Reciprocity in elastic multi-component data

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ABSTRACT

Reciprocity for source and receiver pairs has been routinely adopted in seismic processing to mitigate poor receiver density and improve computational costs. But while its implementation is straightforward for acoustic data, the correct combination of source and receiver components in elastic, multi-component data, can be challenging. We derive the Green's functions for the direct and reciprocal data for a source-receiver pair and show synthetic results demonstrating their equivalence. We also show how an explosive source can be represented as a combination of normal stresses or particle velocities. Finally, we calculate the respective wavelets for each formulation both analytically and by linear inversion.

INTRODUCTION

Spatial reciprocity in elastic systems, also known as Betti's theorem, relates applied forces and observed particle displacements between different points. It states that the Green's function associated with a particle displacement observed at point A due to a force at point B can be related to the Green's function of an equivalent problem where particle displacement and force positions have been interchanged (Aki and Richards, 1980).

Reciprocity can be advantageous in data processing techniques, where differences in spatial sampling of source and receiver domains can be mitigated by exchanging their positions, usually by re-sorting common-shot gathers into common-receiver gathers (Nowack et al., 2003). Such differences in spatial sampling are particularly common in Ocean Bottom Node (OBN) surveys, where the distance between receivers is in the order of hundreds of meters, while the distance between shots is in the order of tens of meters. In areas with low signal-to-noise (S/N) ratios, Regone et al. (2015) have shown that denser receiver grids are favored over denser source grids when producing Pre-Stack Depth Migration images.

Reciprocity in data processing can also lower the computational cost of imaging. When imaging involves propagation of individual shots, such as in Reverse Time Migration (RTM) or other wave field extrapolation techniques, exchanging shots and receivers can lower the cost of computation, since the number of shots in the direct problem is usually an order of magnitude greater than the number of receivers.

While reciprocity can be trivially implemented when dealing with acoustic data, vector data, as is the case of geophones, requires a more careful analysis of reciprocal components. In order to understand that, one should explicitly describe the sources and receivers in their respective forces and displacements, as well as the Green's functions that connect them.

Here, we start by showing the equivalence between normal stresses and particle velocities when describing explosive type sources. We use the velocity-stress formulation of the elastic wave equation (Virieux, 1986) to analytically prove that equivalence. We then show numerically how the same analytical solution is found by a linear inversion of the source wavelet for a particle velocity quadrupole source.

Next, we describe the Green's functions that relate pairs of particle velocities at source and receiver positions and obtain their reciprocal functions. We show that a direct problem with a source in both vertical and horizontal velocities can be simulated by two reciprocal experiments where each particle velocity component is injected separately at the receiver position and recorded as a sum of components at the source. Finally, we show these results for a synthetic experiment in the Marmousi2 elastic model.

METHODOLOGY

In marine acquisition, the physical effect of airguns as seismic sources can be represented as a single point in which the moment tensor is diagonal (Aki and Richards, 1980). Using the velocity-stress formulation of the isotropic elastic wave equation, the injection of this kind of signal into the subsurface is given by the following system of partial differential equations:

$$\rho(x,z)\frac{\partial v_x(x,z,t)}{\partial t} = \frac{\partial \sigma_{xx}(x,z,t)}{\partial x} + \frac{\partial \sigma_{xz}(x,z,t)}{\partial z}, \tag{1}$$

$$\rho(x,z)\frac{\partial v_z(x,z,t)}{\partial t} = \frac{\partial \sigma_{xz}(x,z,t)}{\partial x} + \frac{\partial \sigma_{zz}(x,z,t)}{\partial z}, \tag{2}$$

$$\frac{\partial \sigma_{xx}(x,z,t)}{\partial t} = [\lambda + 2\mu](x,z)\frac{\partial v_x(x,z,t)}{\partial x} + \lambda(x,z)\frac{\partial v_z(x,z,t)}{\partial z} + S_{ex}(x,z,t), \tag{3}$$

