VELOCITY MODEL BUILDING USING
RESIDUAL-MOVEOUT-BASED WAVE-EQUATION MIGRATION
VELOCITY ANALYSIS

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF GEOPHYSICS
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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August 2015
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Abstract

Wave-equation-based velocity estimation is a set of powerful techniques for robust velocity model building for complex subsurface regions, in which ray-based methods are usually ineffective or even unsuccessful. However simply switching from ray-based tomography methods to wave-equation-based ones does not fully solve the problem. In the area of wave-equation migration velocity analysis (WEMVA), although some promising results have been shown, several issues are still not well solved in today’s WEMVA methods, preventing them from becoming the industry standard. Specifically, the main issues include: 1) severe nonlinearity, which causes the cycle-skipping problem under large velocity error; and 2) imprecise objective functions, which wrongly penalizes residuals that are not caused by velocity error but other factors such as uneven subsurface illumination and incomplete acquisition geometry.

In this dissertation, I address these issues by developing a new WEMVA method that uses the residual-moveout (RMO) information of the angle-domain common-image gathers (ADCIG) to quantify the velocity model error. In this RMO-based WEMVA approach, I combine the strengths of the wave-equation and the ray-based tomography by replacing the ray-based tomographic operator with a wave-equation-based one, while keeping the conventional ray-based tomography workflow. In contrast to other WEMVA methods that build their objective functions directly based on the common-image gather amplitudes, this method defines a purely kinematics-based objective function that links to the velocity model through an residual-moveout (RMO) parameter. Since the RMO parameter scales almost linearly with the velocity error, this approach greatly reduces the risk of cycle-skipping in the absence of
low-frequency data. Moreover, focusing on the gather kinematics makes this method insensitive to spatial and angular variations of the gather amplitudes, thus leads to high-quality model gradients. In addition, this method does not require explicit picking of the moveout parameters because it uses the derivative over the velocity-scanning semblances to calculate the moveout perturbation. With promising results, my 2-D examples demonstrate that this RMO-based WEMVA method is very robust against cycle-skipping, can effectively flatten the angle gathers, and does not require moveout parameters picking.

Furthermore, I extend the RMO-based WEMVA method to the 3-D case. To deal with multiple azimuths 3-D ADCIG, I augment my method’s formulation by assigning independent moveout parameters to each azimuth. A simple synthetic example verifies that the 3-D extension of the RMO-based WEMVA is able to invert simultaneously velocity information from multiple azimuths. Finally, I apply my RMO-based WEMVA to a 3-D WATS (Wide Azimuth Towed Streamers) field dataset from GOM (Gulf of Mexico). To make applying WEMVA methods to this large industrial scale dataset computationally affordable on the academic computing resources I have in the school of Earth, Energy and Environmental Sciences, I adopt a target-oriented inversion approach that concentrates on a relatively small target area of interest inside the full physical domain of the dataset. The target-oriented RMO-based WEMVA inversion of this field dataset yields geophysically more consistent models. The inversion results show convincing imaging improvements and enhancements in the flatness of the 3-D ADCIG universally across the target domain and all azimuths.
Preface

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Our testing is currently limited to LINUX 2.6 (using the Intel Fortran90 compiler) and the SEPlib-6.4.6 distribution, but the code should be portable to other architectures. Reader’s suggestions are welcome. For more information on reproducing SEP’s electronic documents, please visit [http://sepwww.stanford.edu/research/redoc/](http://sepwww.stanford.edu/research/redoc/).
Acknowledgments

When I decided to come to the US and attend Stanford six years ago, I had no idea what would be lying ahead of me. Now in retrospect, after going through all the thick and thin, I believe that I made the right choice. I learned a lot of new knowledge and skills, became empowered with critical thinking and independent problem solving, and moreover, had many great moments during my life at Stanford.

First and foremost, I am most grateful to Prof. Biondo Biondi, my Ph.D advisor, who gave me the opportunity to work in SEP (Stanford Exploration Project), as well as encouraged and guided me on my research. His continuous emphasis on the importance of the real 3-D field data application has helped me push my own limits and earn the powerful can-do spirit. I am deeply grateful to Prof. Jon Claerbout, the legend and genius mind behind SEP. I was stunningly impressed by Jon’s incredible insights on geophysical problems, even though I did not fully understand them at the beginning. It was a privilege to work with him and learn from him. I also owe huge credits to Dr. Robert Clapp for helping me with many difficulties I encountered in research and for teaching two excellent courses — Basic Earth Imaging and Introduction to Computational Earth Sciences. He is also the ‘go-to’ person on SEP’s key infrastructure (both hardware and software), without which I would not be able to accomplish many of the works presented in this dissertation. I sincerely thank my thesis committee member Prof. Tapan Mukerji for his well-taught lectures and his mentoring on my secondary project. I also thank Prof. George Hilley for chairing my defense.

I thank my teacher, Prof. Runqiu Wang, for giving me the very first exposure to
the field of Geophysics and convincing me that this field is a fantastic combination of Physics, Mathematics and Computer sciences.

I feel very lucky to be among so many talented past and present SEP students, who made my time at Stanford enjoyable and my research productive. I owe my gratefulness to my long time friend and schoolmate Xukai Shen, who made me aware first time that there is an interesting field called Exploration Seismology out there that is worth pursuing. My research benefits a lot from previous work by Yaxun Tang and Guojian Shan, who mentored me throughout my junior years at SEP and we sparked great discussions on research. Claudio Cardoso, Gboyega Ayeni and Sjoered de Ridder did a great job TAing us on SEP’s imaging and inversion courses, which provided the most important tools for my research. I am very fortunate to be with some wonderful SEP officemates: Yaxun Tang, Sjoered de Ridder, Adam Halpert, Yi Shen and Guillaume Barnier. I enjoyed the atmosphere they created in our office, as well as the discussions and conversations that sparked out from time to time. I received a lot of help from Elita and Mandy Wong, who arrived one year earlier than me, on almost every aspect of SEP life, including research question, coding problem, program requirement, administrative stuff, as well as computer chores.

I want to express my great gratitude to the fellows on the same year as mine: Ali Almomin, Ohad Barak and Chris Leader. We encouraged each other, helped each other, and the peer pressure among us drove me to go extra mile. In addition, Ohad did a great job on managing and maintaining SEP’s Linux computers. I missed the conversations that I had with SEP students I met, whether they were about geophysics or about general aspects of life, they helped me become a better person. Especially, I appreciate the fruitful discussions with Yaxun Tang, Guojian Shan, Ali Almomin and Elita Li. I thank all the students passing through the SEP ranks during my years here, including: Mohammad Maysani, Nader Musa, Kittinat Taweesintananon, Qiang Fu, Jason Chang, Noha Farghal, Musa Mahararrov, Taylor Dahlke, Huy Le, Eileen Martin, Yinbin Ma, Daniel Blatter, Guillaume Barnier, Gustavo Alves, Ettori Biondi and Kaixi Ruan for being great friends and companions.

I learned a lot from my internship experiences and many great mentors. Especially,
I thank Dr. Marta Woodward and Dr. Dave Nichols of Schlumberger for mentoring me on seismic tomography projects, and I thank Dr. Guojian Shan and Dr. Yue Wang of Chevron for supervising me on WEMVA projects.

I thank Schlumberger, especially Dr. Dave Nichols, for providing and allowing the publication of the 3-D field data set used in this dissertation. I also thank BP and SMARTJV consortium for the synthetic models used in this dissertation.

I owe special thanks to Mrs. Mary Mcdevitt (from the technical communication program in the school of Engineering) and Jason Chang who did a great job proofreading my dissertation and improving it significantly.

I am grateful to the geophysics staff for their support. More specifically, Diane Lau for taking care of SEP meetings, travels and research expense reimbursements; Dennis Michael, Manager of CEES HPTC, for providing constant technical support for our HPC facility, which is essential to the completion of my research; Tara Ilich and Nancy Thurlow, the student services managers, for bookkeeping the degree progress of every student in our department.

I thank my dear friends at Stanford: Jingyi Chen, Haohuan Fu, Boxiao Li, Hangyu Li, Tianze Liu, Yao Tong, Yu Xia, Siyao Xu, Alec Yang, Tieyuan Zhu, Lin Zuo, et al. for the happy times brought into our student lives.

In the last, I would like to thank my parents, Duanrong Zhang and Liqin Chen, for their unconditional support and love throughout my life. They originated from a rural prefecture in central China, and neither of them had the opportunities to attend college as their best years were stranded in the long-lasting political turmoil that plagued the entire nation through the late 1970s, but they did whatever they can to ensure that I get the best education possible and be able to fulfill my aspiration to the full extent. Finally, I feel extremely fortunate to have found the love of my life, Yi (Alice) Wu, who alone made my Stanford experience worthwhile; it is her caring and love that accompanied me through this exciting yet strenuous journey, and our commitment to start a family together provided me the inspiration that I needed to finish.
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Chapter 1

Introduction

As seismic exploration has been increasingly deployed in regions with deeper target and more complex geological structures, obtaining accurate 3-D images, which are essential to successful discoveries, becomes very difficult. A tremendous amount of effort has been spent on more advanced imaging techniques. To better simulate the complex wave propagation phenomena in these structural challenging areas, sophisticated wave extrapolation and imaging techniques such as one-way wave-equation and reverse time migration have been developed. However, these high fidelity imaging techniques are more sensitive to the velocity model being used, therefore require high-fidelity velocity models to fulfill their full potential. This remains a major challenge in the field and is an object of intense research in both academia and industry. Aiming to bridge the gap between the status quo model building practice and the high expectation for model accuracy from the imaging side, this thesis presents a novel wave-equation based velocity model building methodology that is accurate, fully automated and very robust against large velocity error.
CHAPTER 1. INTRODUCTION

MODEL BUILDING IN COMPLEX SUBSURFACE STRUCTURE SETTINGS

Earth model building, in particular velocity model building, is intrinsically more difficult than imaging because of its nonlinear and under-determined nature. The challenges are magnified under complex geological settings. One predominant example of complex geology is the subsalt region. The subsalt area is often a focus of current day seismic exploration practice as its structure makes it more likely to trap oil and gas. However, due to the complex geological overburdens (primarily the salt body intrusion) and sharp velocity contrast between a salt body and its surrounding sediments, the wave propagation pattern is very complex, making it very hard to focus the subsalt events coherently unless a very accurate model and imaging algorithm are both in place. Furthermore, seismic energy, while passing through salt, can often be blocked/diverted from illuminating the subsalt sediments, and we thus have less information on the subsalt target in the collected surface seismic data to start with. The limited illumination not only leads to a low SNR (Signal to Noise Ratio) image, but more importantly, the insufficient angular illumination coverage makes the velocity estimation task very difficult.

In the past three decades, there has been steady progress made in tackling the velocity modeling problem. However, such progress falls short compared to the tremendous progress that has been made on the seismic migration front; we are still far from creating a fully automated method that is capable of resolving high-resolution velocity models robustly (Etgen et al., 2009). The work presented in this thesis is motivated and inspired by the need to improve upon the status quo of velocity estimation techniques, which will be briefly reviewed in the following section.

Ray-based vs. Wave-equation-based reflection tomography

Currently, the standard industry practice of velocity model building using reflection seismic data is represented by ray-based tomography (Woodward et al., 2008), which
dates back to the early 90s (Stork, 1992) when computer capabilities are very limited and could only simulate wave propagation by projecting rays in the target domain, a very coarse way of modeling wave phenomena. A typical ray-based reflection tomography workflow is shown in Figure 1.1.

The foundation of all ray-based reflection tomography methods is the data redundancy in the seismic recordings, which originates from the fact that subsurfaces reflectors are illuminated from multiple directions by seismic energy. The ray-based tomography workflow shown in Figure 1.1 is no exception. It extracts the redundancy by computing common-image-point (CIP) gathers, which is an ensemble of common-offset (surface source-receiver offset) seismic images. If the velocity model is accurate, then any given subsurface reflection point will be imaged at the same depth across all images of individual offset. Otherwise, depth variation of reflection events across offset can be observed. The tomography then measures this depth variation and traces rays through the velocity model to determine a model perturbation that will reduce the depth variations. This process is repeated for multiple iterations.

