivers are assumed to be located at the water surface which is assumed to have a reflection coefficient of -1. The two-way travel time is m time units. The deconvolution filter in the Z-domain for the receiver is then

$$G(Z) = 1 + rZ^m \quad ,$$

Robinson and Treitel (1980). To remove reverberation at both the source and receiver, we must use the square of this filter, since the sea bottom is assumed to be horizontal,

$$G(Z) = (1 + rZ^m)^2 = 1 + 2rZ^m + r^2 Z^{2m} \quad .$$

In the time domain, the input-output relation becomes

$$output(t) = input(t) + 2r * input(t - \tau) + r^2 * input(t - 2\tau)$$

DETERMINING THE REFLECTION COEFFICIENT

Least squares optimization requires the minimization of the sum of the squared error. In the case of deconvolution for water-bottom multiples, we can consider the error to be the deconvolved data:

$$e_n(x,t) = data(x,t) + 2r_n data(x, t - \tau) + r_n^2 data(x, t - 2\tau) \tag{1}$$

where τ is the two-way travel time to the water bottom. The reason this works is that our water layer reverberation filter has energy that is greater than unity so the reverberations are minimized when there is the least energy in the deconvolved data.

Our goal is to compute the error (the deconvolved data) e_{n+1} at the $(n+1)$-th iteration from the error $\mathbf{e}_n$ at the n-th iteration. Linearizing the problem means truncating the Taylor series expansion after the first term:

$$\mathbf{e}_{n+1} = \mathbf{e}_n + \frac{\partial \mathbf{e}_n}{\partial r_n} (r_{n+1} - r_n) \tag{2}$$

