# THE EXPLODING REFLECTOR MODEL
# AND LATERALLY VARIABLE MEDIA

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*Abstract*

The validity of the exploding reflector model is considered where the velocity is laterally variable. Although the model is useful for slowly variable velocities, it breaks down when the velocity variations are rapid. Examples for point scatterers are given.

*Introduction*

Most migration and modeling techniques, especially those used in industrial production, use an imaging principle based on the exploding reflector model. Two basic assumptions are involved. The first is that the CMP-stacked section is equivalent to a zero-offset section. The second is that a zero-offset section may be modeled by placing sources on all the reflectors at $t=0$ and continuing the resulting wavefield to the surface, using half the true velocity. This has been discussed by various authors (e.g. Stolt, 1978; and Clayton, SEP-14, p. 21), but it is usually assumed that the velocity depends only on depth. An example where this fails is shown in Figure 1. In this paper we will explore the extent to which the exploding reflector model is valid when lateral variations in velocity are present. The results may be extended to non-zero offsets; this leads to an accurate method for the computation of synthetic seismograms for all offsets. For simple reflectivity structures, this method is very economical.

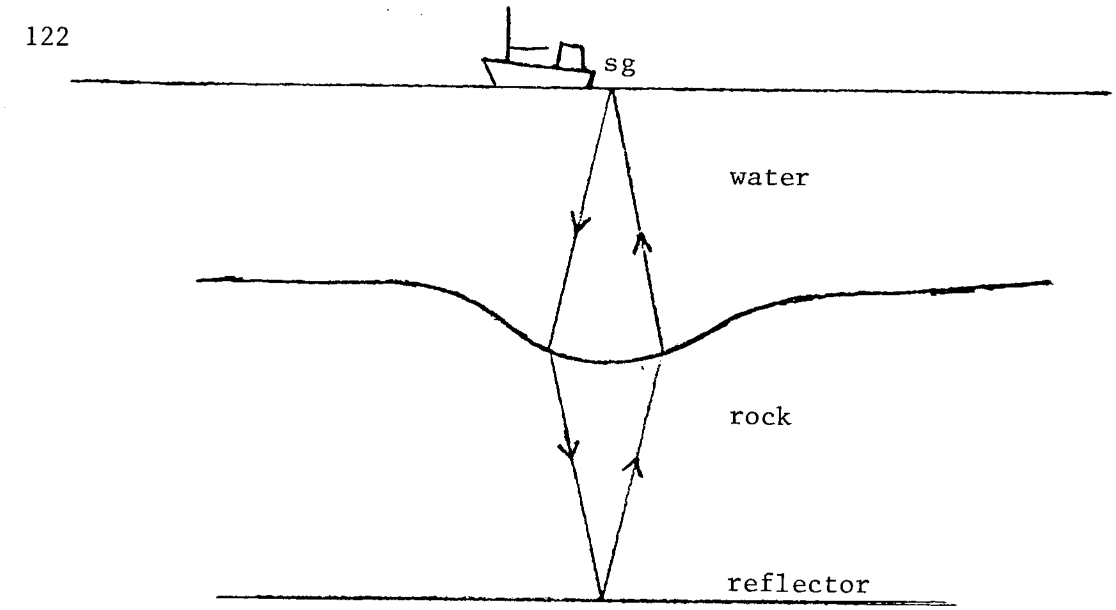


FIGURE 1.--A zero-offset ray path not predicted by the exploding reflector model.