Ray-based methods are very computationally efficient, and are reasonably accurate in simple settings with mainly horizontal reflectors and mild velocity variations. However, the model perturbations computed using these infinite-frequency rays are very distinct from those computed using the wave-equation engine, which much more accurately simulates the finite-frequency wave propagation. Figure 1.2 illustrates the significant difference between the 3-D travel time sensitivity kernels generated using ray-tracing propagation engine and using wave-equation propagation engine. The ray-based kernel is a thin curve connecting the two end points, while the wave-equation-based kernel has the well-known banana-doughnut shape (Woodward, 1989; Marquering et al., 1999). Moreover, ray-based methods often fail in more complex geological settings (for example, salt domes). The complex subsurface structures and strong lateral velocity variations give rise to sophisticated wave-propagation phenomena, which ray-based methods can hardly describe because the theoretical limitation of the infinite-frequency assumption makes them incapable of handling wave effects such as scattering, wavefront triplication, multi-pathing, etc.. This limitation is well
illustrated in Figure 1.3, where the Sigsbee2A model is used to compare a snapshot of the traced rays and a snapshot of the wavefield modeled by acoustic wave-equation. Note that the ray-tracing engine completely fails to characterize the wavefront after the energy passes through the salt body. We can therefore conclude that it is unlikely that ray-based methods would be effective in the subsalt area of this model.

Since wave-equation is much more capable of accurately capturing the wave-propagation physics, to overcome the limitations of ray-based tomography methods, I favor wave-equation based methods, to which the method presented in this thesis belongs.

Figure 1.1: A block diagram illustrating a typical modern day ray-based tomography workflow; figure is from Woodward et al. (2008). [NR] raytomo-workflow
Figure 1.2: Comparison of the 3-D travel time sensitivity kernels using (a) a ray-based tomographic operator and (b) a wave-equation based tomographic operator with a 30 Hz wavelet; A linear $v(z)$ model is used as the background velocity model. Figure is from Rickett (2000).
Figure 1.3: Comparison of the modeling results between the (a) ray tracing method and (b) the wave equation modeling on the Sigsbee2A velocity model; Figure is from Tang (2011). [NR] sigsb2a-rays-vmod,sigsb2a-wave-snapshot
Data space or image space wave-equation tomography

Wave-equation tomography methods fall into two main categories: the first type is the data-domain full waveform inversion (FWI) (Tarantola, 1984), which has the unique advantage of recovering the fine details in the subsurface model; however, this method is very susceptible to cycle-skipping (Pratt, 1999), which leads to divergence in the case of large velocity error. The second type is the image-domain wave-equation migration velocity analysis (WEMVA) (Chavent and Jacewitz, 1995; Biondi and Sava, 1999; Shen et al., 2005). Performing velocity analysis in the image domain has the advantage over the data space method in that the image is easier for users to understand and interpret than the raw seismic recordings, because the reflection events are back-propagated and restored into individual reflectors in the image domain by the imaging algorithm. Therefore, it is easier to analyze the model building results and come up with solutions for the issues encountered, including cycle-skipping. Recently, there has been a trend to combine the data space method (namely FWI) and the image space method (namely WEMVA), initiated by the tomographic full waveform inversion (TFWI) method proposed by Biondi and Almomin (2014). With very promising results, Biondi and Almomin (2014) demonstrate that TFWI is capable of having the best of both worlds — data space and image space. However up to this point, TFWI method requires many more iterations than typical WEMVA or FWI methods (Almomin and Biondi, 2012), making large-scale applications of this method difficult. Therefore, I focus on WEMVA methods in this thesis.

THESIS OVERVIEW

The goal of this thesis is to invent a new WEMVA method that is fully automated, robust against the cycle-skipping problem and can produce high quality velocity estimation results. The remainder of this thesis is organized as follows:
2-D RMO-based WEMVA

In Chapter 2, I describe my new WEMVA method, which I call residual-moveout (RMO) based WEMVA. It combines the advantages of ray-based tomography and wave-equation tomography by replacing the ray-based tomographic operator with a wave-equation-based one while maintaining the conventional ray-based tomography workflow.

Similar to many other WEMVA methods, my RMO-based WEMVA aims to maximize the flatness of the angle-domain common-image gathers (ADCIG). However, rather than directly maximizing the power of angle stack, this method defines the objective function as of the ADCIG’s residual-moveout (RMO) parameters, which characterize the curvatures of the ADCIG. The method then links RMO parameters to the velocity model.

In the first half of this chapter, I present the mathematical formulation of the RMO-based WEMVA objective function, as well as the derivation for gradient computation. Using several synthetic examples including the Marmousi model, I show that my method greatly reduces the risk of cycle-skipping in the absence of low-frequency data, and it produces high-quality gradients by focusing purely on the angle gathers kinematics.

In the second half of the chapter, I revise the assumption that assigns a RMO parameter to every image point in my RMO-based WEMVA formulation. The new assumption assigns a RMO parameter to each event, which is physically more sound and therefore more accurate. I then address the technical issue of devising an automatic event detection module, which is needed in order to build the new assumption into my WEMVA implementation. Last, I show that the new RMO-based WEMVA implementation further improves the inversion accuracy with the Marmousi model and the BP 2006 model examples.
3-D extension of RMO-based WEMVA

In Chapter 3, I establish the theoretic framework to extend the RMO-based WEMVA method to three-dimensions (3-D). Aiming at a straightforward 3-D extension of my WEMVA theory, I propose adding an azimuth axis to the 3-D ADCIG, and the RMO information along each azimuth is treated independently. Then, the RMO-based WEMVA formulation can be easily extended to 3-D with minor modifications. Using a synthetic example, I demonstrate that such a 3-D extension is able to extract variant velocity information from different azimuths.

In addition, I address the practical problem encountered in the 3-D transform between subsurface offset domain and angle-domain common image gathers. Due to the strong irregularity of the mapping relation between subsurface offset wavenumber domain \((k_{hx}, k_{hy})\) and angle domain (azimuth and reflection opening angle), a naively implemented backward (angle to offset) transform will cause severe distortion/artifacts in the converted offset gathers, making the kinematic information contained in the resulted subsurface offset-domain gathers less consistent with that in the input angle-domain gathers. I therefore use a texture-mapping type algorithm in the backward (angle to offset) transform, which is shown to significantly reduce the distortion/artifacts introduced during the transform.

Field data tests — target-oriented approach

In Chapter 4, I describe the workflow I take to apply my RMO-based WEMVA method to an industry scale 3-D marine streamers wide-azimuth data set — E-Octopus III — in the Gulf of Mexico. This 3-D field data set poses many challenges for my implementation, especially for its huge data volume (over 11TB) and complex geological structure (salt bodies). First, I apply careful data regularization and preprocessing, and employ a target-oriented inversion scheme that focuses on the update of sediment velocities in specific regions of interest. Such target-oriented scheme significantly reduces the computational cost, making WEMVA methods affordable on the limited
academic computing resource that I have access to. I then apply my RMO-based WEMVA on several target regions of interest to invert for better velocity models in those regions. The inversion results further improves the angle gathers flatness and the quality of the structural images in the target regions.
Chapter 2

2-D RMO-based WEMVA

Existing wave-equation migration velocity analysis (WEMVA) schemes suffer from problems such as severe nonlinearity (which causes the problem of cycle-skipping) and imprecise objective functions (which can accrue velocity errors by minimizing residuals caused by model complexity and incomplete acquisition). This chapter discusses how I tackle these issues by developing an alternative method to perform WEMVA. I replace the ray-based tomographic operator with a wave-equation-based one, while keeping the conventional ray-based tomography workflow. Similar to some of the existing approaches, my WEMVA method aims to maximize the flatness of the angle-domain common-image gathers (ADCIG). However, rather than directly maximizing the image-stack-power objective function, this method indirectly links the objective function to the velocity model through an intermediate residual-moveout (RMO) parameter. By focusing on the common-image gathers kinematics, this approach greatly reduces the risk of cycle-skipping in the absence of low-frequency data, and it produces high-quality gradients. In addition, the proposed method does not require explicit picking of the moveout parameters because it uses the derivative over the velocity-scanning semblances to calculate the direction of moveout.
INTRODUCTION

Introduced by Gardner (1974) and Sattlegger (1975), migration velocity analysis (MVA) belongs to a family of methods for estimating migration velocity. Instead of focusing on travel-times in the seismic data, MVA extracts the velocity information from the migrated images. Subsequently, Etgen (1990) and van Trier (1990) proposed the first formulations of tomographic MVA using surface offset-domain common-image gathers (ODCIG) obtained by Kirchhoff migration. More recently, the tomographic MVA method has been extended to what is known as wave-equation migration velocity analysis (WEMVA), which uses wave-equation-based physical models rather than rays as the carriers of velocity information (Chavent and Jacewitz, 1995; Biondi and Sava, 1999). These wave-equation-based methods are theoretically more accurate than ray-based methods because the wave-equation better describes wave-propagation physics and provides physically more realistic sensitivity kernels for the velocity update. In practice, it is often observed that the wave-equation methods behave quite differently than the ray-based methods when applied to complex velocity models.

To estimate velocity model, WEMVA methods solve an optimization problem. When forming WEMVA objective functions, evaluating the flatness of the subsurface angle-domain common-image gathers (ADCIG) is currently a popular choice (Biondi and Sava, 1999; Clapp and Biondi, 2000; Biondi and Symes, 2004). The objective function is usually optimized by applying gradient-based algorithms. The computation of the gradient is performed in two steps: 1) computation of a perturbation in the migrated image, and 2) back-projection of the image perturbation into the velocity model using the image-space wave-equation tomographic (ISWET) operator (Sava and Biondi, 2004a). Figure 2.1 shows a typical WEMVA workflow diagram.

Several WEMVA methods have been proposed in the literature. The stack-power maximization method (Chavent and Jacewitz, 1995) directly maximizes the angle stack of the ADCIG, but, similar to the full-waveform inversion (FWI) (Tarantola, 1984) method, it is prone to cycle-skipping when the velocity error is too large (Symes, 2008). The Differential-semblance optimization (DSO) (Symes and Carazzone, 1991;
Shen et al., 2005; Shen and Symes, 2008) method penalizes the first derivative along the angle axis on the ADCIG. This objective function is physically intuitive, easy to implement and can achieve global convergence. However, the DSO objective function is constructed directly on the image gather amplitudes (rather than kinematics) and the differential operator it uses makes it very sensitive to amplitude variation in the ADCIG. This shortcoming has been well-recognized (Vyas and Tang, 2010; Fei and Williamson, 2010), and it leads to a suboptimal velocity gradient with unwanted artifacts that slow down the convergence. Moreover, the DSO method will over-penalize an already flat angle gather with variable amplitudes even when the velocity is accurate, thus force the inversion to converge to an incorrect model. This problem is demonstrated by Yang et al. (2013) who used a subsalt imaging example in which the angle gather has strong amplitude variances due to the highly non-uniform illumination caused by the complex salt body. Sava and Biondi (2004a,b) use prestack Stolt residual migration to help construct the image perturbation. The cycle-skipping problem is avoided in this way, but the user is required to pick a residual migration
parameter at each image point, and the picking is not trivial.

In this chapter, I propose a new method named residual-moveout-based (RMO-based) WEMVA (Zhang and Biondi, 2013) that extracts the velocity information in the angle domain. The key innovation in this method can be summarized as follows:

- Given that the ray-based tomography is much more robust against cycle-skipping than the stack-power maximization WEMVA (because the amount of residual-moveout (RMO) scales almost linearly even with large velocity error), my WEMVA method combines the strengths of the ray-tomography and the wave-equation operators. Instead of directly relating the velocity model to the flatness objective function, my WEMVA method relates the objective function to an intermediate moveout parameter (Al-Yahya, 1989). It then links the moveout parameter to the velocity model.