$$\frac{\partial \sigma_{zz}(x,z,t)}{\partial t} = \lambda(x,z)\frac{\partial v_x(x,z,t)}{\partial x} + [\lambda + 2\mu](x,z)\frac{\partial v_z(x,z,t)}{\partial z} + S_{ex}(x,z,t), \tag{4}$$

$$\frac{\partial \sigma_{xz}(x,z,t)}{\partial t} = \mu(x,z)\left[\frac{\partial v_x(x,z,t)}{\partial z} + \frac{\partial v_x(x,z,t)}{\partial z}\right], \tag{5}$$

where λ, and μ represent the elastic parameters, ρ is the medium density, v_x, and v_z are the particle velocities, and σ_{xx}, σ_{zz}, and σ_{xz} are the propagated stresses. In equations 3 and 4 the forcing term is given by:

$$S_{ex}(x,z,t) = \delta(x - x_s, z - z_s)w(t), \tag{6}$$

where $w(t)$ is the source signature, and $\delta(x - x_s, z - z_s)$ is a spike positioned at the source location. In order to derive the reciprocal experiment based on the elastic reciprocity theorem shown in Aki and Richards (1980), it is easier to find the equivalent body force given by an explosive seismic source. To do so, we have to integrate in time and substitute equations 3 and 4 into 1 and 2, respectively:

$$\begin{aligned}
\rho(x,z)\frac{\partial v_x(x,z,t)}{\partial t} &= \frac{\partial}{\partial x}\int_{-\infty}^{t}\left\{[\lambda+2\mu](x,z)\frac{\partial v_x(x,z,\tau)}{\partial x}+\lambda(x,z)\frac{\partial v_z(x,z,\tau)}{\partial z}\right\}d\tau \\
&+\int_{-\infty}^{t}\frac{\partial S_{ex}(x,z,\tau)}{\partial x}d\tau+\frac{\partial \sigma_{xz}(x,z,t)}{\partial z}, \qquad (7)\\
\rho(x,z)\frac{\partial v_z(x,z,t)}{\partial t} &= \frac{\partial}{\partial x}\int_{-\infty}^{t}\left\{\lambda(x,z)\frac{\partial v_x(x,z,\tau)}{\partial x}+[\lambda+2\mu](x,z)\frac{\partial v_z(x,z,\tau)}{\partial z}\right\}d\tau \\
&+\int_{-\infty}^{t}\frac{\partial S_{ex}(x,z,\tau)}{\partial z}d\tau+\frac{\partial \sigma_{xz}(x,z,t)}{\partial x}. \qquad (8)
\end{aligned}$$

In these two equations we can clearly distinguish the forcing terms in the x and z components:

$$f_x(x,z,t)=\int_{-\infty}^{t}\frac{\partial S_{ex}(x,z,\tau)}{\partial x}d\tau=\frac{\partial\delta(x-x_s,z-z_s)}{\partial x}\int_{-\infty}^{t}w(\tau)d\tau, \qquad (9)$$

$$f_z(x,z,t)=\int_{-\infty}^{t}\frac{\partial S_{ex}(x,z,\tau)}{\partial z}d\tau=\frac{\partial\delta(x-x_s,z-z_s)}{\partial z}\int_{-\infty}^{t}w(\tau)d\tau. \qquad (10)$$

These two body forces have the same time signature that corresponds to the time integral of the original source wavelet of equation 6. On the other hand, the space signature is given by derivatives in the x and z directions of a delta function, respectively. This effect can be approximated by dipoles in the two directions, which is the same result found in the main diagonal of the moment tensor shown by Aki and Richards (1980).