*Theory*

We will assume that the reflection seismogram can be approximated by a distribution of point scatterers, imbedded in a variable velocity medium. This assumption is valid when the reflection coefficients are small and independent of the angle of incidence. When this is the case, seismograms can be computed for arbitrary reflectivity structures by superposition of the seismograms resulting from each reflector point. The seismogram ${}_sS_g$ recorded at g from a source at s may be considered as a convolution of the shot waveform W (as it would be recorded by the recording instrument), the propagation from the source to the reflector ${}_sP_r$, the reflectivity R, and the propagation from the reflector to the geophone ${}_rP_g$ - or in the frequency domain:

$$ {}_sS_g = W \; {}_sP_r \; R \; {}_rP_g \tag{1} $$

In the case of zero offset, reciprocity implies that

$$ {}_sP_r = {}_rP_g \tag{2} $$

and

$$ {}_gS_g = W \; R \; {}_rP_g^{\;2} \tag{3} $$

Reciprocity may also be used to obtain non-zero offsets when ${}_rP_g$ is known for all the reflector points and all surface locations. For a single point scatterer this requires no more computation than the upward continuation of the wavefield due to a point source at depth. When the velocity structure is such that ${}_rP_g$ consists of a single spike, with a traveltime $t_0$ and amplitude $A_0$, it is easy to see that ${}_rP_g^2$ is a spike arriving at $2t_0$ with amplitude $A_0^2$. Thus, one may approximate ${}_rP_g^2$ by stretching ${}_rP_g$ and applying a scale factor (cylindrical or spherical spreading correction).

The condition that ${}_rP_g$ be a spike is not necessary in all cases - for example, if the rate of dissipation is proportional to frequency (Constant Q), a plane wave pulse is broadened in a homogeneous medium, such that the seismogram at any distance is obtained by a stretching and scaling of a single seismogram (Kjartansson, 1979). Because of the frequency-dependence of the velocity the scaling factor is not exactly proportional to distance. This does not apply for other dissipation laws such as that treated by Ricker (1953, 1977).

The observed seismogram is a linear function of the reflectivity in the subsurface (for small reflection coefficients). The convolution of a trace on itself, however, is a non-linear operation and must therefore be performed for each reflector point separately. The time-stretching is a linear operation so it can be performed for all the traces together, implicitly by using half the true velocity. When a linear operator such as the wave equation is used to compute ${}_rP_g$, a great saving in computational effort can result from superposing the reflectors before the wave extrapolation.

The convolution cannot be replaced by the time-stretching when more than two ray paths connect the reflector and the surface point. This can happen when $\partial^2 v/\partial x^2 \neq 0$, or when the interfaces between layers of different velocities are curved.

*Examples*

We will show two such examples. Figure 2 shows the exploding reflector seismogram for a point source near a vertical interface. The velocity on the left of the interface is one-third what it is on the right. The depth of the point scatterer is five times its distance from the vertical interface. Figure 3 shows the result computed using