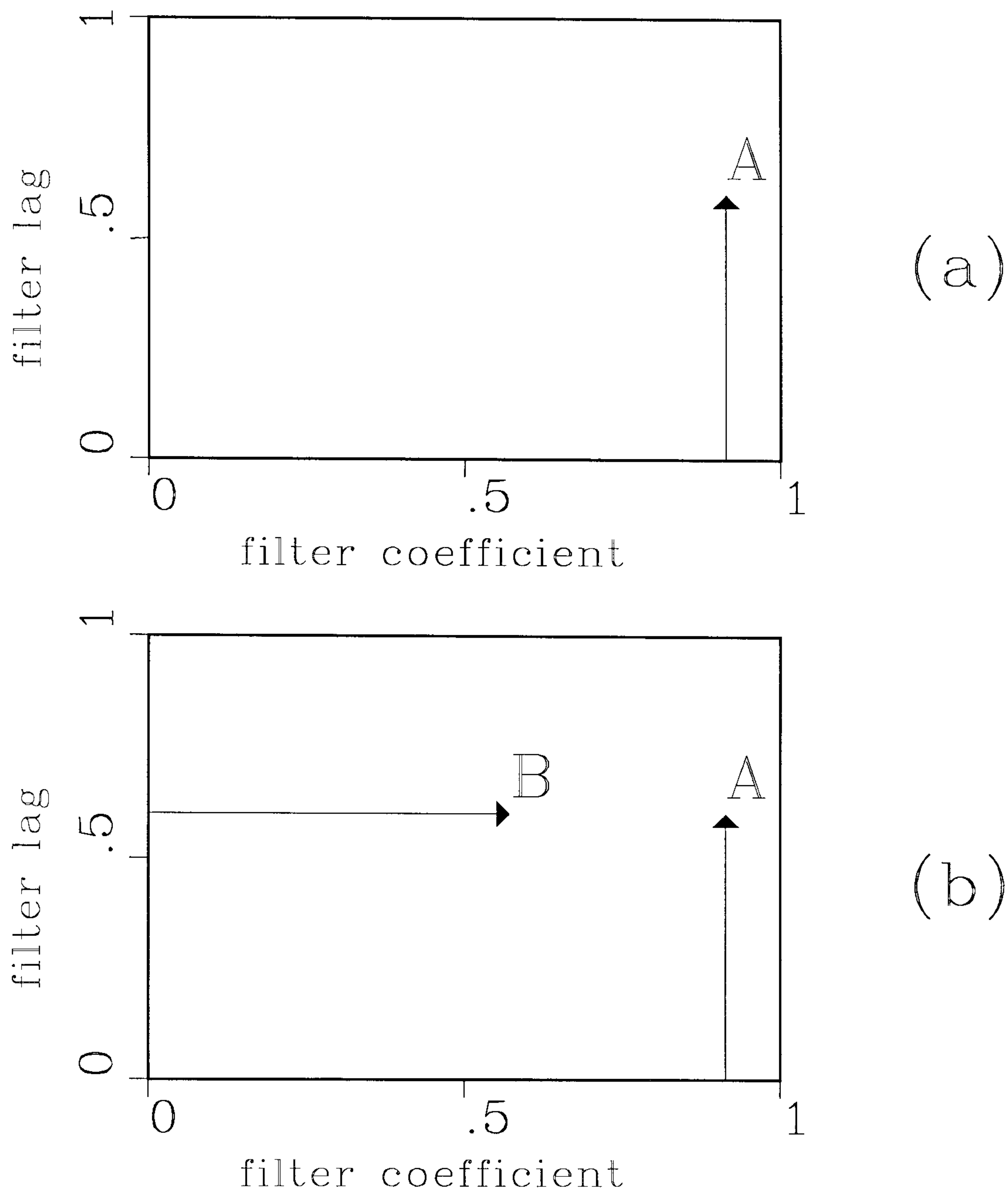


FIG. 2. (a) Fixing the reflection coefficient at a value close to 1 and then searching vertically till point A is reached. (b) Using the filter lag determined by A, a search for the filter coefficient is done, reaching point B. Scale is for illustration only and is not significant.

Actually, seismic data is a function of both x and t but this can still be arranged into one column vector by putting one trace on top of another and so on.

To solve for the least squares value of r, we minimize E,

$$\frac{\partial E_{n+1}}{\partial r_{n+1}} = 0 \tag{3}$$

where

$$E_{n+1} = \mathbf{e}_{n+1}^T \mathbf{e}_{n+1} = \sum_x \sum_t \left(\mathbf{e}_{n+1}(x,t)\right)^2 \tag{4}$$

Gain

If we gain the error (i.e. deconvolved data) by the function t^γ, then equation (1) can be written as

$$e_n(x,t) = t^\gamma \left[data(x,t) + 2r_n data(x,t-\tau) + r_n^2 data(x,t-2\tau) \right] \tag{5}$$

The gain parameter γ weights the errors. For example, if γ were greater than zero, the error would tend to be larger at later times so the least squares algorithm would tend to pay more heed to decreasing error at the later times (i.e. removing reverberations at later times).

From equations (3) and (4),

$$\frac{\partial \mathbf{e}_{n+1}^T}{\partial r_{n+1}} \mathbf{e}_{n+1} = 0 \tag{6}$$

Substituting from equation (2) into equation (6), we get

$$\frac{\partial \mathbf{e}_n^T}{\partial r_n} \mathbf{e}_{n+1} = 0$$

$$\frac{\partial \mathbf{e}_n^T}{\partial r_n} \left(\mathbf{e}_n + \frac{\partial \mathbf{e}_n}{\partial r_n} (r_{n+1} - r_n) \right) = 0 \quad ,$$

from which we can obtain this iterative equation for the reflection coefficient,

$$r_{n+1} = r_n - \left(\frac{\partial \mathbf{e}_n^T}{\partial r_n} \frac{\partial \mathbf{e}_n}{\partial r_n} \right)^{-1} \frac{\partial \mathbf{e}_n^T}{\partial r_n} \mathbf{e}_n ,$$

where all partial derivatives are obtained from equation (5).