When considering a multi-component marine ocean-bottom acquisition we have to be careful to separate the various recorded components to create the correct reciprocal experiments. In fact, it is important to understand that the hydrophone is coupled to the water; whereas, the geophones are coupled to the sea bed. Because of this differential coupling we have at least two reciprocal experiments to consider, one for an acoustic medium and a different one for an elastic subsurface. We are going to see that in reality we need three reciprocal propagations in this acquisition scenario. For the hydrophone component the true experiment can be represented by a time convolution of the explosive source response with the acoustic Green's function, as follows:

$$\begin{aligned}
p(x_{r_h},z_{r_h},t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_{r_h},z_{r_h},t;x,z)\star S_{ex}(x,z,t)dxdz \\
&= g(x_{r_h},z_{r_h},t;x_s,z_s)\star w(t), \qquad (11)
\end{aligned}$$

where (x_{r_h},z_{r_h}) represents the hydrophone position, and $\star$ the time convolution operator. In this case the reciprocal experiment is given by interchanging the source position with the receiver location:

$$p(x_{r_h},z_{r_h},t)=g(x_s,z_s,t;x_{r_h},z_{r_h})\star w(t). \qquad (12)$$

On the other hand, for the geophone component we have to employ the following property of elastic Green's functions (Aki and Richards, 1980):

$$g_{ij}(x_r, z_r, t; x_s, z_s) = g_{ji}(x_s, z_s, t; x_r, z_r), \tag{13}$$

where $g_{ij}(x_r, z_r, t; x_s, z_s)$ is the Green's function of a body force injected in the j-th direction from (x_s, z_s) and recorded in the i-th axis at (x_r, z_r). Using the representation theorem and the forcing terms of equations 9 and 10 we can describe the recorded particle velocities as:

$$\begin{aligned} v_x(x_{r_g}, z_{r_g}) = & \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[g_{xx}(x_{r_g}, z_{r_g}, t; x, z) \star \dot{f}_x(x, z, t) \\ & + g_{xz}(x_{r_g}, z_{r_g}, t; x, z) \star \dot{f}_z(x, z, t)]dxdz \\ = & [g_{xx}(x_{r_g}, z_{r_g}, t; x_s + dx, z_s) - g_{xx}(x_{r_g}, z_{r_g}, t; x_s - dx, z_s) \\ & + g_{xz}(x_{r_g}, z_{r_g}, t; x_s, z_s + dz) - g_{xz}(x_{r_g}, z_{r_g}, t; x_s, z_s - dz)] \star w(t), \end{aligned} \tag{14}$$

$$\begin{aligned} v_z(x_{r_g}, z_{r_g}) = & [g_{zz}(x_{r_g}, z_{r_g}, t; x_s, z_s + dz) - g_{zz}(x_{r_g}, z_{r_g}, t; x_s, z_s - dz) \\ & + g_{zx}(x_{r_g}, z_{r_g}, t; x_s + dx, z_s) - g_{zx}(x_{r_g}, z_{r_g}, t; x_s - dx, z_s)] \star w(t), \end{aligned} \tag{15}$$

in which (x_{r_g}, z_{r_g}) is the geophone position, and where we have approximated the derivatives of a delta function by:

$$\frac{\partial \delta(x, z)}{\partial x} \approx [\delta(x + dx, z) - \delta(x - dx, z)]/dx, \tag{16}$$

$$\frac{\partial \delta(x, z)}{\partial z} \approx [\delta(x, z + dz) - \delta(x, z - dz)]/dz. \tag{17}$$

By employing the result of equation 13 in relations 14 and 15 we have:

$$\begin{aligned} v_x(x_{r_g}, z_{r_g}) = & [g_{xx}(x_s + dx, z_s, t; x_{r_g}, z_{r_g}) - g_{xx}(x_s - dx, z_s, t; x_{r_g}, z_{r_g}) \\ & + g_{zx}(x_s, z_s + dz, t; x_{r_g}, z_{r_g}) - g_{zx}(x_s, z_s - dz, t; x_{r_g}, z_{r_g})] \star w(t), \end{aligned} \tag{18}$$

