Short Note: PEF swapping positive with negative lags

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ABSTRACT

Jon Claerbout recently hypothesized that inserting negative filter lags opposite a gap in positive filter lags could produce interesting and useful filters. Here I prove that, indeed, such filters lead to controlled autocorrelation width and may provide properties beyond those of conventional gapped deconvolution.

INTRODUCTION

In SEP 150, Claerbout and Guitton (Claerbout and Guitton, 2013) found that adding a small number of anti-causal filter lags via the cepstral domain produced a pleasing deconvolution result. As Jon pointed out in PVI (Claerbout, 2004), simply adding negative lags to a time-domain PEF in order to obtain an interpolation filter is not generally a good idea as, in the limit, it produces an output which has a spectrum inverse to the input rather than white. This summer Jon speculated that moving a few positive (causal) lags to their negative (anti-causal) counterpart locations could perhaps produce nice looking PEF output, that is, output that does not arbitrarily boost all frequencies to and beyond whiteness, irrespective of their level of noise. I took up that challenge and calculated what swapping just the first lag would produce in the PEF's autocorrelation.

THEORY

Following the PEF whiteness proof of Levin et al. (2013), I minimize $||\mathbf{r}||_2^2$ by adjusting filter coefficients a_m in the residual

$$\mathbf{r} = a_{-1}Z^{-1}\mathbf{d} + \mathbf{d} + 0 + a_2 Z^2\mathbf{d} + a_3 Z^3\mathbf{d} + \ldots \tag{1}$$

so that

$$\begin{aligned} 0 &= \frac{1}{2}\frac{d}{d\,a_m}(\mathbf{r}\cdot\mathbf{r}) \\ &= \mathbf{r}\cdot\frac{d\,\mathbf{r}}{d\,a_m} = \mathbf{r}\cdot Z^m\mathbf{d}\ . \end{aligned}$$

Let us now examine for an integer k the autocorrelation term $\mathbf{r} \cdot Z^k \mathbf{r}$.

$$
\begin{aligned}
k = 0 : \mathbf{r} \cdot Z^0 \mathbf{r} &= \cancel{a_{-1} \mathbf{r} \cdot Z^{-1} \mathbf{d}} + \mathbf{r} \cdot \mathbf{d} + \cancel{a_2 \mathbf{r} \cdot Z^2 \mathbf{d}} + \dots \\
k = 1 : \mathbf{r} \cdot Z^1 \mathbf{r} &= a_{-1} \mathbf{r} \cdot \mathbf{d} + \mathbf{r} \cdot Z \mathbf{d} + \cancel{a_2 \mathbf{r} \cdot Z^3 \mathbf{d}} + \dots \\
k = 2 : \mathbf{r} \cdot Z^2 \mathbf{r} &= a_{-1} \mathbf{r} \cdot Z \mathbf{d} + \cancel{\mathbf{r} \cdot Z^2 \mathbf{d}} + \cancel{a_2 \mathbf{r} \cdot Z^4 \mathbf{d}} + \dots \\
k > 2 : \mathbf{r} \cdot Z^k \mathbf{r} &= \cancel{a_{-1} \mathbf{r} \cdot Z^{k-1} \mathbf{d}} + \cancel{\mathbf{r} \cdot Z^k \mathbf{d}} + \cancel{a_2 \mathbf{r} \cdot Z^{k+2} \mathbf{d}} + \dots
\end{aligned}
$$

which says that the autocorrelation vanishes outside the first couple of lags. By comparison, a singly gapped causal prediction error filter has one nonzero autocorrelation lag. I leave to the reader to see that swapping the first n PEF coefficients leads to $2n$ nonzero autocorrelation lags.

DISCUSSION

As noted in the previously cited work, these calculations allow for an infinitely long prediction error filter and do not guarantee comparable behavior for finite length filters but should yield reasonable filter behavior for seismic data. Whether and when this approach might be superior to simple causal gapped prediction error filtering will be explored in future work. We do know that conventional gapped deconvolution reduces boosting of high frequency noise. The Claerbout and Guitton paper seems to suggest it will produce more symmetrical output as well and so could be an alternative to, e.g., phase-only Q compensation.

REFERENCES

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