Implementing Wave-Equation Migration Velocity Analysis Within Linearized Waveform Inversion with Velocity Updating: Considerations and Challenges

Alejandro Cabrales-Vargas

ABSTRACT

Wave-Equation Migration Velocity Analysis is one of the fundamental processes for performing Linearized Waveform Inversion with Velocity Updating, and also the most computationally intense. We recently proposed the implementation of the former by means of employing Random Boundary Conditions for storage alleviation, at the cost of performing extra wavefield propagations. We show the result of this implementation. However, the scattered source wavefield and the scattered receiver wavefield depend on the direction of propagation of the corresponding wavefields that originate them. Therefore, the source wavefield must propagate forward in time when scattering. Likewise, the receiver wavefield must propagate backward in time when scattering. This restriction leads to the fact that we require twelve propagations per iteration plus one, instead of the eight iterations plus one that we had initially expected. Additionally, Random Boundary Conditions can introduce random noise that could potentially harm the inversion result if they are not properly implemented.

INTRODUCTION

Some colleagues and I recently proposed Linearized Waveform Inversion with Velocity Updating (LWIVU) (Cabrales-Vargas et al., 2016a,b, 2017) as a new inversion technique aimed at improving the subsurface reflectivity, which allows the subsurface velocity model to vary. Such variability is not intended to correct reflector positioning, but to improve amplitudes affected by the accumulated effect of inaccuracies in velocity or slowness, therefore yielding more confindence in the estimation of the subsurface reflectivity. Cabrales-Vargas et al. (2017) discuss some aspects of the LWIVU processing components, such as the Gauss-Newton Hessian construction by means of point-spread functions. Gauss-Newton Hessian can be precomputed, stored, and applied "on the fly," interpolating as needed. On the contrary, Wave-Equation Migration Velocity Analysis (WEMVA) (Biondi and Sava, 1999; Biondi, 2006) has to be performed twice at each iteration during the inversion. Cabrales-Vargas et al. (2017) propose the use of Random Boundary Conditions (RBC) (Clapp, 2009) in

WEMVA to prevent saving wavefields in disk and the corresponding I/O access. In Reverse-Time Migration (RTM), the price of using such RBC is an extra propagation of the source wavefield. On the same grounds, the estimated number of wavefield propagations within LWIVU was four for each WEMVA step (Cabrales-Vargas et al., 2016a,b, 2017).

However, I have found that a single application of WEMVA demands seven propagations when implemented with RBC, not five (counting an initial propagation of the source wavefield). It signifies twelve propagations per LWIVU iteration instead of eight (WEMVA is performed twice per iteration.) In the first section of this report I discuss the reason for the additional propagations. Next, I implement the WEMVA operator in a simple two-layer model to verify the effects of the RBC, and an alternative implementation using Energy Imaging Conditions (EIC) (Rocha et al., 2016).

WAVE-EQUATION MIGRATION VELOCITY ANALYSIS WITH RANDOM BOUNDARY CONDITIONS

The WEMVA process represents a linear operator that maps perturbations in the slowness squared field into perturbations in the migrated image. If the process is performed using zero subsurface offset, the WEMVA operator is self adjoint; thus, the same operator retrieves a perturbation in slowness squared from a perturbation in the image.

The WEMVA process can be split into the following steps:

- Forward propagation of the source wavefield in background slowness field
- Backward propagation of the receiver wavefield in background slowness field
- Scattering of the source wavefield upon the perturbation in the image or in the background model
- Scattering of the receiver wavefield upon the perturbation in the image or in the background model
- Zero-lag time cross-correlation of the source wavefield and scattered receiver wavefield
- Zero-lag time cross-correlation of the receiver wavefield and scattered source wavefield

Let us assume that we can store disk the propagated wavefields. We can execute the WEMVA process as shown in Algoritm 1. This procedure demands four propagations (indicated in italics): two propagations in the background model, and two propagations after scattering. Notice that only the source and the receiver wavefields need to be stored, not the scattered wavefields.

Algorithm 1 WEMVA implementation saving both source and receiver wavefields

- Forward *propagate* the source wavefield and store; then, scatter upon perturbation and forward *propagate* the scattered source wavefield.
- Backward *propagate* the receiver wavefield and store; then, scatter upon perturbation and backward *propagate* the scattered receiver wavefield.
- Perform cross-correlations.

Now let us assume that we can only store one wavefield. We begin with the source wavefield for simplicity. In this case we can proceed as indicated in Algorithm 2. Notice that now we need to perform two extra propagations compared to the previous case. This is the price the we must pay for preventing the storage of more than one wavefield at a time.