- My WEMVA method introduces a new way to construct the image perturbation from this moveout parameter, based on the fact that the moveout parameter describes the kinematics change in the ADCIG caused by the model velocity change. After the image perturbation is constructed, it is then back-projected into the model space by the ISWET operator.

The remainder of this chapter details this innovation in the following subsections that:

1. Explain the theoretical framework of the proposed RMO-based WEMVA.

2. Verify the theory with several simple synthetic examples.

3. Raise the practical issue of accurately handling multiple events and propose the use of image processing techniques to improve the workflow.

4. Demonstrate the effectiveness of the improved workflow with a couple of more geologically complex examples.
2-D THEORY OF RESIDUAL-MOVEOUT-BASED WEMVA

For simplicity, in this chapter I assume that the physical domain is two dimensional (2-D) in the following derivation and implementation. I parameterize the model space as slowness rather than velocity because kinematic perturbations are more linear with respect to slowness than with respect to velocity; the same situation applies to the ISWET operator (image perturbations are more linear with respect to slowness).

Introducing the RMO-based WEMVA objective function

We start from the classical stack-power maximization objective function:

\[ J(s) = \sum_x \sum_z \left[ \sum_{\gamma} I(z, \gamma, x; s) \right]^2, \]  

(2.1)

where \( s \) is the model slowness, \((z, x)\) are the depth and horizontal axes, respectively, \( \gamma \) is the reflection opening angle, and \( I(z, \gamma, x; s) \) is the prestack image in the reflection-angle domain obtained by migration using the slowness \( s \).

Note that the objective function 2.1 favors stronger reflectors over weaker ones. To make it independent of reflector strengths, I adopt the same approach used in velocity scan semblances, i.e., normalize the stack-power at each image point by dividing it with the energy of its surrounding event within a local depth window. I therefore introduce a local depth-window variable \( z_w \) to represent the average width of the wavelet for all events, and the normalized objective function becomes:

\[ J^N(s) = \sum_x \sum_z \frac{\sum_{z_w} \left[ \sum_{\gamma} I(z + z_w, \gamma, x; s) \right]^2}{\sum_{z_w} \sum_{\gamma} I^2(z + z_w, \gamma, x; s)}. \]  

(2.2)

In the above equation, I use a local window of length \( L \) for \( z_w \) (through this derivation, I assume the summation interval for variable \( z_w \) is always \([-L/2, L/2]\)).
choosing a value for $L$, the rule of thumb is that $L$ should not be smaller than the span of the event’s wavelet, and it should not be too large such that it contains more than one event in each window. Although it is true that such an ideal case cannot always be achieved, as long as $L$ is set as a reasonable value, my experiences show that the inversion results are not very sensitive to the value of $L$.

As stated in the introduction, objective functions defined by simply stacking all angles are prone to cycle-skipping, thus producing incorrect model gradients when velocity error is large. Recalling that the conventional ray-based reflection tomography does not show such disadvantages, we can approximate objective function 2.2 at some initial slowness $s_0$, with an alternative one that focuses on kinematic changes of the ADCIG. Given $I(s_0)$ and $I(s)$ as the prestack images with initial and updated slowness, respectively, I first define a residual moveout operator parameterized by $\rho$, $M_{\rho}$, which will describe (as accurately as possible) the kinematic difference between $I(s)$ and $I(s_0)$. In other words, $M_{\rho}I(s_0)$ (the initial image after moveout) will have a shape similar to that of $I(s)$ (the image migrated with new slowness). Hence $\rho$ is a function of both $s$ and $s_0$, since $s_0$ will remain fixed during the calculation of the model gradient, I can then denote the dependence of $\rho$ simply with $\rho(s)$. I assign independent $\rho$ to the ADCIG at each image point $(z, x)$ because there can be many events in the gathers, and each event can have a different moveout.

Next, I replace $I(s)$ in equation 2.2 with its kinematic approximation $M_{\rho}I(s_0)$, thus creating an alternative objective function that associates with the model $s$ through the RMO parameter $\rho$:

$$J(\rho(s)) = \sum_x \sum_z \left( \sum_{\gamma} M_{\rho(s)}I(z, \gamma, x; s_0) \right)^2 \times \sum_{\gamma} \left[ M_{\rho(s)}I(z, \gamma, x; s_0) \right]^2.$$  \hspace{1cm} (2.3)

The actual RMO function of $M_{\rho}$ is subject to the user’s own design. I use $\rho \tan^2 \gamma$ as the moveout function because it is the analytic solution of the RMO in the simple case of constant slowness and flat reflectors (Biondi, 2006). Thus we have

$$M_{\rho}I(z, \gamma) = I(z + \rho \tan^2 \gamma, \gamma).$$  \hspace{1cm} (2.4)
and equation 2.3 now becomes

\[ J(\rho(s)) = \sum_x \sum_z \left( \frac{\sum_w I(z + z_w + \rho \tan^2 \gamma, \gamma, x; s_0)}{\sum_w \sum_\gamma I^2(z + z_w + \rho \tan^2 \gamma, \gamma, x; s_0)} \right)^2. \]  
(2.5)

Note that the term computed in objective function 2.5 resembles the form of a moveout semblance, we can simplify the expression of the objective function by defining a moveout semblance:

\[ S_m(\rho, z, x; s_0) = \frac{\sum_w \left[ \sum_\gamma I(z + z_w + \rho \tan^2 \gamma, \gamma, x; s_0) \right]^2}{\sum_w \sum_\gamma I^2(z + z_w + \rho \tan^2 \gamma, \gamma, x; s_0)}. \]  
(2.6)

Then, objective function 2.5 can be rewritten as

\[ J_{S_m}(\rho(s)) = \sum_x \sum_z S_m(\rho(z, x), z, x; s_0). \]  
(2.7)

I use gradient-based methods to solve this optimization problem, and the model update given by the gradient of objective function 2.7 is

\[ \frac{\partial J_{S_m}}{\partial s} = \sum_x \sum_z \frac{\partial \rho(z, x)}{\partial s} \frac{\partial S_m(\rho(z, x), z, x; s_0)}{\partial \rho(z, x)}. \]  
(2.8)

In the above equation, \( \partial \rho/\partial s \) describes the relation between slowness perturbation and moveout parameter perturbation, and \( \partial S_m/\partial \rho \) indicates the search for better moveout parameters to flatten the gathers.

**Gradient calculation of the RMO-based objective function**

In this section, I explain how to compute the gradient shown in equation 2.8. First, \( \partial S_m/\partial \rho \) can be easily calculated by taking the numerical derivative by finite differences along the \( \rho \) axis of the semblance panel \( S_m \). Note that \( \rho(s = s_0) = 0 \) (because there is zero kinematic difference between \( I(s) \) and \( I(s_0) \) in this case); thus \( \partial S_m/\partial \rho \)
needs to be evaluated only at \( \rho = 0 \). In cases where the velocity error is large, the actual curvature of the gather may not be felt by the derivative at \( \rho = 0 \). To ensure that \( \partial S_m/\partial \rho \) detects the correct sign of the curvature of the ADCIG, I apply Gaussian kernel smoothing along axis \( \rho \) to the semblance before taking the derivative. (The use of this kind of smoothing was first introduced by Toldi (1985).)

As for the term \( \partial \rho/\partial s \), recall that in my formulation, I assume “\( \mathbf{M}_\rho I(s_0) \) is kinematically similar to \( I(s) \)”. Justifying this statement mathematically is the key step in deriving the operator \( \partial \rho/\partial s \). In principle, the methodology I use in the derivation is analogous to the one used by Luo and Schuster (1991) for the (so called) “travel-time wave-equation tomography” method. While I still use the Born approximation to describe wavefield perturbation with respect to slowness perturbation, I overcome the limitations of the Born approximation in that the kinematics rather than amplitudes are linearized with respect to slowness. A similar concept is also presented by Marquering et al. (1998).

My method splits the \( \partial \rho/\partial s \) calculation into two steps: first it converts the perturbation of moveout curvature parameter, \( \Delta \rho \), to the vertical shift of the ADCIG event, \( \Delta \zeta \), at each reflection angle through a least-squares fitting formula; then it follows the chain \( \Delta \rho \rightarrow \Delta \zeta \rightarrow \Delta s \) to compute \( \Delta s \).

**Relationship \( \Delta \rho \rightarrow \Delta \zeta \)**

Let us denote \( N_\gamma \) as the number of samples along the reflection angle axis \( \gamma \). Suppose we focus on one event in the ADCIG at a certain location. Recall the RMO-based objective function 2.7 in which we are interested in the kinematic differences between the new ADCIG \( I(s) \) and the initial ADCIG \( I(s_0) \); we denote such differences with \( \{(\gamma_i, \zeta_i) \mid i = 1, 2, ..., N_\gamma \} \), where \( \zeta_i \) is the depth difference of the same event at angle \( \gamma_i \) between \( I(s) \) and \( I(s_0) \). Then, using the chain rule in calculus, we have:

\[
\frac{\partial \rho}{\partial s} = \sum_{i=1}^{N_\gamma} \frac{\partial \rho}{\partial \zeta_i} \frac{\partial \zeta_i}{\partial s}, \tag{2.9}
\]
To find out how $\rho$ and $\zeta$ are associated, note that the RMO curvature parameter $\rho$ also characterizes the kinematic differences between $I(s)$ and $I(s_0)$; we thus can fit $(\gamma_i, \zeta_i)$ with the parabolic moveout function: $\zeta(\gamma) = \zeta_0 + \rho \tan^2 \gamma$. We define the best-fitting zero-angle depth and curvature $(\zeta_0$ and $\rho$) values as follows:

$$\langle \zeta_0, \rho \rangle = \arg\min_{\zeta_0, \rho} \sum_{i=1}^{N_\gamma} \{ (\zeta_i - \rho \tan^2 \gamma_i - \zeta_0)^2 \}. \quad (2.10)$$

$\zeta_0$ and $\rho$ is very easy to solve with simple calculus. We denote $\bar{x} = \sum_i x_i$ (the sum of quantity $x$ over all reflection angles $\gamma_i$), and then we have:

$$\rho = \frac{N_\gamma \zeta \tan^2 \gamma - \tan^2 \gamma \zeta}{N_\gamma \tan^4 \gamma - (\tan^2 \gamma)^2}.$$  

By taking the partial derivative $\partial \rho / \partial \zeta_i$, it is easy to find $\Delta \rho$ if there is a depth perturbation $\Delta \zeta_i$:

$$\frac{\partial \rho}{\partial \zeta_i} = \frac{N_\gamma \tan^2 \gamma_i - \tan^2 \gamma}{N_\gamma \tan^4 \gamma - (\tan^2 \gamma)^2}. \quad (2.11)$$

**Relationship $\Delta \zeta \rightarrow \Delta s$**

As for $\partial \zeta / \partial s$, since $\zeta$ is defined as the event depth difference between $I(s_0)$ and $I(s)$ at certain reflection angle $\gamma$, the key step is to translate the following statement into mathematical form:

If I shift the event in the initial image by the amount of $\zeta$, the resulting image trace $I(z + \zeta, \gamma, x; s_0)$ will agree with the new one $I(z, \gamma, x; s)$ in terms of kinematics.

We can achieve this translation by finding the maxima of the following auxiliary objective function (Zhang and Biondi, 2013):

$$J_{aux}(\zeta) = \sum_{z_w} I(z + z_w + \zeta, \gamma, x; s_0) I(z + z_w, \gamma, x; s) \quad \text{for each } z, x, \gamma. \quad (2.12)$$
in which \( z_w \) again represents the summation window ranging from \([-L/2, L/2]\), assuming \( L \) is the average event window size.