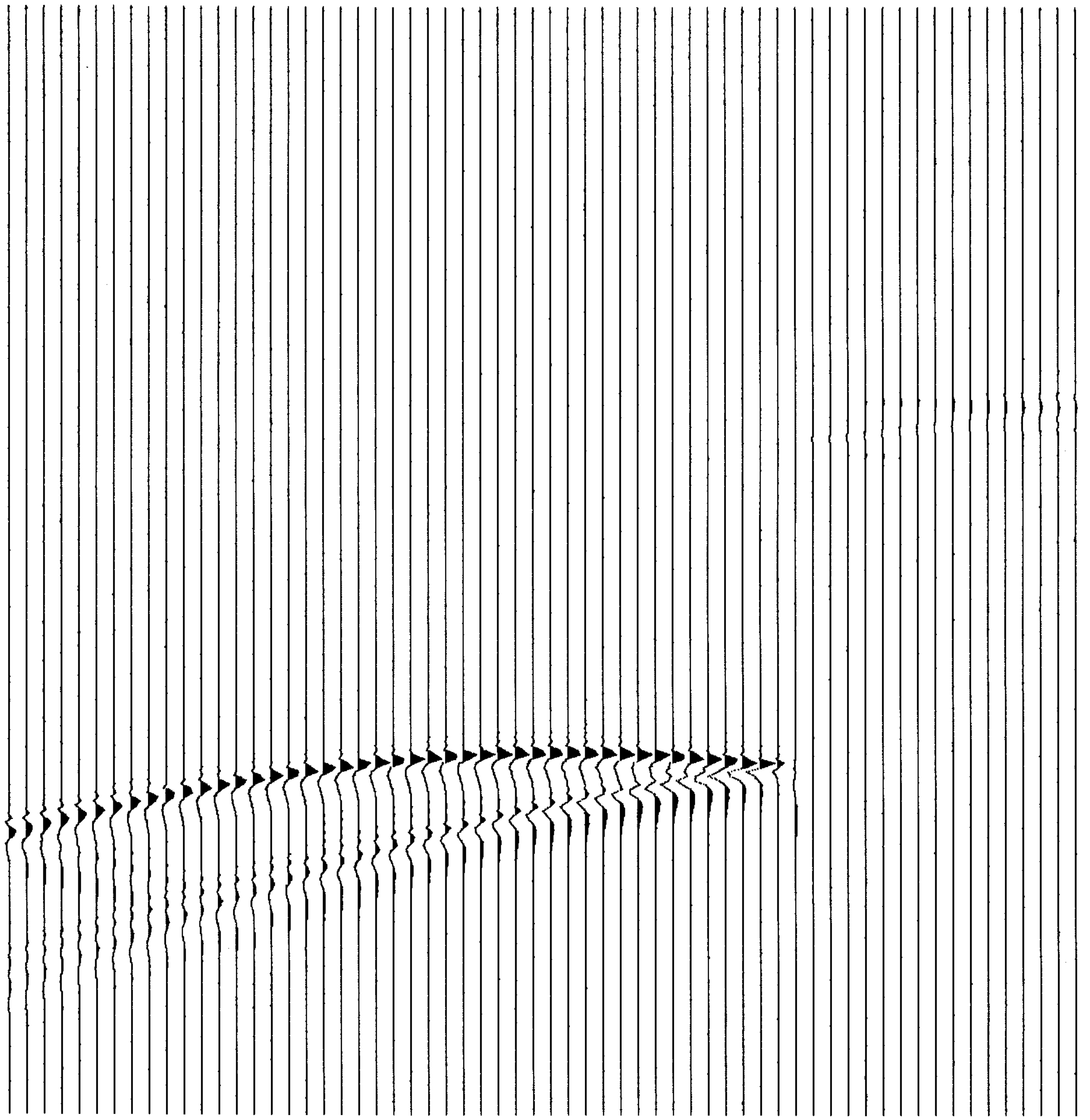

FIGURE 2.--Exploding reflector zero-offset section for a point scatterer at a depth of five times its distance from a vertical velocity contrast of 3 to 1. Computed using monochromatic 45-degree wave equation (Kjartansson, SEP-15, pp. 1-20). Ray paths are shown in Figure 4, a and b.