Algorithm 2 WEMVA implementation storing one wavefield at a time

- Forward *propagate* the source wavefield and store it.
- Backward *propagate* the receiver wavefield and scatter upon perturbation; then, backward *propagate* the scattered receiver wavefield "on the fly."
- Crosscorrelate the scattered receiver wavefield with the stored source wavefield as the former is propagated.
- Delete the source wavefield.
- Backward *propagate* the receiver wavefield and store it.
- Forward *propagate* the source wavefield and scatter upon perturbation; then, forward *propagate* the scattered source wavefield "on the fly."
- Crosscorrelate the scattered source wavefield with the stored receiver wavefield as the former is propagated.

We can go further and prevent the storage of the propagated wavefields whatsoever using RBC to ensure the reversibility of propagations, similar to the RTM case (Clapp, 2009). In such a case, we proceed as indicated in Algorithm 3. Notice that by using RBC we have to pay the price of performing an extra propagation with respect to storing one wavefield, or three with respect to storing both wavefields. This is the implementation that I will employ for the WEMVA step in LWIVU. In the last reports (Cabrales-Vargas et al., 2016a,b, 2017) we had estimated fewer propagations because we had assumed that propagation of the scattered wavefields was independent of the time direction. In other words, we (**incorrectly**) reasoned as indicated in

Algorithm 3 WEMVA implementation storing none of the wavefields (using RBC)

- Forward *propagate* the source wavefield; then save the last two time frames.
- Backward *propagate* the receiver wavefield and scatter upon perturbation; then, backward *propagate* the scattered receiver wavefield "on the fly."
- At the same time, backward *repropagate* the source wavefield and cross-correlate with the scattered receiver wavefield. Save the last two frames of the receiver wavefield.
- Forward *propagate* the source wavefield and scatter upon perturbation; then, forward *propagate* the scattered source wavefield "on the fly."
- At the same time, forward *re-propagate* the receiver wavefield and cross-correlate with the scattered source wavefield.

Algorithm 4.

Algorithm 4 Wrong implementation

- Forward *propagate* the source wavefield; then, save the last two time frames.
- Backward *propagate* the receiver wavefield and scatter upon perturbation; then, backward *propagate* the scattered receiver wavefield "on the fly."
- At the same time, backward *repropagate* the source wavefield and scatter upon perturbation; then, backward propagate the scattered source wavefield. Cross-correlate corresponding wavefields as they are propagated backwards in time.

In summary, considering that the LWIVU process demands two WEMVA implementations (one for the gradient in model space and the other for its projection onto the data space), we obtain the following number of propagations for each case:

- Storing both wavefields: Two propagations per iteration + two initial propagations of the source and the receiver wavefields.
- Storing one wavefield: Eight propagations per iteration + one initial propagation of the source wavefield.
- Storing none of the wavefields (using RBC): Twelve propagations per iteration + one initial propagation of the source wavefield.

The significant increase in the number of wavefield propagations when avoiding their storage is because storing prevents their recomputation throughout the process.

Only the scattered wavefields are recomputed because the perturbations change as iterations progress. On the contrary, storing none of the wavefields demands recomputation of all wavefields as needed.

SYNTHETIC EXAMPLES

In this section I show the application of the WEMVA operator and the adjoint using a simple two-layer model with a Gaussian positive-velocity anomaly (Figure 1). I first isolate the anomaly to obtain the corresponding perturbation in background slowness squared (Figure 2), which is negative. Then, I apply forward WEMVA and obtain the corresponding perturbation in the image (Figure 3). Notice the presence of the low-wavenumber tomographic component, as well as the virtual absence of random artifacts which can potentially be produced by the RBC. Next, I apply adjoint WEMVA to the perturbation in the image for recovering an approximation to the original perturbation in the background (Figure 4). The perturbation in the image maps back into a shape that resembles the original Gaussian anomaly, although the amplitude is wrong because an inversion process is needed for recovering the original amplitudes. Notice that the aforementioned tomographic component maps onto a reflector resembling the perturbation in the image, which amplitude obscures the approximated anomaly. These results are similar to those obtained with tapering boundary conditions (Cabrales-Vargas et al., 2016b), although I have not yet set forth an inversion in the present case.