Essentially, the auxiliary objective function 2.12 determines the relation between \( s \) and \( \zeta \). This methodology is analogous to the one used by Luo and Schuster (1991) for the cross-well tomography problem. While I still use Born approximation as a component in this approach, I overcome the limitation of Born approximation and benefit from the advantages of Rytov approximation. The Born approximation does not change the image’s moveout, but only rotates the phase of the event by \( \pm 90^\circ \) (Woodward, 1992). In contrast, if we use the Rytov approximation, slowness perturbation will result in the shift of the image events, which is exactly what is desired. However, there is a serious limitation of the Rytov approximation: it cannot handle multiple arrivals. To enjoy the best of both worlds, the auxiliary objective function approach I use can predict the shift of gather events given model perturbation and meanwhile is not subject to the shortcoming of the Rytov approximation with regard to handling multiple events.

From the auxiliary objective function 2.12, we can find \( \partial \zeta / \partial s \) using the rule of partial derivatives for implicit functions. Since \( \zeta \) maximizes objective function 2.12,

\[
\frac{\partial J_{\text{aux}}}{\partial \zeta} = 0. \tag{2.13}
\]

Then we can differentiate equation 2.13 with respect to \( \zeta \) and \( s \), which yields

\[
\frac{\partial^2 J_{\text{aux}}}{\partial \zeta^2} \frac{\partial \zeta}{\partial s} - \frac{\partial^2 J_{\text{aux}}}{\partial \zeta \partial s} \frac{\partial \zeta}{\partial s} = -\frac{\partial^2 J_{\text{aux}}}{\partial \zeta \partial s} \frac{\partial^2 J_{\text{aux}}}{\partial \zeta^2}. \tag{2.14}
\]
From equation 2.12 we can find

\[
\frac{\partial J_{\text{aux}}}{\partial \zeta} = \sum_{z_w} \dot{I}(z + z_w + \zeta, \gamma, x; s_0)I(z + z_w, \gamma, x; s) \\
\frac{\partial^2 J_{\text{aux}}}{\partial \zeta^2} = \sum_{z_w} \ddot{I}(z + z_w + \zeta, \gamma, x; s_0)I(z + z_w, \gamma, x; s) \\
\frac{\partial^2 J_{\text{aux}}}{\partial \zeta \partial s} = \sum_{z_w} \frac{\partial I(z + z_w, \gamma, x; s)}{\partial s} \dot{I}(z + z_w + \zeta, \gamma, x; s_0) \\
\]

in which \( \dot{I} \) and \( \ddot{I} \) indicate the first and second derivatives of the image gathers along axis \( z \) (depth), respectively. Then we substitute the above three equations into equation 2.14 at \( s = s_0, \zeta = 0 \):

\[
\left. \frac{\partial \zeta}{\partial s} \right|_{s=s_0} = -\frac{\sum_{z_w} \frac{\partial I(z + z_w, \gamma, x; s)}{\partial s} \dot{I}(z + z_w, \gamma, x; s_0)}{\sum_{z_w} \dot{I}(z + z_w, \gamma, x; s_0)I(z + z_w, \gamma, x; s_0)}. \tag{2.15}
\]

Each part in equation 2.15 has a clear physical interpretation: 1) the denominator term normalizes the amplitude of different events; 2) the \( \frac{\partial I(z + z_w, \gamma, x; s)}{\partial s} / \partial s \) term is indeed the wave-equation image-space tomographic operator (Biondi and Sava, 1999) that converts the image perturbation into the slowness model perturbation by linearizing the wave-equation; 3) the back-projected image, \( \dot{I}(z + z_w + \zeta, \gamma, x; s_0) \) is built based on the initial image and has a first-order \( z \) derivative that introduces a proper 90° phase shift, ensuring a well-behaved slowness gradient from the tomographic operator.

To illustrate the sensitivity kernel defined in equation 2.15, I use a simple example in which there is a uniform background velocity of 2000 m/s and one single reflector. The slowness sensitivity kernel is calculated by back-projecting a shift perturbation \( \Delta \zeta(\gamma, x) \) that has one single spike at \( \gamma = 30^\circ, x = 0 \). Figure 2.2a shows the sensitivity kernel when the reflector is flat, and Figure 2.2b shows the sensitivity kernel with a dipping reflector (dip angle = 20°). It is clearly shown in these two plots that this operator will project the slowness perturbation along the corresponding wave path based on the location and reflection angle of the image shift.
Now I can visually illustrate the $\partial \rho / \partial s$ sensitivity kernel described in equation 2.9. The calculated sensitivity kernel $\partial \rho / \partial s$ is shown in Figure 2.3, in which I place a single perturbation of $\rho$ at the midpoint of the reflector and project the perturbation back to slowness using the chain relation $\Delta \rho \rightarrow \Delta \zeta \rightarrow \Delta s$. As with the Toldi operator (Toldi, 1985), the characteristic shape of such a sensitivity kernel is a center lobe with two side lobes of opposite polarity, which reaffirms the well-known fact that velocity perturbation at the center and at the two sides will change the curvature of ADCIG toward opposite directions. Yet, the lateral average of the sensitivity kernel is positive at all depths, which would give the correct update in case of a bulk-shift slowness error.

If we review the gradient calculation in equation 2.8, the success of my method simply relies on the proper behavior of the two components in the formula: $\partial J_{s_m}/\partial \rho$ needs to correctly detect the curvature of the ADCIG so that the inversion will choose a moveout direction that properly flattens the gathers; $\partial \rho / \partial s$ needs to properly convert the curvature perturbation to the update in slowness model space.

The inversion workflow

My RMO-based WEMVA formulation can be summarized in the following way: first, it defines the amplitude-normalized angle-stack objective function 2.2; but because the model gradient computed directly from this objective function is susceptible to the cycle skipping problem, my method then, at every iteration, approximates the objective function 2.2 with the RMO-based alternative objective function 2.7, which is robust against cycle-skipping and produces a much better behaved gradient (as will be shown in the remaining chapter). Therefore, in the iterative inversion setup, my method uses objective function 2.2 as the optimization goal while using the search direction computed from the RMO-based alternative objective function 2.7 at each iteration. Although in theory the normalized angle-stack objective function 2.2 might still be affected by cycle-skipping and trapped in local maxima, my experiences from working on all of the examples presented in this dissertation allow me to conclude
Figure 2.2: Slowness sensitivity kernel at incident angle $\gamma = 30^\circ$ for a flat reflector (a) and a $20^\circ$ dipping reflector (b). The reflector depth is around 850 m.
confidently that the objective function 2.2 is able to achieve global convergence under large velocity errors thanks to superior model gradients provided by the approximated RMO-based objective function 2.7.

In terms of the nonlinear solver in my inversion workflow, I use the Polak-Riberie variant of the nonlinear conjugate-gradient (CG) algorithm. Li (2014) presents a detailed recipe for this algorithm. For each iteration, I set the current slowness model to $s_0$ and migrate the recorded data to obtain $I(s_0)$, and then calculate the gradient using equation 2.8. Then, the search direction is computed from the gradient using the formula proposed in the Polak-Riberie CG algorithm. Consequently, I perform a line search on objective function 2.2 using the obtained search direction. After that, I update the model $s$ with my estimate of best model perturbation, and then move onto the next iteration.

The line search I implement (Vigh and Starr, 2008) evaluates the objective function at the current model point and two more model points along the search direction.
Given a step size $\alpha$, search direction $g$ and the current model $s$, the line search algorithm generates and evaluates two extra model points at $s + \alpha g$ and $s + 2\alpha g$. Then the algorithm fits a parabola based on those three points and finds the optimal point (apex) on the parabola. Last, it evaluates the objective function at that optimal point, compares all four model points it has evaluated, and selects the best one as the updated model. Note that during each iteration (except the first one), the objective function value for the first evaluation is already available from the previous iteration; therefore, computationally only three function evaluations are required for each iteration. As for how to choose the step size $\alpha$, the starting step size in the first iteration is set such that the distances between the starting model and each of the two line search model points are 0.05 and 0.1 times of the $\ell_2$ norm of the starting model. For later iterations, the step size at that iteration is set by the optimal step size from its previous iteration.

**NUMERICAL EXAMPLES**

I created several simple synthetic examples to verify the effectiveness of the proposed method. In these examples, I use one-way propagators in modeling and inversion, for the sake of computational efficiency. To control the resolution of the inversion among iterations, I re-parameterize the model space using coarsely sampled B-spline nodes. By gradually increasing the number of spline nodes as the number of iterations increases, I constrain the inversion to resolve the low-wavenumber part of the model first, then gradually move up to a higher wavenumber to retrieve more model details (Biondi, 1990). For comparison, I also implement the stack-power maximization method (objective function 2.1). I use the same spline node setting and the same optimization parameters for both methods. For each figure in the examples, I use the same clip for all subplots.
Simple synthetic models

In these experiments, the model is 16 km in \( x \) and 1.2 km in \( z \); the grid sampling is 20 m in \( x \) and 10 m in \( z \). The survey geometry follows a marine streamer acquisition pattern with 4 km cable length, receiver spacing of 20 m and a total of 150 shots simulated from \(-6\) km to \(+6\) km with a spacing of 80 m. A total of 106 frequencies are calculated, ranging from 5 Hz to 40 Hz. Unless explicitly mentioned, there is one flat reflector at a depth of 800 m and a constant initial velocity of 2000 m/s.

In the first example, the true model is a constant background velocity (2000 m/s, same as the starting velocity) with a 1.0 km-wide Gaussian anomaly at the center. The peak value of the Gaussian anomaly is 4000 m/s. Figure 2.4a shows the true model (in slowness), and Figure 2.6a shows the migrated zero subsurface-offset image using a constant initial velocity of 2000 m/s. The originally flat reflector is pulled up significantly at the center due to the velocity error.

Figure 2.5 shows the first slowness update using the two methods: 1) my RMO-based WEMVA and 2) stack-power maximization WEMVA. My method generates a better model update. The anomaly is correctly located, and the target region is more uniformly updated, except for the small holes within the “W” shape due to the poorer wave path coverage caused by the curved reflector. The stack-power maximization method suffers from cycle skipping and is not able to locate the area that needs updating. Figure 2.4b shows the inverted model (in slowness) using the RMO-based WEMVA method. The stack-power maximization WEMVA inversion does not converge due to the poor model update shown in Figure 2.5b; therefore, that inversion result is not shown. If we compare the true model (Figure 2.4a) and the inverted model (Figure 2.4b), note that the peak velocity value (~2700 m/s here and following) of the anomaly in the inverted model is much weaker than the true value (4000 m/s). No known tomographic method can recover exactly the true model due to the well-known under-determination of the reflection tomography problem.

Figure 2.6 shows a comparison of the migrated images using the initial model, the inverted model by the RMO-based WEMVA and the true model. The corresponding
angle gathers in these three cases are shown in Figure 2.7. The RMO-based WEMVA inversion results in an improved image, although the improved image is not the same as the true image (flat reflector). This is consistent with our observation on the slowness models. As we can see from the angle gathers plot (Figure 2.7), the initial angle gathers are not flat, but after the inversion all the gathers then become flat, meaning that WEMVA cannot do better unless more information is provided to resolve the null-space in the model.

In the second example, the true velocity model has a horizontal gradient, where velocity increases linearly from 1650 m/s to 2500 m/s. Figure 2.8a shows the true

![Figure 2.4: (a) The true slowness model with a Gaussian anomaly; (b) the inverted slowness model using the RMO-based WEMVA.](image_url)
Figure 2.5: The first slowness update direction from (a) the RMO-based WEMVA and from (b) the direct stack-power maximization WEMVA; refer to Figure 2.4b for the true model.
Figure 2.6: The migrated images using (a) the initial slowness model, (b) the inverted model from the RMO-based WEMVA and (c) the true model; refer to Figure 2.4 for the models.
Figure 2.7: The angle gathers corresponding to the images shown in Figure 2.6. The gathers shown are extracted at positions $x = 0.0, 1.0$ and 2.5 km.
model (in slowness) and Figure 2.10a shows the migrated zero subsurface-offset image using the initial constant slowness model. The originally flat reflector is tilted in the image because of the horizontal slowness gradient. Figure 2.9 shows the first slowness updates using the two methods. Again my method presents a better behaved gradient. The amplitude of the model update is proportional to the actual magnitude of the slowness error. The stack-power maximization method is able to predict correctly when the slowness error is small (around \( x = 0.0 \) km). However, as the slowness error reaches a certain threshold, cycle-skipping happens and the model update becomes incorrect. The result computed by the RMO-based WEMVA method does not suffer from the cycle-skipping problem. Figure 2.8b shows the inverted model (in slowness) by the RMO-based WEMVA method. Figure 2.10 shows a comparison of the migrated images using the initial model, the inverted model and the true model. The corresponding angle gathers in the three cases are shown in Figure 2.11. The improvement on the angle gather flatness can be easily observed.