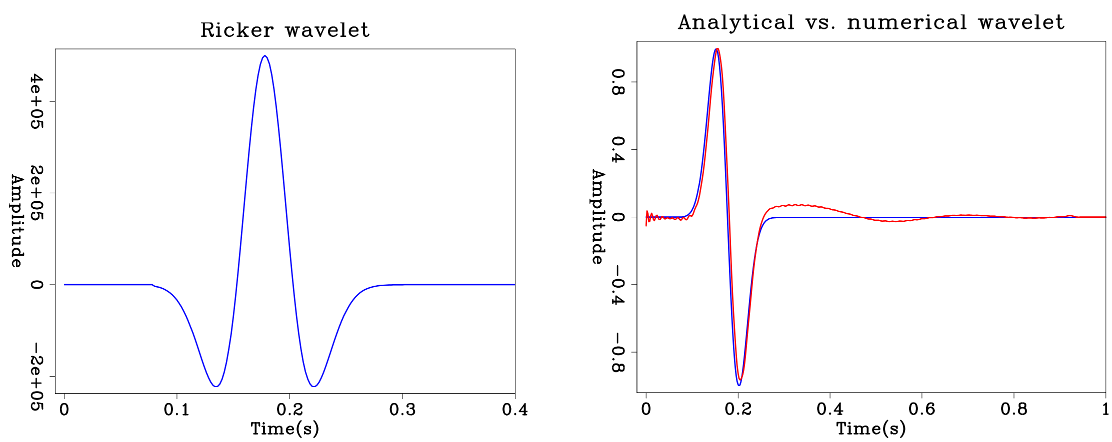
$$\begin{aligned} v_z(x_{r_g}, z_{r_g}) = & [g_{zz}(x_s, z_s + dz, t; x_{r_g}, z_{r_g}) - g_{zz}(x_s, z_s - dz, t; x_{r_g}, z_{r_g}) \\ & + g_{xz}(x_s + dx, z_s, t; x_{r_g}, z_{r_g}) - g_{xz}(x_s - dx, z_s, t; x_{r_g}, z_{r_g})] \star w(t). \end{aligned} \tag{19}$$

These two equations describe the actual reciprocal experiment when an explosive source is employed and particle velocities are acquired. We notice that in both components we are injecting a force at the receiver location x_{r_g}, z_{r_g} only in one direction, that is in x and z, respectively. However, the recording of the particle velocities is performed using a quadrupole. For instance, the particle velocity $v_x(x_{r_g}, z_{r_g})$ component can be obtained by injecting a force in the horizontal axis and recording the propagated v_x and v_z with a dipole along the corresponding direction. The same is true for the vertical component but with an injection in the z axis. Because we are injecting a force along one axis per component in the reciprocal experiment, we have to perform two different propagations. This observation means that in order to perform the reciprocal experiment for an ocean-bottom multi-component survey we have to propagate three independent propagations, that is one for the pressure component and two for the geophone components. Therefore, as long as the number of receivers is less than a third the number of sources, the reciprocal experiment involves fewer propagations than the true acquisition. Despite this apparent issue we clearly see the advantage in this acquisition scenario. In fact, in this case we usually have thousands of sources and just few hundreds of receivers such that the aforementioned condition is potentially fulfilled.

RESULTS

Equivalence between stress and velocity explosive sources

We started by generating multi-component elastic data in a constant background model by applying an explosive source in the normal stress components of the wavefield, as described by equations 3 and 4. The source was a Ricker type wavelet with peak frequency 10Hz, as shown in Figure 1a. Next, we run a linear inversion of the source wavelet, but now instead of finding a source for the stress components, we look for a wavelet that minimizes the objective function and is described by a quadrupole in the velocity components of the wavefield, as we have shown in equations 9 and 10. According to our analytical solution, the quadrupole wavelet should be the time integral of the original Ricker. Figure 1b shows a comparison of the analytical and inverted solutions and figure 2 shows the objective function for the linear inversion problem.