EXAMPLE

We demonstrate the proposed method with field data. Figure 3 shows a marine shot gather showing clear water bottom multiples. The gather has NMO applied to flatten reflections in order that the reverberation period be constant for the entire data set. The lag between water-bottom reflections is about .12 sec. Figure 4 shows the square error as a function of lag when the reflection coefficient is fixed at .8. The figure shows a clear minimum at .128 sec, which we take as the filter lag. We are now at point A of Figure 2(a). Figure 5 shows the reflection coefficient as a function of iteration using the descent of equation (7), where the reflection coefficient converges to .34. We are now at point B in Figure 2(b). The filter lag has been fixed to be .128 sec. The deconvolved gather corresponding to the last iteration is shown in Figure 6, where most of the peg-legs have disappeared. The solution corresponding to point A in Figure 2(a) is shown in Figure 7, which is inferior to Figure 6 because the gain in the initial step did not compensate exactly for the water bottom reflection coefficient.

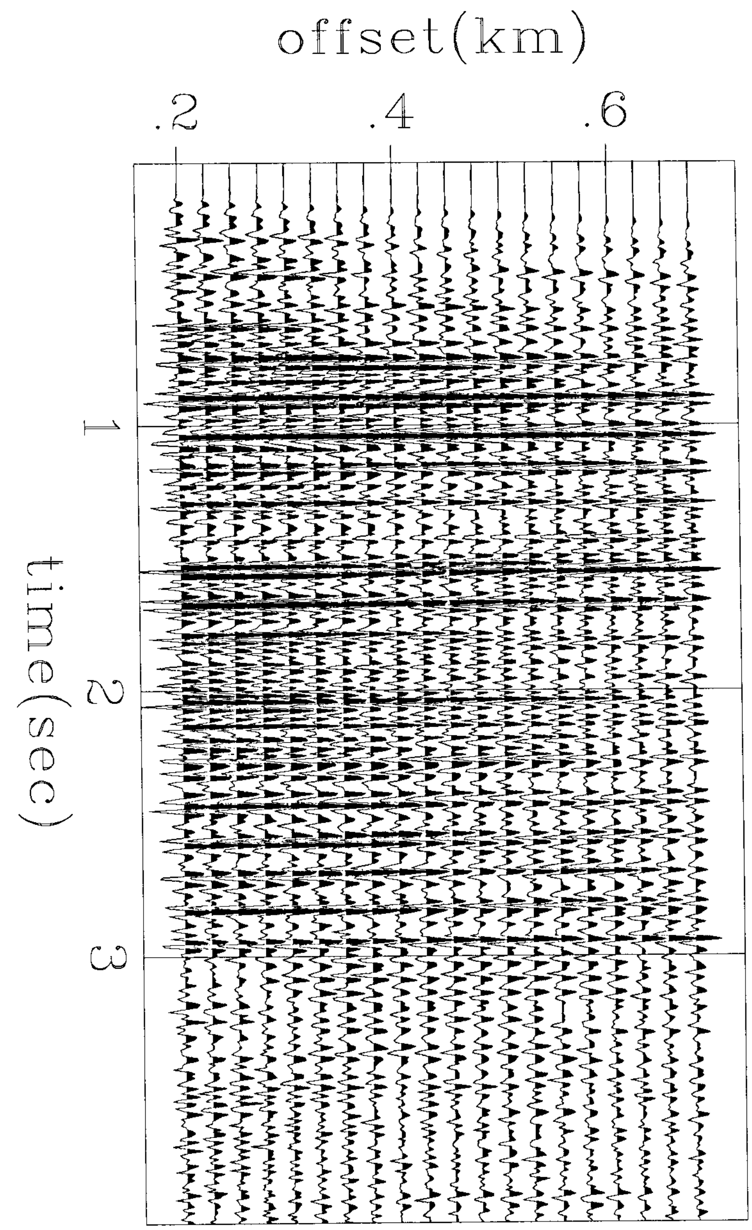


FIG. 3. Marine shot gather. NMO has been applied with constant water velocity to align reflections.