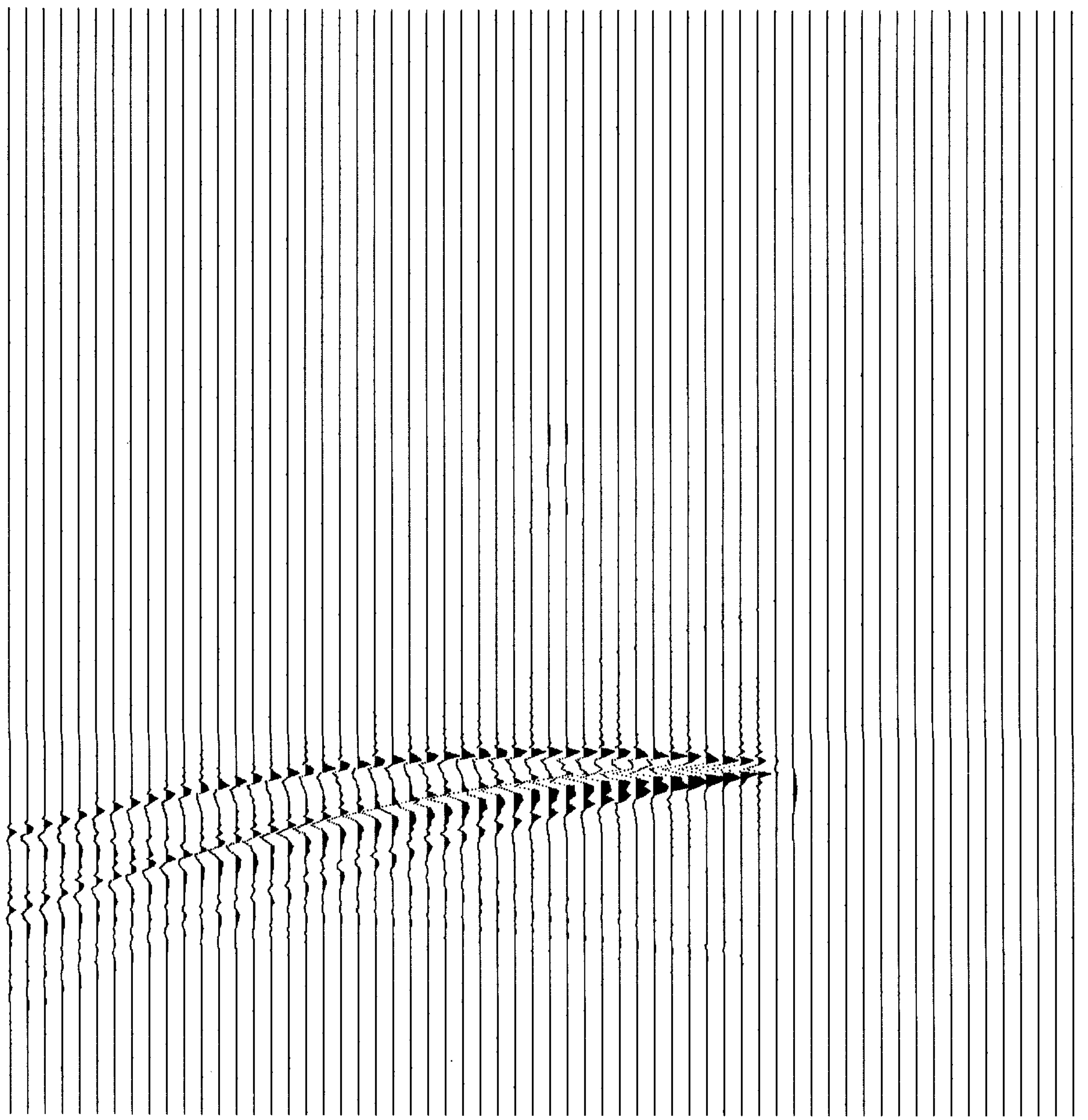

FIGURE 3.--Zero-offset section for same earth model as Figure 2, computed using Equation (3). All the ray paths shown in Figure 4 are now included.

Equation (3) for the same earth structure. The strongest arrival on Figure 3 is not present at all in the exploding reflector approximation. Note also that the relative amplitudes are different between the two figures. Figure 4 shows the ray paths included in Figures 2 and 3. The velocity structures used in these figures are probably not too realistic from a geologic point of view. Figure 5 shows a somewhat more realistic velocity model. For computational convenience we have assumed a velocity that is independent of depth, but very similar effects would be expected for point scatterers below a curved interface between two layers with different velocities, e.g. below a depression in the seafloor. Figure 6 shows the exploding reflector result for the velocity function shown in Figure 5. The source is at a depth that is 1.3 times the width of the computed model. Figure 7 shows the correct zero-offset section. As before, there are significant differences.