(a) (b)

Figure 1: (a) Ricker wavelet used to generate data and (b) comparison between the time integral of the Ricker wavelet (blue) and the numerical approximation obtained by linear inversion (red). [**CR**]

Next, we run two synthetic experiments using the 2D elastic Marmousi model (Martin et al., 2002). In the first experiment, we inject an explosive source using the stress formulation in equations 3 and 4. In the second experiment, we repeat

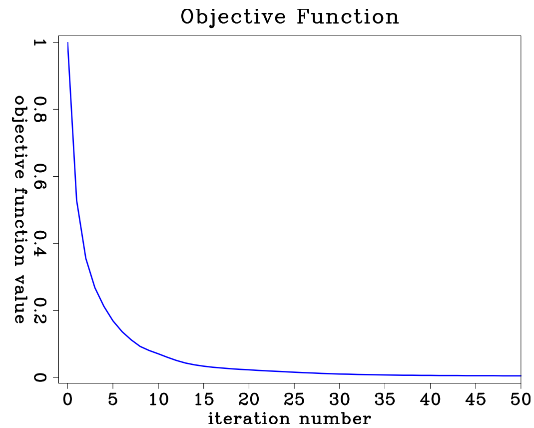


Figure 2: Objective function for the first 50 iterations of the linear inversion of the wavelet. The value at the 50th iteration is 0.5% of the initial value. [**CR**]

the positions for source and receiver, but now inject an explosive source according to equations 9 and 10. Our goal is to show that multi-component elastic data recorded at the ocean-bottom interface is equal for both source formulations, even with a fairly complex model. Figure 3 shows all data components overlaid for (red) stress sources and (blue) particle velocity sources.

Reciprocity in multi-component data

The 2D elastic Marmousi model is a fairly complex model even for shallow targets, which allows us to observe many reflections and mode conversions in the synthetic data and study the validity of the reciprocal data for different types of events. Figure 4 shows the elastic parameters for this model.

In order to get the direct data, we place an initial explosive source close to the water surface, representing an air-gun. The synthetic source is modeled using the quadrupole in particle velocities. We position the receiver at the ocean-bottom interface, so it can mimic the complexity of the elastic coupling to the solid medium observed in ocean-bottom data. Source and receivers are not aligned in depth, so that amplitude versus angle relations are observed.

As we described in the previous section, we need three independent experiments to generate the reciprocal data for a pair of source and receiver. The first experiment injects the source at the reciprocal position as a quadrupole in velocities. This experiment generates the traditional acoustic reciprocity, giving us the hydrophone reciprocal data. Next, we run two modelings, a vertical and a horizontal velocity source and record the data as a quadrupole at the new receiver position. We summarize our results in figure 5, that shows the direct and reciprocal multi-component data. It can be seen that both data sets overlay almost perfectly. There are minor differences that we believe can be attributed to numerical accuracy.