For the third example, the true velocity is a constant 1900 m/s background plus a high velocity anomaly at \( x = 2000 \) m with peak value 2850 m/s. As for previous example, the starting model is constant and equals to 2000 m/s. I designed a hump-shaped reflector to test my RMO-based WEMVA method’s ability to handle reflectors with variable geological dips. Figure 2.12a shows the true model (in slowness) and Figure 2.14a shows the migrated zero subsurface-offset image. Figure 2.13 shows the first slowness update using my method. The effect of the high velocity anomaly is partially cancelled by the lower constant part (1900 m/s). Nonetheless, the overall result still requires a negative update at the anomaly’s location. In the presence of a variably dipping reflector, my method still yields a satisfying result. Figure 2.12b shows the inverted model by the RMO-based WEMVA. My method recovers both the velocity anomaly at \( x = 2.0 \) km and the bulk-shift velocity difference in the remaining part of the velocity model. Figure 2.14 shows a comparison of the migrated images using the initial model, the inverted model and the true model. Figure 2.15 compares the ADCIG in these three cases. Again, the angle gathers are fully flattened but not exactly the same as the true angle gathers. Specifically, note that the true gathers at \( x = 2.0 \) km has narrower angular range due to the lense effect of the overhang
Figure 2.8: (a) The true slowness model in the second example, with a horizontal gradient from 1650 m/s to 2500 m/s; (b) the inverted slowness models using the RMO-based WEMVA. [CR] vel2.hg, vel2.hginv
Figure 2.9: The first slowness update directions of (a) the RMO-based WEMVA and of (b) the stack-power maximization WEMVA. The true model refers to Figure 2.8a.

[CR] dS21w-hg,dS21w-hg-1
Figure 2.10: The migrated images using (a) the initial slowness model, (b) the inverted model from the RMO-based WEMVA and (c) the true model; refer to Figure 2.8 for the models.
Figure 2.11: The angle gathers corresponding to the images shown in Figure 2.10. The gathers shown are extracted at positions $x = -4.5, -2.5, 0.0, 2.0$ and $5.0$ km.
Gaussian anomaly; while in the inverted model, the shape of the anomaly is not exactly recovered, and therefore the angle gather at $x = 2.0$ km does not appear to have narrow range. More information is required in order to further improve the inverted model.

Figure 2.12: (a) The true slowness model in the third example, with both a constant slower velocity 1900 m/s and a high velocity anomaly at $x = 2000$ m. The anomaly’s peak velocity is 2850 m/s. (b) The inverted slowness model using the RMO-based WEMVA. [CR] vel2.bump,vel2.bumpinv
Marmousi Example

I also test my method on a more geologically realistic example: the Marmousi model. The model is 6 km in $x$ and 1.6 km in $z$. The spatial sampling is 20 m. The survey geometry follows the land acquisition pattern with receivers at every surface location on the top. I simulate 51 shots, covering the whole lateral span of the model with a spacing of 120 m. I run 30 nonlinear iterations for the inversion.

Figure 2.16a shows the true model (in velocity) and 2.16b shows the starting model, which has a vertical gradient increasing from 1600 m/s and 3200 m/s. Note here that I plot velocity rather than slowness, since in literatures this well-known model is usually shown in velocity. Figure 2.16c shows the inverted velocity model using the RMO-based WEMVA method. Figure 2.17 shows a comparison of the migrated images using the initial model, the true model and the inverted model. It is clear that the inverted velocity model improves the focusing of the migrated image and positions the reflectors closer to their true locations. The markings highlight the improvements. Figure 2.18 shows the comparison between the initial ADCIG and the ADCIG migrated using the inverted model. The improvement of gather flatness is also observed. Note one limitation of the proposed method: there are a few “W” and “M” shaped events present in the gathers shown in Figure 2.18b. Because I use a
Figure 2.14: The migrated images using (a) the initial slowness model, (b) the inverted model by the RMO-based WEMVA, and (c) the true model; refer to Figure 2.12 for the models. [CR] img-bump, img-bumpinv, img-bumpT
Figure 2.15: The angle gathers corresponding to the images shown in Figure 2.14. The gathers shown are extracted at positions $x = -4.6, -3.0, 2.0,$ and $4.0$ km.
single moveout parameter that can only describe a parabolic moveout \((\rho \tan^2 \gamma)\), my method can not further flatten these “W” or “M” shaped moveouts.

Figure 2.16: (a) The slightly smoothed true velocity model of Marmousi, (b) the starting velocity model and (c) the inverted velocity model using the RMO-based WEMVA method. [CR] [vels-marm-2]

IMPLEMENTATION REFINEMENT FOR RMO-BASED WEMVA

The examples shown in the previous sections have verified the validity of the proposed RMO-based WEMVA method. During the actual implementation and experimentation, I have noticed a major opportunity for improvement: If we recap the gradient calculation in equation 2.7 and 2.8: I made an assumption that every image point \((z, x)\) has an independent moveout parameter \(\rho\). I also assumed the windows
Figure 2.17: The migrated zero subsurface offset images using the velocity models shown in Figure 2.16, i.e., (a) the true model, (b) the starting model and (c) the inverted model.
Figure 2.18: Comparison between the ADCIG migrated using (a) the initial model and (b) the inverted model by the RMO-based WEMVA. The angle gathers are extracted from the inline locations of $x = 7.0, 8.2, 9.2, 10.2$ and $11.2$ km. [CR] adcigs.marm.zg,adcigs.marm.bf
(span over z axis) of all reflector events are roughly of equal length and therefore can be summarized with an average event window size $L$. The main rationale behind making such assumptions is for the simplicity of formulation and coding implementation. However, assigning $\rho$ for every image points is not only unnecessary but also inconsistent with my method’s physical interpretation. Indeed, I should assign moveout parameter $\rho$ to individual events instead of individual image point locations; moreover, I should assign a different window size for each event because there can be large variations of the window sizes among different events.

Switching the assumption from moveout per image point to moveout per event is expected to improve the accuracy of the computed model gradient and also marginally reduce the proposed method’s computation cost since it cuts the computation of image perturbation. However, it also brings additional complexity for implementation because now I need to identify individual events in the image gathers.

Before I dive into the details on creating an event detection module, let me start with the design objectives. First and foremost, the event detection method has to be fully automatic with minimum number of parameters that requires manual intervention. Otherwise, it will require human intervention for every iteration, which is impractical. Second, the event detection method needs to be simple to explain and simple to implement. Last, although it is impossible to design an automatic event detection method that is 100% accurate, the inversion can tolerate some inaccuracies. Because the gradient is formed by the combined influences from all identified events, as long as the majority of events are correctly identified and have their RMO information correctly extracted, the global convergence will be retained. On the other hand, in some challenging situations where part of the image has very poor quality, there is no known correct answer because even the human user cannot confidently identify events.

**Automatic event detection**

I propose a straightforward workflow for automatic event detection as follows:
• My program first detects the center locations of individual events in the image \((z, x)\) as “anchor points”, instead of blindly assuming that every image sample is its own event.

• For each anchor point detected, my program detects the event width by measuring the event signature (waveform) compactness around the anchor point location.

Anchor points detection

The basic idea for my anchor point detection workflow is inspired by the methodology proposed in Cullison (2011). First my method uses several image processing techniques to extract all potential event anchor point candidates, and then filter out a portion of candidates based on several geophysical criteria before outputting the final set of anchor points. Let me denote the stacked image (zero subsurface offset image) \(I(z, x)\). To find candidate anchor points, I perform the following steps:

1. Apply automatic gain control (AGC) to the stacked image \(I_{bal}(z, x)\), in order to broadly balance the amplitude of reflectors in the image.

2. Take energy envelope (magnitude of the input’s Hilbert signal) along \(z\) axis on \(I_{bal}(z, x)\), denoting the output as \(I_E(z, x)\).

3. Apply non-maximum suppression (NMS) (along \(z\) axis) to \(I_E(z, x)\). As its name suggests, non-maximum suppression will suppress all samples that are not local maxima in the input image to zero. It will produce a binary image of the input size, and only the locations of local maxima in the input image are set to 1, while all the rest of the output are set to 0. From this binary image, I extract the locations of all non-zero values and place them in the candidates set of the anchor points, \(S_{cand}\).

However, not all candidate points in \(S_{cand}\) are well-suited for WEMVA gradient back-projection. In other words, extra quality check (QC) is helpful to prevent noisy
data leaking to the gradient calculation. Specifically, my implementation filters out any candidate points if they do not meet one of the criteria below:

1. Because specular reflection is a basic assumption of reflection tomography methods, I screen out all candidate points that do not satisfy specular reflection requirement.

   I achieve this screening by computing *linearity coefficients* over all candidate points and threshold the values of linearity coefficients.

   First introduced into the field of exploration Geophysics by Hale (2009a,b), the *linearity coefficient* is a good measure to quantify how specular a reflection point is. Briefly speaking, if the linear coefficient of the target reflection point is high (close to 1.0), it indicates that there is a strong local linear reflector structure around that point. Thus, we know that the specular reflector assumption holds well. Similarly, if the linear coefficient is low (close to 0.0), it indicates that there is no locally linear reflector structure around that point (for example, the target point is on fault boundary, or is just an isolated diffractor). Therefore, the specular reflector assumption does not hold.

   The local linearity coefficients of the stacked image can be conveniently estimated using structure tensor (Cullison, 2011).

2. I also filter out candidate points that have a large dipping angle in my examples) because neither the theory nor my implementation of this RMO-based WEMVA handles steeply dipping reflectors well. The reason is explained as follows:

   - My derivation uses the vertical shift of the ADCIG to predict the velocity perturbation. In fact, the gathers always shift along the normal direction (perpendicular to the corresponding dipping interface) rather than the vertical direction. In the case of reflector dips being large, the amount of vertical shift in the gathers is significant smaller than the amount of shift along normal direction, making the RMO information extraction less sensitive and less accurate.
On the implementation side, note that I compute the ADCIG indirectly by converting from subsurface offset common-image gathers (Sava and Fomel, 2003; Biondi and Tisserant, 2004). The horizontal subsurface offset ($h_x$ in 2-D, $(h_x, h_y)$ in 3-D) gathers are known to lose resolution and accuracy for angle-domain gathers generation on steeply dipping reflectors (Biondi, 2006).

Therefore my workflow also filters out steep reflectors ($\geq 45^\circ$ in my examples). To determine the steepness of reflectors, we can again use structure tensors to conveniently estimate the local dipping angles of the stacked image (Cullison, 2011).

**Variable event window size**

After I identify the set of event anchor points, I can further detect the actual window size of each event instead of using a preset constant window size uniformly:

1. For each anchor point, extract the amplitude at that anchor location from the energy envelope image $I_E(z, x)$.

2. We can infer that each anchor point is positioned approximately at the center of the event it belongs to, because the anchor point samples the peak amplitude of the energy envelope. Therefore, my code starts from the anchor point’s location, then searches in both directions ($\pm z$) for the tails of the energy envelope.