*Conclusions*

We have found that the exploding reflector model can accurately predict traveltimes when the velocity is smoothly varying with $x$. When the velocity is sufficiently variable so that different rays from subsurface reflectors cross each other on the way to the surface, however, the strongest events may be completely missing on the computed section. Any migration scheme based on the exploding reflector model will treat those events incorrectly. It seems that in order to image this kind of data correctly, one must downward-continue the entire experiment (see Lynn, SEP-14, pp. 5-13).

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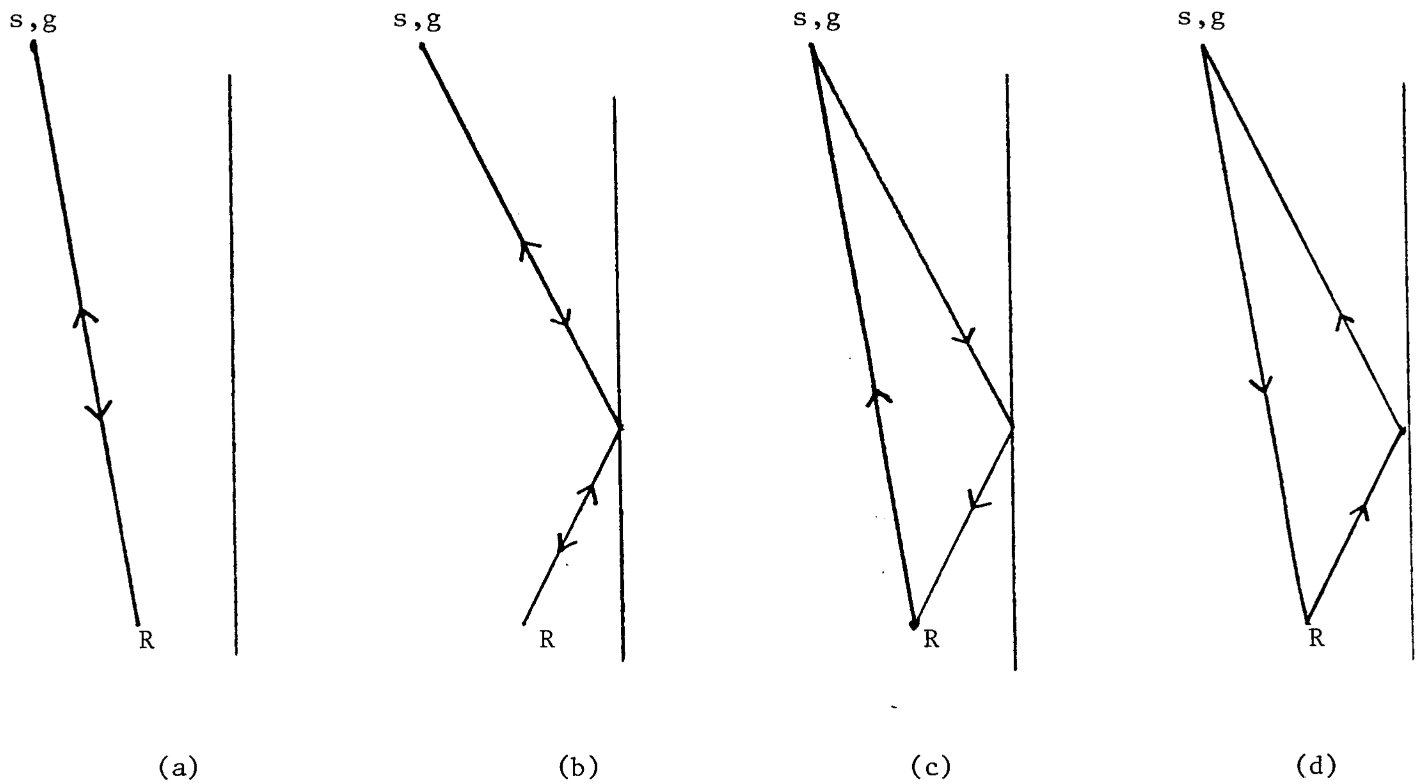



FIGURE 4.--Ray paths for the model in Figures 2 and 3. The exploding reflector model includes only (a) and (b).