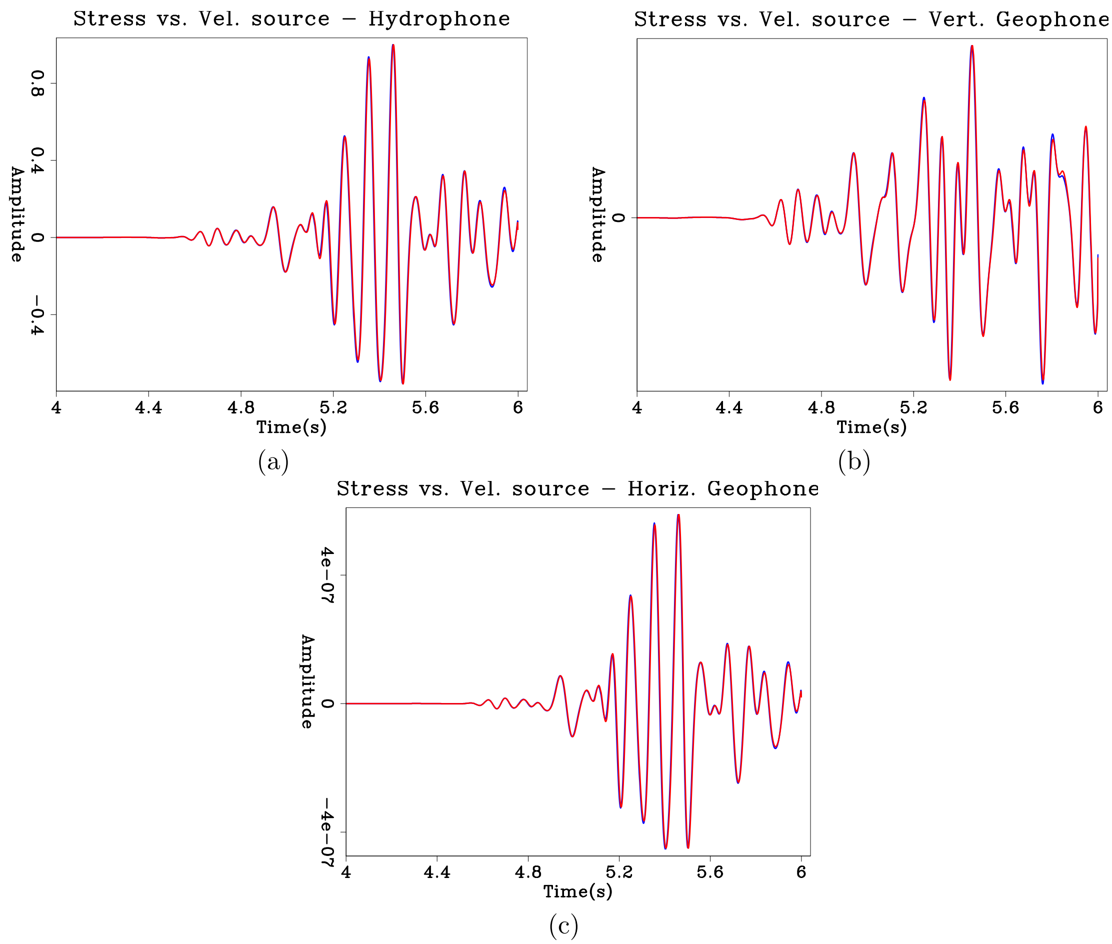


Figure 3: Data recorded for one receiver due to an explosive source injected in the stress components (red) or as a particle velocity quadrupole (blue). Graphs show (a) hydrophone, (b) vertical geophone and (c) horizontal geophone data. [**ER**]

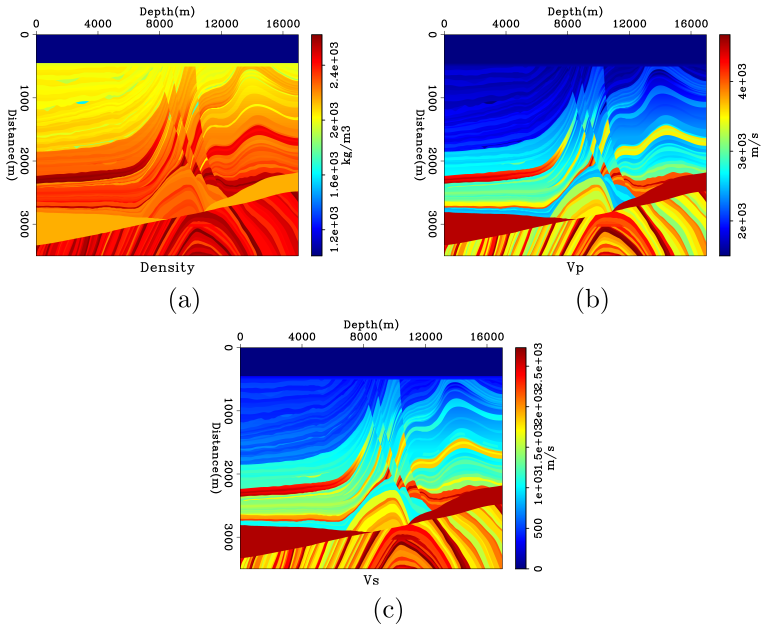


Figure 4: (a) Density, (b) V_p and (c) V_s for the 2D Marmousi 2 elastic model. [**ER**]