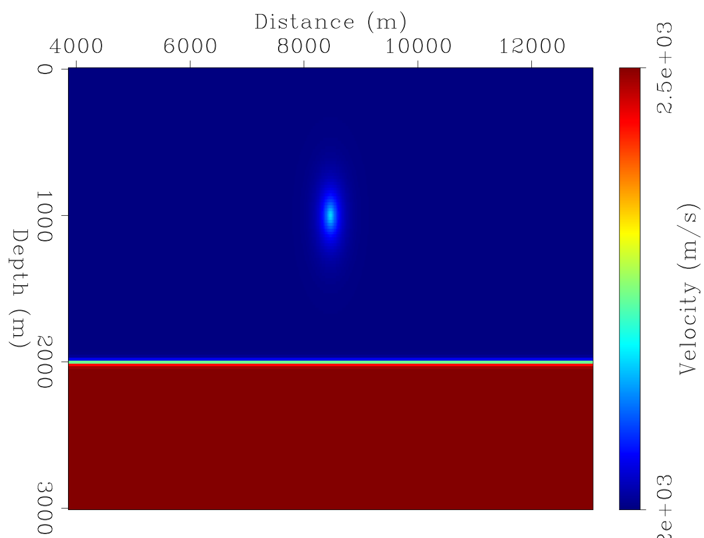

Figure 1:]
Two-layer velocity model with Gaussian anomaly. [**ER**]

I repeat the experiment replacing the cross-correlation imaging conditions (CIC) (Claerbout, 1992) with EIC. Figure 5 shows that the tomographic component has been attenuated in forward WEMVA. Likewise, after applying adjoint WEMVA with EIC the unwanted reflector was virtually removed (Figure 6), although some mild random artifacts can be seen now that the reflector's amplitude no longer obscures the estimated anomaly. This separation of tomographic and reflectivity components can

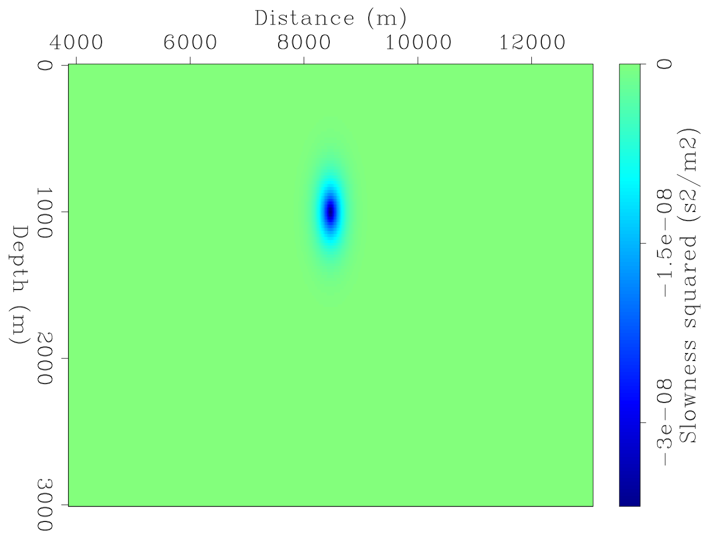

Figure 2:]
Gaussian anomaly expressed in slowness squared. [**ER**]

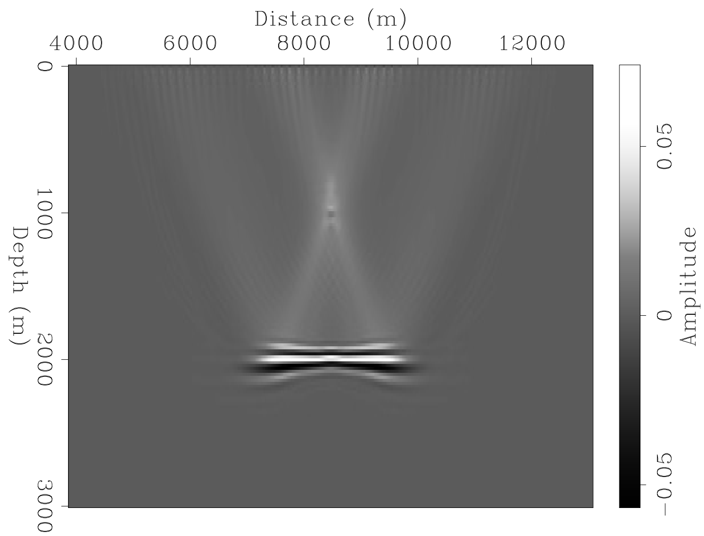

Figure 3:]
Perturbation in the image after applying forward WEMVA to the Gaussian anomaly. [**CR**]