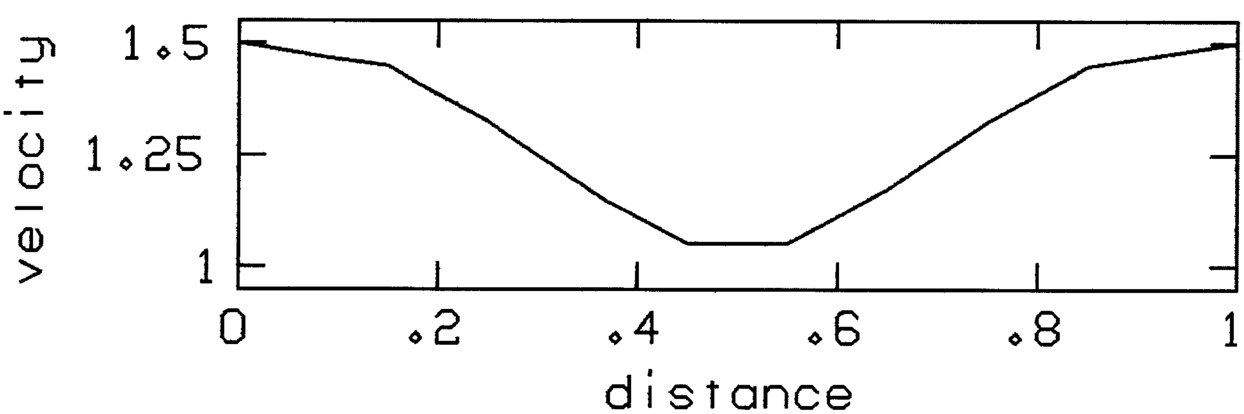



FIGURE 5.--Velocity along the sections in Figures 6 and 7. Velocity is independent of depth.