CONCLUSION

We show that equivalent synthetic acoustic data can be generated in the velocity-stress formulation by either sources in the normal stress components or in the particle velocity components. When dealing with multi-component data, this is advantageous because a single source wavelet can be used for generating reciprocal data for both scalar (hydrophone) and vector (geophone) components.

We describe the Green's functions that relate pairs of particle velocities at the source to pairs of particle velocities at the receiver. Furthermore, we describe the reciprocal equations and show that, for a two-component source in the direct problem, the reciprocal problem is described by two independent solutions obtained by injecting single-component particle velocities at the receiver position and extracting a sum of both components at the source. While this increases the number of simulations from one in the direct problem to three in the reciprocal problem (acoustic data plus two vector components), we believe that such tradeoff is still computationally advantageous when the ratio between number of sources and number of receivers is greater than three. Also, as mentioned in the Introduction, spatial sampling in source and receiver domains can justify the use of reciprocal data.

ACKNOWLEDGEMENTS

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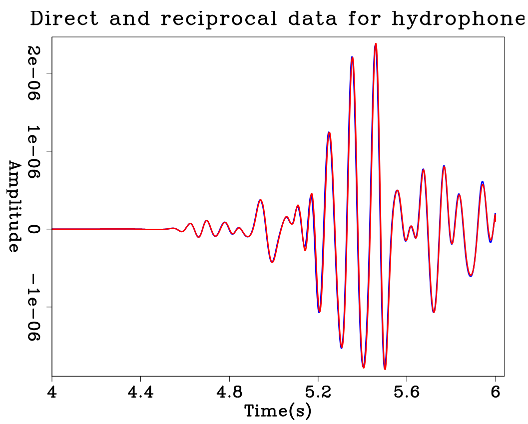
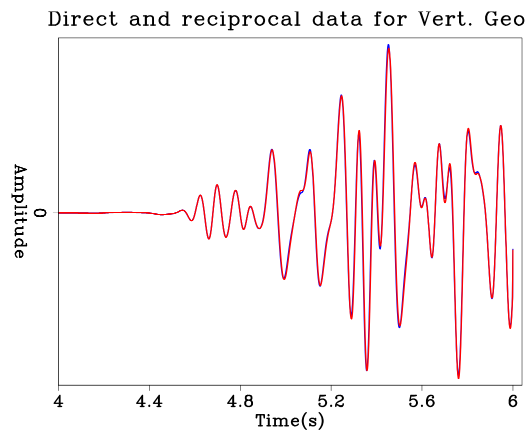


(a)



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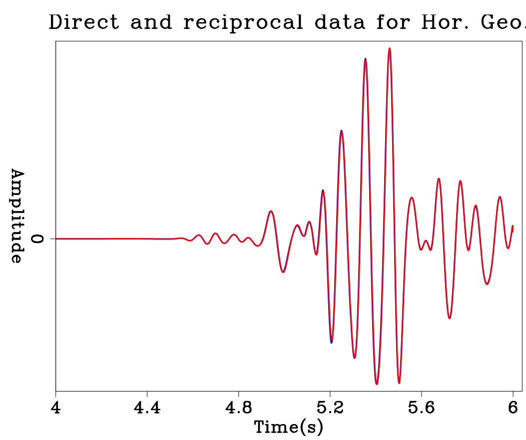

(c)

Figure 5: Direct (blue) and reciprocal (red) data for the (a) hydrophone, (b) vertical-geophone and (c) horizontal geophone. [**ER**]

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