**Illustrative example on automatic event detection**

As an intuitive explanation for the algorithms used in my automatic event detection module, I present a simplified event detection example on a 1-D seismic trace input. In the RMO-based WEMVA case, we can think of the input as an image trace along depth direction at a certain horizontal location, $I(z, x = x_0, y = y_0)$. 
Figure 2.19a shows the input 1-D signal. The goal is to detect the individual events accurately and efficiently. As we can visually identify from the plot, I deliberately create 4 distinct events in the input, each with a different signature (waveform):

1. A typical Ricker2 wavelet (second derivative of Gaussian).
2. A Ricker2 wavelet with opposite polarity.
3. A Ricker1 wavelet (first derivative of Gaussian). This wavelet is asymmetric and has a 90° phase difference compared to the Ricker2 wavelet.
4. A typical Ormsby wavelet (Ryan, 1994). This wavelet usually has bigger side lobes (more rippling) compared to the Ricker wavelet due to the steeper tapering effect on the edges of the spectrum.

These events possess different signal characteristics (variant amplitude, polarity, phase and spectrum shape) that help to demonstrate the merits of the proposed approach.

Figure 2.19b shows the energy envelope overlaying the input. As we can see, the envelope strips out many of the complexities in the original signal (like asymmetric phase, negative polarity, vibrating waveform, etc.), and singles out the “wave packet” information for each event. The simplicity brought by the energy envelope makes it a much better choice for anchor point detection instead of the original signal, as demonstrated by the following example.

Figure 2.20 shows the anchor points found by applying non-maximum suppression to different input signals. More specifically, Figure 2.20a uses the original signal as detection input, and Figure 2.20b uses the energy envelope. For comparison, I add the result using the absolute value of the original signal as input, shown in Figure 2.20c. Apparently, the result using the energy envelope (Figure 2.20b) is the most accurate. Figure 2.20a result fails on the second event that has a negative polarity. Because of this negative polarity, the two local maxima it finds are indeed the peaks of two side lobes rather than the main lobe. Figure 2.20a result also fails on the last event due to the severely vibrating waveform, that the computer program picks up
both the peak of main lobe and the first two strong side lobes. Figure 2.20c result is able to handle negative polarity in the second event because it uses the absolute value of the original signal as input. However, it fails on the third and fourth event. The result contains several false positives because the absolute value function converts local minima into local maxima.

After locating the event anchor points, Figure 2.21 shows the detected event window sizes of each anchor point for the corresponding test cases in Figure 2.20. Again from top to bottom, it shows the results using the (a) original signal, (b) the energy envelope and (c) the absolute value of the signal as input, respectively. The detected event windows are drawn as box functions with dash lines. As we can see, the result using energy envelope is the most accurate.

**Formulation change after introducing event detection**

Once we figure out how to do automatic event detection, the modification on implementation becomes simple. In general, I only need to slightly modify the summation indices in the gradient calculation derivations above. Instead of summing over every image point, my method now sums over each identified event anchor point.

For example, the semblance-based objective function 2.7 now becomes

\[
J_{S_m}(\rho(s)) = \sum_{k=1}^{\left|S_{\text{anchor}}\right|} S_m(\rho(z_k, x_k), z, x; s_0),
\]  

(2.16)

in which \(S_{\text{anchor}} = \{(z_k, x_k) | k = 1, 2, \ldots, \}\) is the set of event anchor points, and \(\left|S_{\text{anchor}}\right|\) denotes the size of this set. Similarly, the gradient formula 2.8 becomes

\[
\frac{\partial J_{S_m}}{\partial s} = \sum_{k=1}^{\left|S_{\text{anchor}}\right|} \frac{\partial \rho(z_k, x_k)}{\partial s} \frac{\partial S_m(\rho(z_k, x_k), z, x; s_0)}{\partial \rho(z_k, x_k)}.
\]  

(2.17)

In addition, I change the summation bounds of the local window variable \(z_w\) in the semblance formula 2.6. Originally the bounds are fixed at \([-L/2, +L/2]\), but now the
Figure 2.19: An example to illustrate the event detection workflow. (a): the input 1-D signal; (b): the input with its energy envelop overlaid. [CR] trace-input,trace-envelop-overlay
Figure 2.20: The results of detected anchor points (shown as × marks) using different input signals: (a) the original signal as input; (b) the energy envelope as input; (c) the absolute value of the original signal as input.

[CR]
ori-maxima, env-maxima, abs-maxima
Figure 2.21: The results of detected event windows (shown as dash lines) using different inputs: (a) the original signal as input; (b) the energy envelope as input; (c) the absolute value of the original signal as input.
event window sizes are determined on individual bases. In other words, the bounds are set by each event’s window size detected by the proposed algorithm.

These aforementioned formulation changes are very easy to implement in computer software.

**INVERSION EXAMPLES USING THE REFINED IMPLEMENTATION**

With the improved workflow to better handle multiple events in the RMO-based WEMVA workflow, I test my implementation on a couple of more complex examples.

**Marmousi model: refined implementation**

In this example, the experiment setup (true and starting models, acquisition settings, inversion parameters, etc.) is exactly the same as the Marmousi example shown previously. The only difference is that the improved gradient implementation is used.

Figure 2.22 demonstrates the proposed automatic event anchor point detection workflow, which starts with the migrated zero subsurface-offset image, and then applies a sequence of image processing techniques (automatic gain control (AGC), energy envelop and non-maximum suppression (NMS)) to the input to obtain an image containing only the anchor point candidates. As can be seen from the figure, the extraction has achieved good accuracy. More importantly, the whole process is fully automatic and no manual picking is involved.

Figure 2.23 compares (a) the true model, (b) the starting model with vertical velocity gradient, (c) the inverted model using the previous implementation of RMO-based WEMVA (without event detection) and (d) the inverted model using the refined implementation. It is clear that the result with the new implementation is closer to the true model than the previous result is. The shapes of the velocity features in the new result are much more consistent with those in the true model. Figure
2.24 compares the migrated images using the initial model, the true model and the inverted model. With the aids of the grid lines and the markers, we can conclude that reflector positions in the new inversion result are more correct than those in the previous inversion result. Figure 2.25 shows the comparison between the ADCIG migrated using the previous inverted model and the ADCIG migrated using the newly inverted model. We are able to observe the further improvements in gather flatness from the new inversion result.

In addition, note that the new result (Figure 2.23d) is obtained with only 20 nonlinear iterations, fewer than the 30 iterations used for the previous inversion result (Figure 2.23c). The above comparisons of velocity models, images and angle gathers allow me to conclude that even with fewer iterations, the refined implementation results in a better model.

**BP synthetic model**

I also test this new implementation on a portion of the BP synthetic model. The model is extracted from the upper-left part of the original BP model, with a size of 25 km in $x$ and 8 km in $z$. The spatial sampling of the model is 25 m in $x$ and 12.5 m in $z$. I synthesize several flat reflectors to simulate the seismic data, and use marine streamer geometry to simulate 200 shots on the top of the model, with receiver streamers towing from left to right. The shots start from $x = -5$ km, and the spacing is 125 m. There are 401 receivers on the 10 km long streamer with a spacing of 25 m. A total of 201 frequencies are modeled using the one-way wave-equation, ranging from 5 Hz to 30 Hz.

Figure 2.26a shows the true model (in velocity) and Figure 2.26b shows the starting model, in which I assume the complex salt overburden has been well resolved but the sediments velocity below the salt follows an over-simplified linear vertical trend from 3.4 km/s to 4.4 km/s. Because I assume the velocity above the salt is accurate, I apply a mask during the inversion so that only the subsalt region of the model is updated.
Figure 2.22: An illustration of the proposed automatic event point detection workflow: it starts with the migrated zero subsurface-offset image, and then a sequence of image processing techniques (Automatic Gain Control (AGC), Energy Envelop and Non-Maximum Suppression (NMS)) are applied to the input. The final processed image contains the detected event anchor point candidates as non-zero values.
Figure 2.23: (a) The slightly smoothed true velocity model of Marmousi; (b) the starting velocity model; (c) the inverted velocity models using the previous implementation of RMO-based WEMVA (without event detection); (d) the inverted velocity models using the new refined implementation of RMO-based WEMVA. [CR] vels-marm-refined-4
Figure 2.24: The migrated zero subsurface offset images using the velocity models shown in Figure 2.23, i.e., (a) the true model, (b) the starting model, (c) the previously inverted model and (d) the newly inverted model by the refined RMO-based WEMVA implementation. With the aids of the grid lines and the markers, we can see that image (d) has less positioning error than image (c) does with regard to the true image (a). For example, the circled reflector in the previously inverted image (c) is shallower than its counterparts in image (a) and (d); and the arrow on the top points to the zero-crossing of a flat reflector on both image (a) and (d), while its counterpart in image (c) is aligned with the positive peak of the same reflector. [CR] LaTEX command for images-marm-refined-4
Figure 2.25: Comparison among the ADCIG migrated using (a) the previously inverted model with the RMO-based WEMVA and (b) the inverted model by the refined RMO-based WEMVA implementation (Figure 2.23c,d). The angle gathers are extracted from the inline locations of $x = 7.0, 8.0, 9.0, 10.1$ and $11.2$ km. [CR] adcigs2.marm.bf,adcigs2.marm.bfae
Figure 2.26c shows the inverted velocity model after 40 iterations. The inversion result shows good convergence to the true model; The major velocity features in the true model are all recovered in the inverted model, as highlighted by the markers.

Figure 2.27 compares the migrated images using the velocity models in Figure 2.26. These images are plotted using the same clip, so that we can easily observe the improvement of the reflector coherence by the inversion. Especially in the region indicated by the arrows and boxes, the reflectors become much more continuous and better positioned (being flat). Moreover, Figure 2.28 compares the ADCIG between the starting velocity model, the true model and the inverted velocity model. We can observe significant improvement of gathers flatness by the inversion. Even in the circled area where it is not accurate to use parabolic moveout to characterize the shapes of the initial gathers, my RMO-based WEMVA still manages to further flatten these gathers; mainly because in this example, the inversion has abundant reflection events to utilize, and therefore, the overall model gradient maintains good quality in spite of the RMO characterization inaccuracies in some events.

In this example, the complex salt overburden causes very uneven subsalt illumination, which in turn leads to amplitude variations along the reflection angles in the ADCIG. As can be seen from the ADCIG shown in Figure 2.28, even with the true velocity model, some angle gathers have narrower angular ranges and some have illumination holes at certain angles. Such amplitude variations would have caused trouble to WEMVA methods that build objective functions directly based on gather amplitudes as they try to compensate for the variations by perturbing the slowness model. However, the RMO-based WEMVA method is not adversely affected by these variations because this method is purely based on kinematics, therefore it does not wrongly try to balance the amplitude variations of gathers.

CONCLUSION

In this chapter, I have presented a new method to perform wave-equation migration velocity analysis named RMO-based WEMVA. It can robustly capture the moveout
Figure 2.26: (a) The true velocity model of the upper-left part of the BP 2006 model, (b) the starting velocity model, and (c) the inverted velocity model using my refined RMO-based WEMVA implementation. The markers highlight the model improvement by the inversion. [CR] [bp2Vels]
Figure 2.27: The migrated images using the velocity models shown in Figure 2.26. The markers highlight the image improvement by the inversion. [CR] [bp2imgs]
Figure 2.28: The ADCIG migrated using the velocity models shown in Figure 2.26. The angle axis ranges from $-45^\circ$ to $45^\circ$, with a sampling of $1.5^\circ$. We can see that the flatness of ADCIG improves significantly from the initial model to the inverted model.
information from the ADCIG and properly back-projecting that information into the velocity model space. I demonstrated this promising approach using several examples. As shown by my examples, this method does not suffer from cycle-skipping, does not require moveout parameters picking, and can robustly improve the flatness of the angle gathers. This chapter only describes the two-dimensional version of the RMO-based WEMVA method. I will present the 3-D extension in the next chapter.
Chapter 3

3-D extension of RMO-based WEMVA

In the previous chapter, I have demonstrated that my proposed RMO-based WEMVA approach is robust to the cycle-skipping problem, and that it produces well-behaved gradients which lead to high-quality inversion results. In this chapter, I address the major theoretical and practical challenges of extending this method to three-dimensions (3-D). Specifically, I propose an upgraded parameterization for the 3-D angle-domain residual moveout by treating each azimuth angle independently. This results in a simple 3-D RMO-based WEMVA theory framework. I also address the practical issue in 3-D transforms between subsurface offset and angle-domain image gathers.