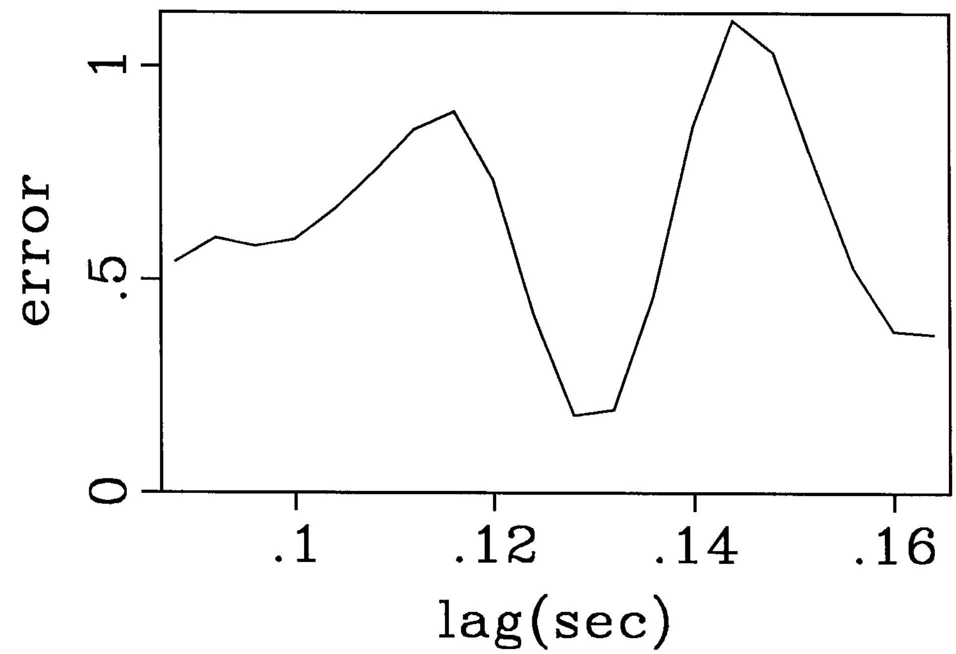


FIG. 4. Square error as a function of lag for the deconvolution of the data in Figure 3. The data was first time-gained and a reflection coefficient of .8 was used.