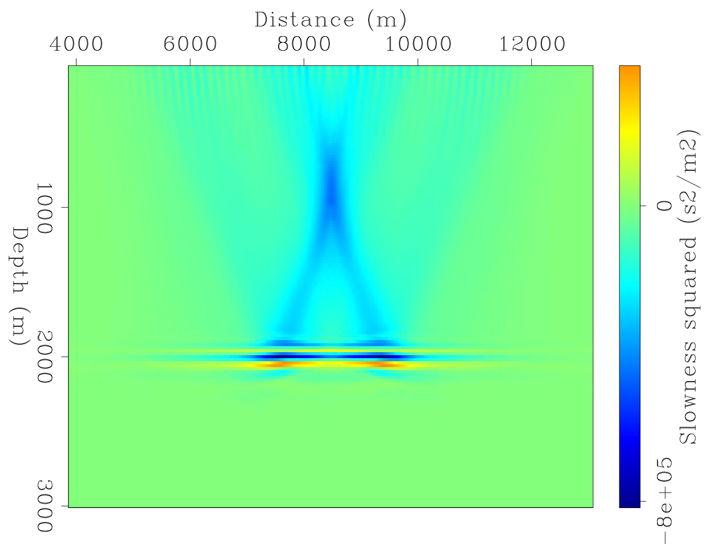

Figure 4:]
Approximation to the perturbation in the background slowness after applying adjoint WEMVA to the perturbation in the image (Figure 3). [**CR**]

potentially help in producing more reliable results during the inversion. Nonetheless, my implementation of WEMVA using EIC has not passed the dot-product test yet.

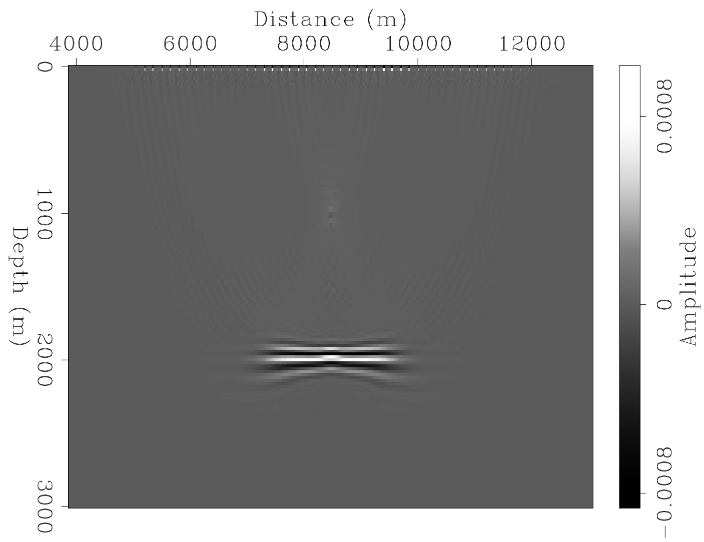

Figure 5:] Perturbation in the image after applying forward WEMVA to the Gaussian anomaly using EIC. [**CR**]

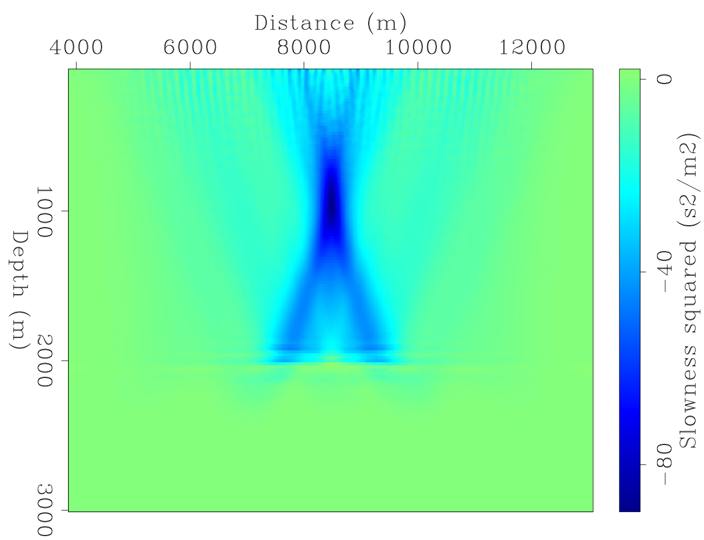

Figure 6:] Approximation to the perturbation in the background slowness after applying adjoint WEMVA to the perturbation in the image (Figure 5), using EIC. [**CR**]

CONCLUSIONS

I rectified the number of propagations by iteration that LWIVU requires when the WEMVA step is performed with RBC. The restriction of time directionality during the computation of the scattered wavefields makes the number of propagations rise to seven for a single application of WEMVA, and twelve per iteration plus one within LWIVU, where two WEMVA applications are required.

Using RBC does not introduce significant random artifacts to the WEMVA results. The incorporation of EIC attenuates both the tomographic component in the perturbation in the image and the reflectivity component in the perturbation in the background model. I still have to revisit this variation to verify whether it passes the dot-product test.

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