INTRODUCTION

The advancement of acquisition technologies like wide-azimuth (WAZ) recording and coil-shooting has been proved to yield significant imaging improvements in geologically challenging areas (Michell et al., 2006; Barley and Summers, 2007; Michele Buia, 2008). This ongoing trend necessitates the extension of my proposed RMO-based WEMVA framework to three-dimensions (3-D). Through this extension, my method...
becomes able to invert simultaneously velocity information from multiple azimuths.

The are only a few published studies of WEMVA method extensions to 3-D. This is primarily because the majority of WEMVA research is carried out in academia, and very few academic institutions have the technical capability and computing resources to perform 3-D WEMVA inversion. Among the limited literature, the majority of the studies addressing this topic come from researchers in SEP (Stanford Exploration Project). Li (2014) implemented a 3-D version of VTI (Vertical Transverse Isotropy) WEMVA using the stack-power maximization objective function. Although the stack-power maximization objective function has serious shortcomings (as explained in the previous chapters), the advantage of using this objective function is that it is very simple (almost trivial) to extend into 3-D. Since the method simply computes the zero subsurface offset image \( I(h = 0) \), this becomes \( I(h_x = 0, h_y = 0) \) in the 3-D case. Tang (2011) implemented a 3-D version of WEMVA using the Differential Semblance Optimization (DSO) objective function. This extension is also quite straightforward, as \( \| I(h) \|_2^2 = \| I(h_x, h_y) \|_2^2 \). It is worth noting that in practice, the 3-D DSO method is computationally more expensive than the 3-D stack-power method maximization because the extra cost of computing and storing the full 5-D \((z, x, y, h_x, h_y)\) subsurface offset gathers becomes non-trivial.

In this chapter, I also aim to present a straightforward 3-D extension of my proposed RMO-based WEMVA approach. I will show that by simply adding an independent azimuth axis, the theory can be made applicable to 3-D under minor modifications. The examples I show verifies that such 3-D extension is able to extract variant velocity information over different azimuths.

**EXTENSION TO 3-D: THEORY**

I first review the key derivations of the RMO-based WEMVA theory in the 2-D case, and then present the 3-D generalization of the theory by analogy. To maintain the consistency of notation with previous chapters, the model space is parameterized as slowness unless otherwise specified.
As we shift from 2-D physical domains to 3-D ones, two extra dimensions (crossline axis $y$ and subsurface azimuth $\phi$) are added to the angle-domain common-image gathers (ADCIG). Therefore, $I(z, \gamma, \phi, x, y)$ denotes the prestack image, assuming there are $m$ samples along the $\phi$ axis, then $\phi = \{\phi_i : i = 1, 2, ..., m\}$.

The original 2-D amplitude-normalized stack-power maximization objective function

$$J_{2D}^N(s) = \sum_x \sum_z \sum z_w \left[ \sum_{\gamma} I(z + z_w, \gamma, x; s) \right]^2 \sum z_w \sum_{\gamma} I^2(z + z_w, \gamma, x; s)$$

(3.1)

can be generalized to

$$J^N(s) = \sum_{x,y} \sum_z \sum_{i} \sum z_w \left[ \sum_{\gamma} I(z + z_w, \gamma, \phi_i, x, y; s) \right]^2 \sum z_w \sum_{\gamma} I^2(z + z_w, \gamma, \phi_i, x, y; s),$$

(3.2)

in which we try to maximize the power of angle-stack not only for each image location $(z, x, y)$, but also for each azimuth $\phi_i$.

**RMO-based WEMVA objective function and its gradient**

The next step is to choose a proper residual moveout (RMO) parameterization for the 3-D ADCIG, in which the moveout is a 2-D surface (defined by $(\gamma, \phi)$) rather than a 1-D curve (of $\gamma$). I choose a straightforward approach in which the moveout surface is separated into individual curves by azimuth $\phi$. For each azimuth angle $\phi_i$, I independently assign the curvature parameter $\rho_i$, i.e., $\rho = \{\rho_i : i = 1, 2, ..., m\}$.

Under this parameterization, we can again approximate objective function 3.2 with an alternative RMO semblance based objective function:

$$J_{S_m}(\rho(s)) = \sum_{x,y} \sum_z \sum_{i} S_m(\rho_i, \phi_i, z, x, y; s_0),$$

(3.3)
in which

\[
S_m(\rho_i, \phi_i, z, x, y; s_0) = \frac{\sum_{z,w} \left( \sum_\gamma I(z + z_w + \rho_i \tan^2 \gamma, \gamma, \phi_i, x, y; s_0) \right)^2}{\sum_{z,w} \sum_\gamma I(z + z_w + \rho_i \tan^2 \gamma, \gamma, \phi_i, x, y; s_0)}.
\]  (3.4)

To avoid clutter, the meaning of the undefined variables (e.g. \(z_w, s_0\)) are the same as in the 2-D cases (see the explanations of equation 2.6 and 2.7 in chapter 2). The gradient formula now turns into

\[
\frac{\partial J_{S_m}}{\partial s} = \sum_{x,y} \sum_z \sum_i \frac{\partial \rho_i}{\partial s} \frac{\partial J_{S_m}}{\partial \rho_i}.
\]  (3.5)

Because each azimuth \(\phi_i\) and its corresponding RMO curvature \(\rho_i\) are treated separately, we can compute \(\partial J_{S_m}/\partial s\) in the same way we do in the 2-D case, i.e.,

- \(\partial J_{S_m}/\partial \rho_i\) is calculated by taking the first order derivative by finite differences along \(\rho_i\) on the semblance panel \(S_m(\rho_i, \phi_i, z, x, y; s_0)\).

- \(\partial \rho_i/\partial s\) is calculated following the derivation in equation 2.9:

\[
\frac{\partial \rho_i}{\partial s} = \sum_{j=1}^{N_\gamma} \frac{\partial \rho_i}{\partial \zeta_j} \frac{\partial \zeta_j}{\partial s},
\]  (3.6)

in which index \(\zeta_j\) corresponds to the angle gather shift at individual reflection angle \(\gamma_j\), \(N_\gamma\) is the number of samples along \(\gamma\) axis, and \(\partial \rho_i/\partial \zeta_j, \partial \zeta_j/\partial s\) are defined the same way they are in equation 2.11 and 2.15 presented previously, i.e.,

\[
\frac{\partial \rho_i}{\partial \zeta_j} = \frac{N_\gamma \tan^2 \gamma_j - \tan^2 \gamma}{N_\gamma \tan^4 \gamma - (\tan^2 \gamma)^2},
\]  (3.7)

and

\[
\left. \frac{\partial \zeta_j}{\partial s} \right|_{s=s_0} = -\frac{\sum_{z,w} \frac{\partial I(z + z_w, \gamma_j, \phi_i, x, y; s_0)}{\partial s} \bigg|_{s=s_0} \dot{I}(z + z_w, \gamma_j, \phi_i, x, y; s_0)}{\sum_{z,w} \dot{I}(z + z_w, \gamma_j, \phi_i, x, y; s_0) \dot{I}(z + z_w, \gamma_j, \phi_i, x, y; s_0)}.
\]  (3.8)
Refined formulation for 3-D RMO-based WEMVA using event detection

We can refine the 3-D RMO-based WEMVA formulation using the same approach as I have done in the 2-D case, by assigning a moveout parameter $\rho$ to each detected event instead of every image point location. It is simple to modify my 3-D WEMVA formulation after introducing event detection. First, the semblance-based objective function (3.3) now becomes

$$J_{Sm}(\rho(s)) = \sum_{k=1}^{|S_{anchor}|} \sum_i S_{m}(\rho_i(z_k, x_k, y_k), \phi_i, z, x, y; s_0), \quad (3.9)$$

in which $S_{anchor} = \{(z_k, x_k, y_k)|k = 1, 2, \ldots\}$ is the set of event anchor points, and $|S_{anchor}|$ denotes the size of this set. Similarly, the gradient formula (3.5) becomes

$$\frac{\partial J_{Sm}}{\partial s} = \sum_{k=1}^{|S_{anchor}|} \sum_i \frac{\partial \rho_i(z_k, x_k, y_k)}{\partial s} \frac{\partial J_{Sm}}{\partial \rho_i(z_k, x_k, y_k)}. \quad (3.10)$$

In addition, the summation bounds of the local window variable $z_w$ in the semblance formula (3.4) is no longer a fixed range $[-L/2, +L/2]$. Instead, the new summation bound for each event will reflect the automatically detected window size for that event.

Gaussian anomaly example

As a proof of concept of my straightforward 3-D extension of the 2-D theory of the RMO-based WEMVA, I created the following synthetic example to test the 3-D extension. In this example, the model is $2.4\text{ km}$ in $x$, $2.4\text{ km}$ in $y$ and $1.0\text{ km}$ in $z$; the grid sampling is $20\text{ m}$ in $x$ and $y$, $10\text{ m}$ in $z$. The receivers are fixed during the entire survey, spanning a rectangular area from $(-0.8\text{ km}, -0.8\text{ km})$ to $(+0.8\text{ km}, +0.8\text{ km})$ with receiver spacing of $20\text{ m}$ in both $x$ and $y$. The shot locations cover the same region as the receivers do, and the shot spacing is $100\text{ m}$. This leads to a survey of
81 \times 81 receivers and 17 \times 17 shots along both x and y. I computed the synthetic data using 32 frequencies, ranging from 5 Hz to 40 Hz. There is one single flat reflector at a depth of 0.6 km. The true model is a constant background velocity of 2 km/s, (the same as the starting velocity), but with a 0.6 km-wide, 0.25 km-tall Gaussian anomaly at the center, with a peak value of 3 km/s, as shown in 3.1. A constant 2.0 km/s velocity is used in the starting model.

First, I computed the initial subsurface offset domain common-image gathers (OD-CIG) with the horizontal subsurface offsets \((h_x, h_y)\), which range between ±100 m. The acquisition geometry used here is able to provide full azimuth for the inner portion of the model. Therefore, I chose to generate angle-domain common-image gathers (ADCIG) along three azimuth angles: 0°, 30° and 60°.

Figure 3.2 shows the derivatives of the semblance objective function over the moveout parameter \(\rho_i\), with each panel representing a distinct azimuth: 0, 30 and 60 degrees. Because there is only one single event along the depth axis, \(\rho(z, x, y)\) simplifies to \(\rho(x, y)\). Therefore, \(\partial J_{sm}/\partial \rho_i(x, y)\) can be plotted in Figure 3.2 as 2-D fields. Since the migration velocity is slower than the true one, most of the ADCIG are curving upward (meaning negative \(\rho\)). Therefore, a proper objective function should sense mostly negative curvature. Figure 3.2 verifies the consistency between the numerical results and the theoretical expectation. Note that in each plot there are some positive values in the direction perpendicular to the azimuth orientation. This is caused by the fact that at these locations, the seismic energy traveling through the designated azimuth probes a much smaller anomaly. (Imagine we have a spherical anomaly centered around the origin \((x^2 + y^2 + z^2 = 1)\), and that the strength of the anomaly decreases with distance from the center. If we intercept the anomaly with vertical plane \(y = 0\), the anomaly seen on the section is strong because the section passes through the center of the sphere. However, if we intercept the sphere with a plane defined by \(y = 0.7\), the anomaly seen on that plane will be much weaker and smaller because this part of the anomaly is far away from the center.) In this case, the ADCIG at near angles are affected by the anomaly but the far angles are not. Therefore, the ADCIG show the opposite curvature. This phenomenon has been well
studied in the literature (Toldi, 1985; Biondi, 2006), and it might hinder the tomography algorithm from finding a good update. Fortunately, with more than one azimuth, the curvatures observed from the alternative azimuth would offset this effect. This example shows one of the advantages of having multiple azimuth image gathers.