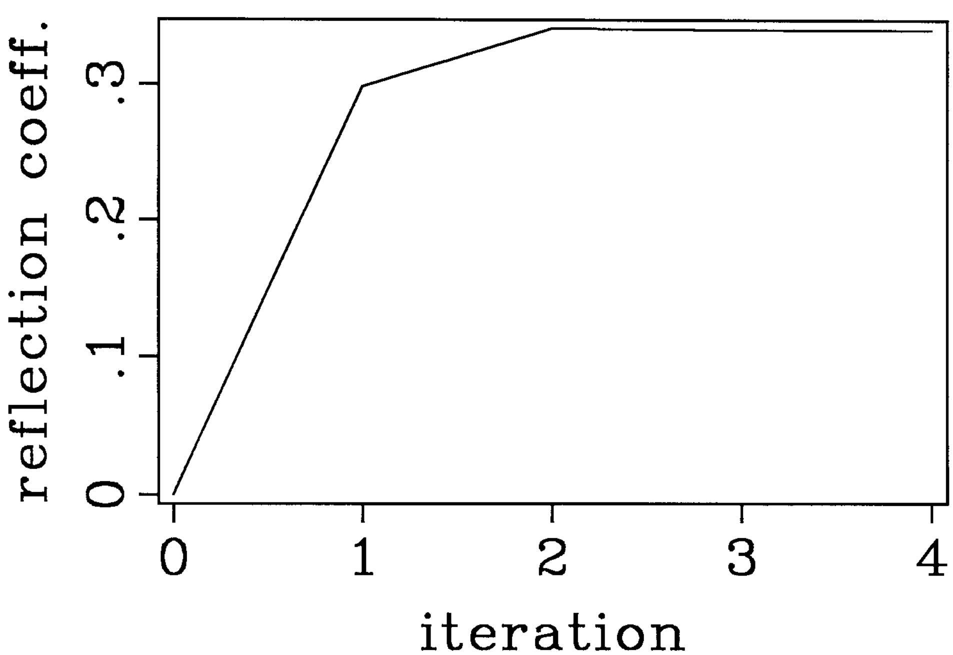


FIG. 5. Reflection coefficient as a function of iteration for the data in Figure 3. The filter lag was determined from Figure 4 to be .128 sec. Notice the rapid rate of convergence.

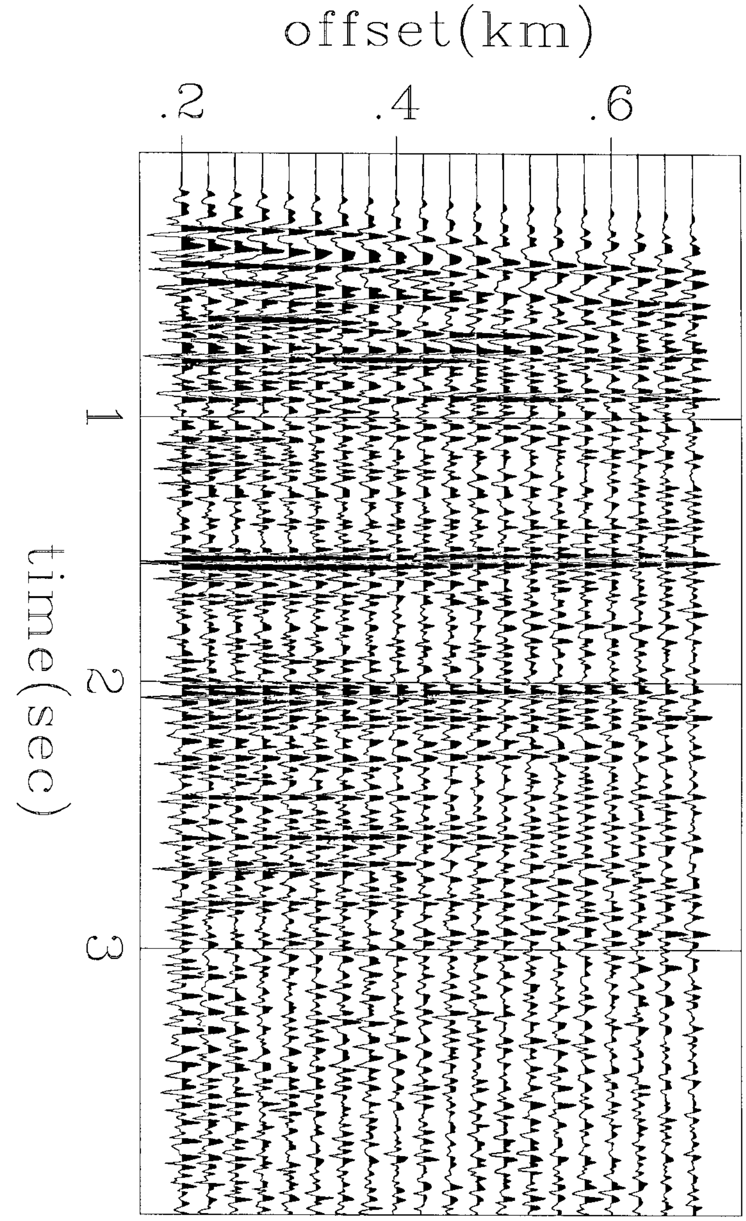


FIG. 6. The result of deconvolving the data in Figure 3 using a lag of .128 sec (from Figure 4) and a reflection coefficient of .34 (from Figure 5).