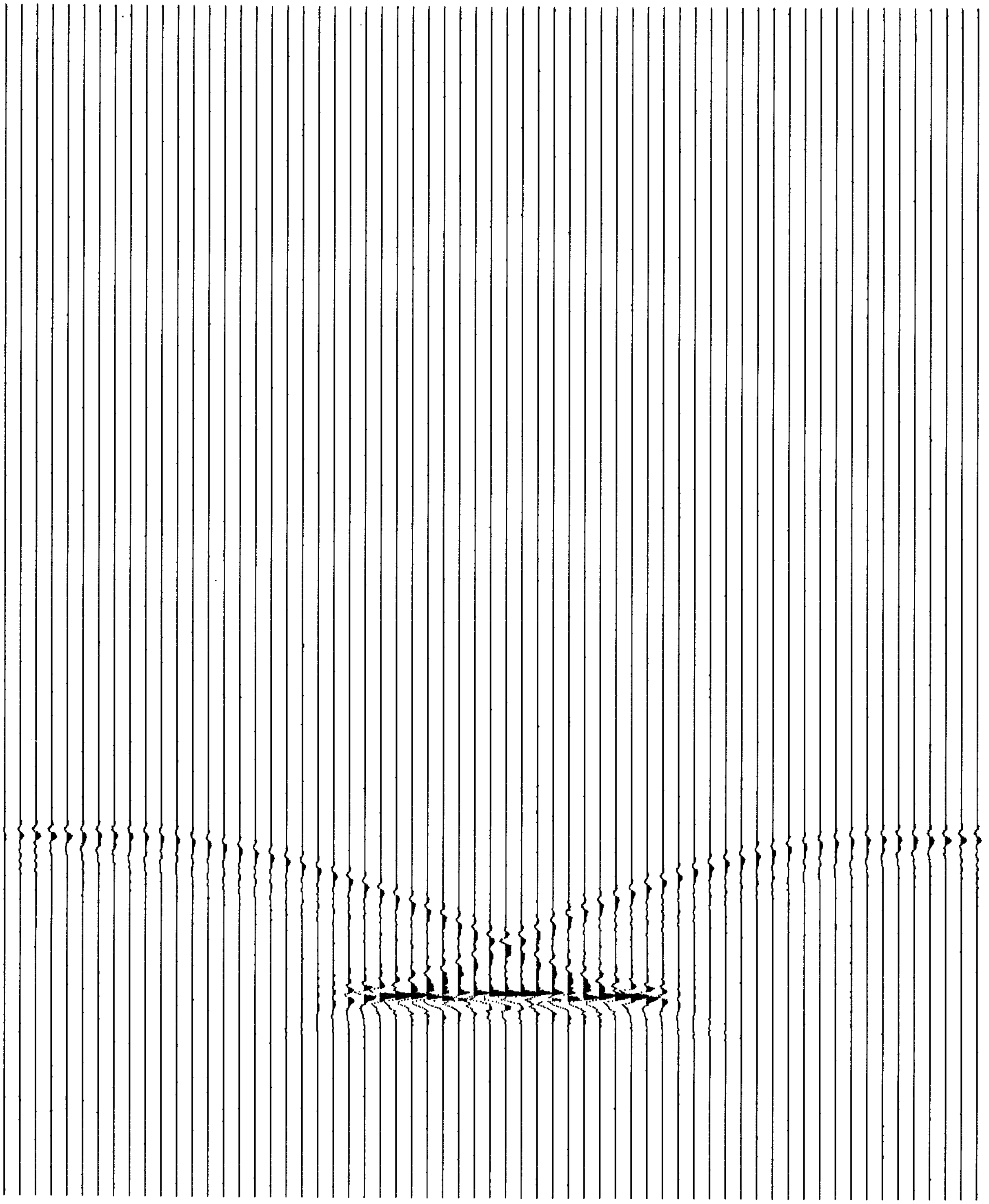


FIGURE 6.--Exploding reflector model zero-offset section for a point scatterer at a depth of 1.3 times the width of the section. Velocity is independent of depth and varies as shown in Figure 5 along the section.