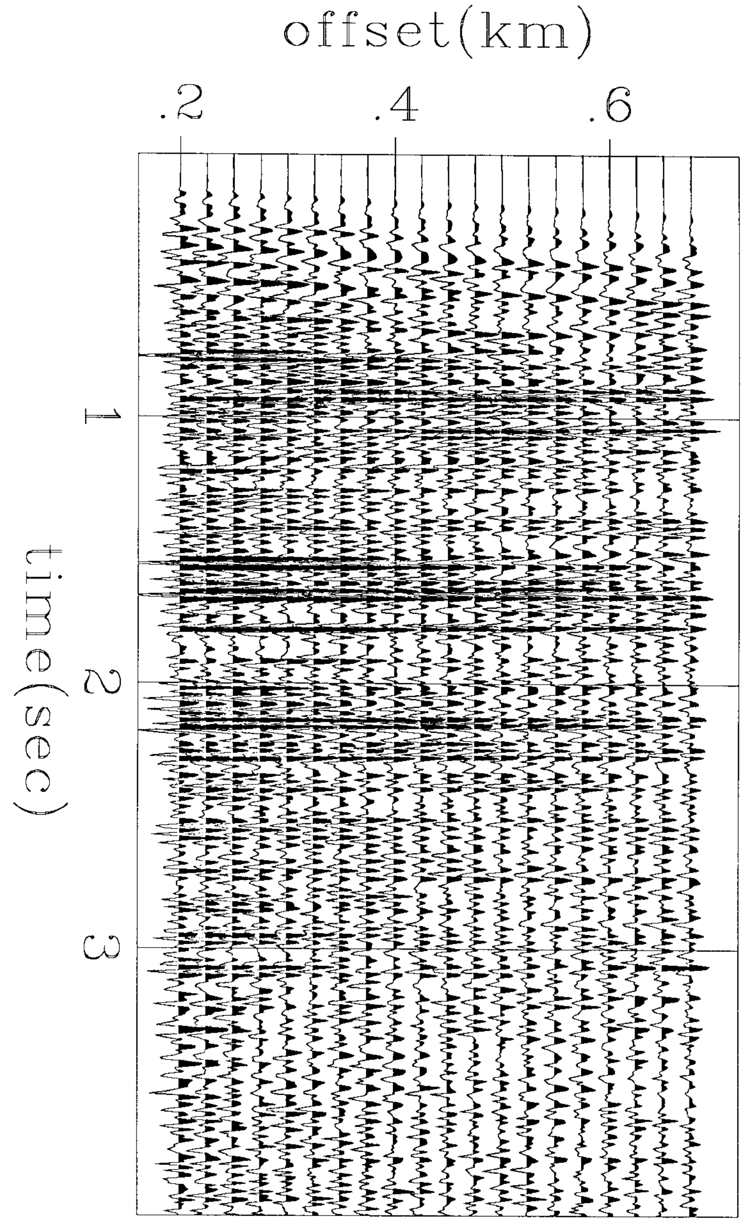


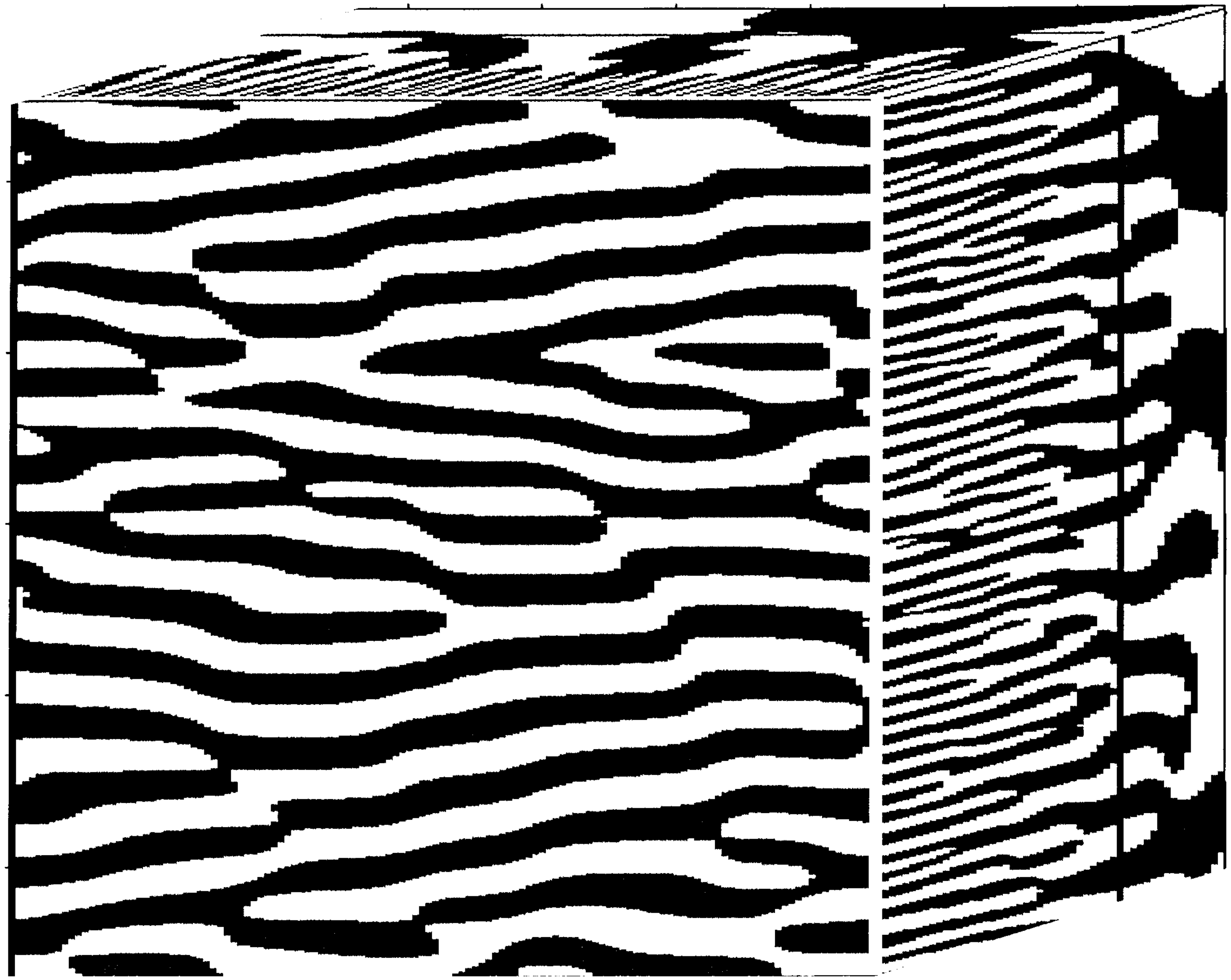
FIG. 7. The result of deconvolving the data in Figure 3 using a lag of .128 sec (from Figure 4) and a reflection coefficient of .8 after gain to compensate for transmission and reflection losses.

CONCLUSIONS

The scheme we presented was successful in suppressing water bottom multiples. Its cost is very small so its routine use is feasible. The only non-automatic portion of the scheme is the picking step to determine the approximate reverberation period. This could be easily automated if desired at the expense of some of the efficiency. As a by-product, the algorithm estimates the reflection coefficient of the sea-floor which may possibly be useful.

REFERENCES

Robinson, E.A. and Treitel, S., 1980, Geophysical signal analysis, Princeton-Hall, Inc., p. 24.



Scale-space extremum map of seismic image for a paper appearing in the next SEP report. -R.O.