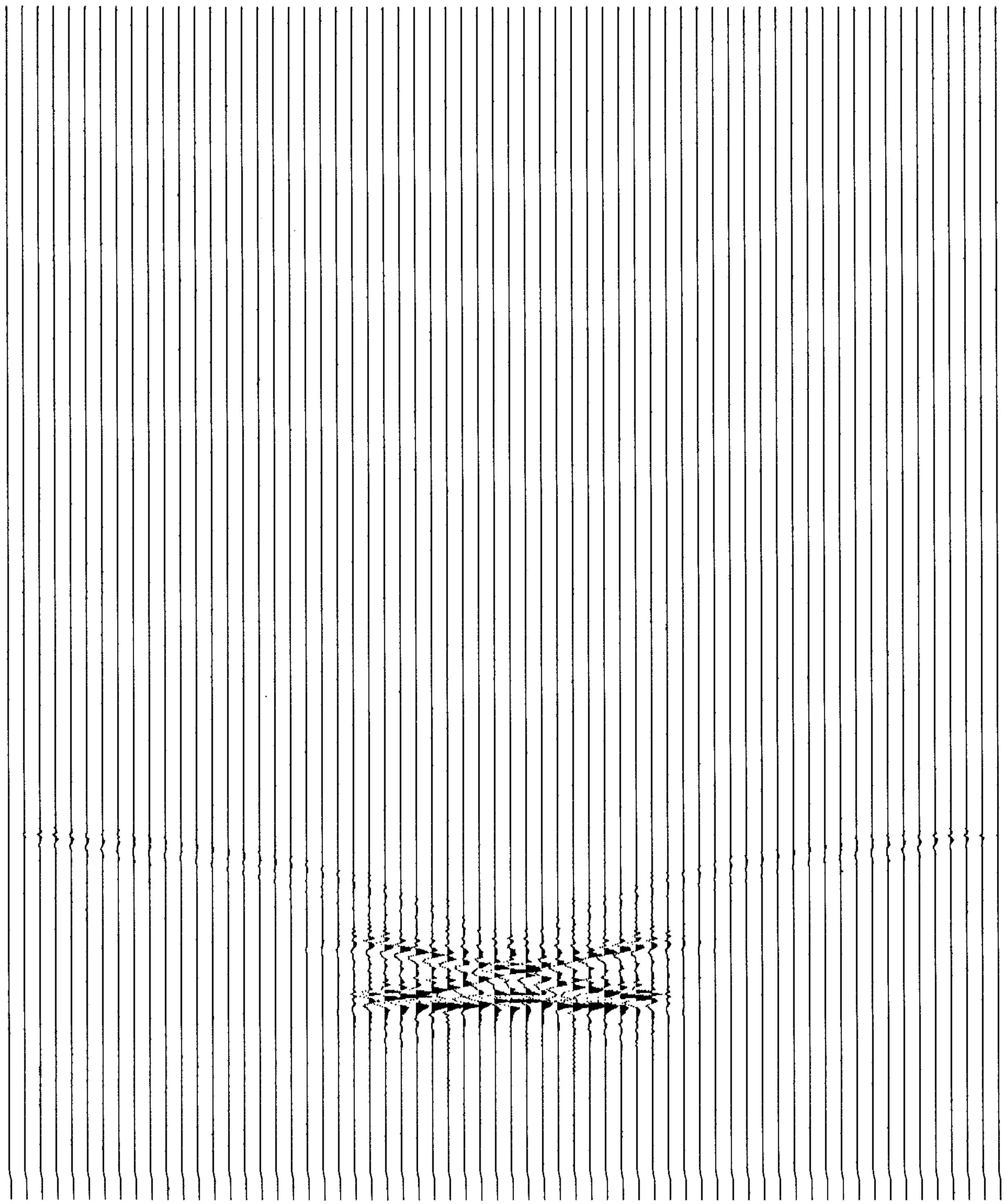
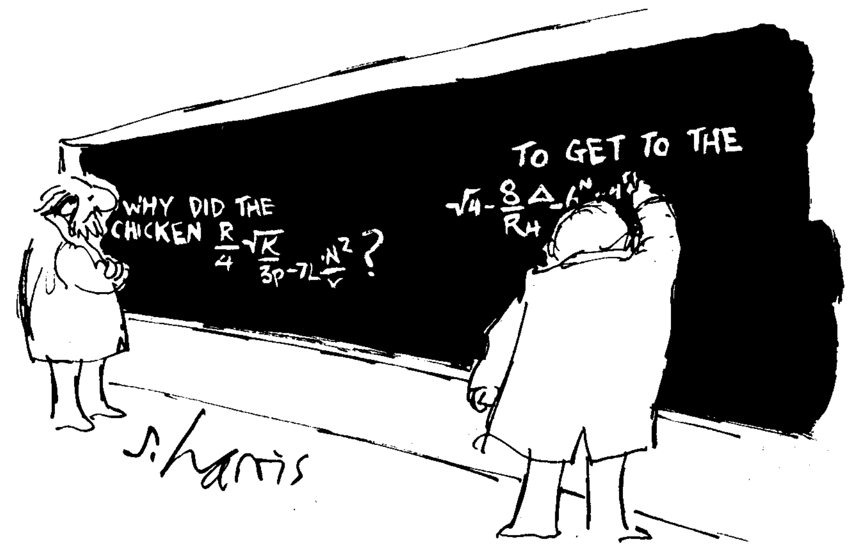


FIGURE 7.--Zero-offset section computed using Equation (3) for the same earth structure as in Figure 6.



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