Figure 3.3 shows the first model update (in velocity) calculated using the proposed 3-D RMO-based WEMVA approach. This method works as expected that the update is concentrated around the anomaly’s location, and the sign of the update is correct.

![Diagram of the true velocity model](image)

**Figure 3.1:** The true velocity model, with positive Gaussian anomaly in the center. The velocity is constant 2.0 km/s in the background, and with a 0.6 km wide (in both x and y), 0.25 km tall Gaussian anomaly at the center. The peak value in the center of the anomaly is 3.0 km/s.
Figure 3.2: The $\partial J_{sm}/\partial \rho_i$ term for (a) $\phi = 0^\circ$, (b) $\phi = 30^\circ$ and (c) $\phi = 60^\circ$. The values correlate well with the curvatures of the ADCIG at each $(x, y)$ location. Because there is only one single event along the depth $(z)$ axis, $\rho$ can be simply plotted for each $(x, y)$ instead of $(z, x, y)$. [CR] \text{dJdu0, dJdu30, dJdu60}
Figure 3.3: First update of the velocity model calculated by my 3-D RMO-based WEMVA implementation.
3-D TRANSFORM BETWEEN SUBSURFACE OFFSET AND ANGLE DOMAIN COMMON IMAGE GATHERS

In my implementation, I have to add an extra component on top of the theory presented above: the transform between subsurface offset and angle domain common-image gathers (CIG). Although the theory of RMO-based WEMVA method operates exclusively on the angle-domain CIG, from a practical point of view, the subsurface offset domain CIG are much simpler than angle domain CIG for computer implementation. The image-space tomographic operator on offset domain CIG also has a simpler formula and can be straightforwardly implemented by horizontally shifting the source and receiver wavefields along opposite $x, y$ directions before applying the imaging condition (Tang et al., 2008). Therefore transforming between subsurface offset domain CIG and angle domain CIG is required for my implementation.

Sava and Fomel (2003) discovered the mathematical relationship between the two domains in the 2-D case, and it turns out that this 2-D case transform can be simplified to be independent of the local structural dip information, and can be implemented using a simple slant-stack operator. Unfortunately, such simplification does not hold in the 3-D case. As shown by Tisserant and Biondi (2003) and Biondi and Tisserant (2004), the theory for 3-D transform between the subsurface offset and the angle domain is more complex than it is in 2-D, which brings some extra technical challenges.

In this section, I first briefly review the theory of the 3-D offset to angle transform, and I then explain the associated technical challenges and, last, my best effort to overcome them.

Theory of 3-D offset $\leftrightarrow$ angle transform

Let us begin with a 3-D subsurface offset CIG $I(h_x, h_y, x, y, z)$, in which $x, y, z$ are the physical dimensions and $h_x, h_y$ corresponds to the horizontal subsurface offset axes along $x$ and $y$ directions. The steps to perform the offset to angle transform are as follows:
1. Perform Fourier transform $I(h_x, h_y, x, y, z) \rightarrow I(h_x, h_y, k_x, k_y, k_z)$.

2. For each $(k_x, k_y, k_z)$,
   - apply Fourier transform $I(h_x, h_y) \rightarrow I(k_{hx}, k_{hy})$
   - map $I(k_{hx}, k_{hy}) \rightarrow I(\gamma, \phi)$ based on the following relations (Tisserant and Biondi, 2003):
     
     \[
     \begin{bmatrix}
     k'_x \\
     k'_y
     \end{bmatrix} = \begin{bmatrix}
     \cos \phi & -\sin \phi \\
     \sin \phi & \cos \phi
     \end{bmatrix} \begin{bmatrix}
     k_x \\
     k_y
     \end{bmatrix}
     \]

     \[
     k'_{hx} = k_z \sqrt{1 + (k'_y/k_z)^2} \tan \gamma
     \]

     \[
     k'_{hy} = \frac{k'_y k'_2 k'_{hx}}{k_y^2 + k_z^2}
     \]

     \[
     \begin{bmatrix}
     k_{hy} \\
     k_{hy}
     \end{bmatrix} = \begin{bmatrix}
     \cos \phi & \sin \phi \\
     -\sin \phi & \cos \phi
     \end{bmatrix} \begin{bmatrix}
     k'_{hx} \\
     k'_{hy}
     \end{bmatrix}
     \]

     (3.11)

3. Apply inverse Fourier transform $I(\gamma, \phi, k_x, k_y, k_z) \rightarrow I(\gamma, \phi, x, y, z)$.

Similarly, the backward transform (angle to offset) is done by reversing the order of the above procedures.

As we can see, in contrast to the 2-D case, here the offset ↔ angle relationship depends on the local structural dip $(k_x/k_z$ and $k_y/k_z)$; therefore, in total a 5-D Fourier transform has to be applied over the entire CIG cube. More importantly, the mapping process between $(\gamma, \phi)$ and $(k_{hx}, k_{hy})$ is highly irregular (Biondi, 2003), which introduces severe distortion and degrades the gather quality after the transform is applied. To better illustrate this, the mapping from a regularly sampled $(\gamma, \phi)$ mesh to the $(k_{hx}, k_{hy})$ domain given fixed $k_z = 1/2$, $k_x = 1/4$ and $k_y = 1/4$ (assuming the Nyquist wave number is 1.0) is shown in Figure 3.4. We can see that the mapping is very ill-posed (a rectangular block in $(\gamma, \phi)$ maps to a fan shape with uncovered angle ranges), and the density of the samples on $(k_{hx}, k_{hy})$ becomes very non-uniform (the inner fan is very dense while the outer fan is sparse). Because my method has to sample the fields in both domains on a regular grid, we can conjecture that if we map
a field of values from \((k_{hx}, k_{hy})\) to \((\gamma, \phi)\) and then map this field back to \((k_{hx}, k_{hy})\), there would certainly be a significant discrepancy between the final output and the original input. Therefore, a good mapping scheme is very important to reducing the artifacts caused by such mapping irregularity.

The problem of irregular and coarse angle to offset mapping

To pinpoint the problem during the domain mapping process, we must first discuss the implementation details of such mapping. The forward mapping (offset to angle) is relatively easy because usually the input domain (offset) is very well sampled, and the output domain (angle) sampling is relatively coarse. A pull-type (Claerbout, 2009) operator plus 2-D linear interpolation would be sufficient. The precise algorithm is presented in Algorithm 1.

\begin{algorithm}[h]
\caption{Offset domain to angle domain mapping}
\begin{algorithmic}
\State Given \(k_x, k_y, k_z\), and the corresponding 2-D Fourier image slice \(I(k_{hx}, k_{hy})\) at the given \(k_x, k_y, k_z\);
\For {each azimuth \(\phi\) to be sampled}
  \For {each reflection angle \(\gamma\) to be sampled}
    \begin{itemize}
    \item Calculate \(k_{hx}, k_{hy}\) using the given \(\gamma, \phi\) based on formula 3.11;
    \item Sample the Fourier domain image value at \(k_{hx}, k_{hy}\) from the regularly sampled slice \(I(k_{hx}, h_{hy})\). Since \(k_{hx}, k_{hy}\) may fall between neighboring grid points, a 2-D linear interpolation will be used;
    \end{itemize}
  \EndFor
\EndFor
\end{algorithmic}
\end{algorithm}

However, an issue arises for the backward transform (angle to offset). As just stated, the angle domain common-image gathers (especially azimuth angle \(\phi\)) is usually not sufficiently sampled. Simply implementing the backward operator as the adjoint operator of the forward mapping (defined in algorithm 1) is equal to placing all available samples from the \((\gamma, \phi)\) plane onto their mapped locations on the \((k_{hx}, k_{hy})\) plane. Such approach would leave many holes unfilled in the output \((k_{hx}, k_{hy})\) plane because of the coarseness in the input \((\gamma, \phi)\) domain. In turn, the unfilled holes in
Figure 3.4: Graphical illustration of the mapping from the \((\gamma, \phi)\) plane to the \((k_{hx}, k_{hy})\) plane, a key step in 3-D transform between subsurface offset and angle domain common image gathers. Each dot in (a) maps to a corresponding dot in (b); similarly, each quadrilateral patch in (a) maps to a corresponding patch in (b). [CR]
would cause distortion in the Fourier transformed \((h_x, h_y)\) domain, especially at far offset.

Therefore, to avoid having unfilled holes in the \((\gamma, \phi) \rightarrow (k_{hx}, k_{hy})\) mapping, one potential approach would be to use a pull-type operator for the backward mapping. In other words, instead of looping over the input \((\gamma, \phi)\) domain, we would iterate over all points in the output \((k_{hx}, k_{hy})\) domain, find the corresponding \((\gamma, \phi)\) coordinates, and fetch the values at these coordinates from the input. Unfortunately, this approach is not easy to implement under the mapping relation defined in equation 3.11. Because of the specific algebra in equation 3.11, although it is easy to map from a given \((\gamma, \phi)\) value to \((k_{hx}, k_{hy})\), it is difficult to do the reverse.

**Texture-mapping scheme**

Given the difficulty articulated above, in my method I choose to use a simple yet effective scheme to perform this angle to offset mapping by an algorithm that ensures the mapping result is free of unfilled holes. This scheme is a simplified version of a typical texture-mapping algorithm that can be found on any modern computer graphics textbook. In computer graphics, texture-mapping is a well-known and well-studied topic; specifically, it involves the addition of detailed surface texture (a rectangular and uniformly sampled image) to the surface of an arbitrary shaped computer-generated 3-D object model. Texture-mapping task is very similar to the irregular mapping problem we encounter here. The idea of my mapping algorithm is very simple: instead of mapping from sample points to sample points (shown as dots in Figure 3.4), I map from rectangular patches in the \((\gamma, \phi)\) domain to quadrilateral patches in the \((k_{hx}, k_{hy})\) domain, with the assumption that each patch contains uniformly the value of the sample located in its center. Because the quadrilateral patches in \((k_{hx}, k_{hy})\) domain can have an arbitrary shape and orientation, I use a classic polygon-filling algorithm (Foley et al., 2013) to fill each individual quadrilateral \((k_{hx}, k_{hy})\) patch, as shown in algorithm 2.
Algorithm 2 Angle domain to offset domain mapping

Given \( k_x, k_y, k_z, \) the corresponding 2-D Fourier image slice \( I(\gamma, \phi) \) at the given \( k_x, k_y, k_z, \) and the sampling interval for the angle axes \( \Delta \phi, \Delta \gamma; \)

\[
\text{for each sampled azimuth } \phi \text{ do}
\]
\[
\text{for each sampled reflection angle } \gamma \text{ do}
\]
\[
\begin{align*}
(\phi - \Delta \phi/2, \gamma - \Delta \gamma/2) & \rightarrow k_{hxy}^{(1)} \\
(\phi - \Delta \phi/2, \gamma + \Delta \gamma/2) & \rightarrow k_{hxy}^{(2)} \\
(\phi + \Delta \phi/2, \gamma + \Delta \gamma/2) & \rightarrow k_{hxy}^{(3)} \\
(\phi + \Delta \phi/2, \gamma - \Delta \gamma/2) & \rightarrow k_{hxy}^{(4)}
\end{align*}
\]

- Using formula 3.11, calculate the \( k_{hx}, k_{hy} \) coordinates that corresponds to the four corner points surrounding the input sample at \( (\phi, \gamma) \), i.e., let

\[
(\phi - \Delta \phi/2, \gamma - \Delta \gamma/2) \rightarrow k_{hxy}^{(1)}
\]
\[
(\phi - \Delta \phi/2, \gamma + \Delta \gamma/2) \rightarrow k_{hxy}^{(2)}
\]
\[
(\phi + \Delta \phi/2, \gamma + \Delta \gamma/2) \rightarrow k_{hxy}^{(3)}
\]
\[
(\phi + \Delta \phi/2, \gamma - \Delta \gamma/2) \rightarrow k_{hxy}^{(4)}
\]

- Using the image value at \( I(\gamma, \phi) \) to fill the polygon defined by the four vertices in the output \( (k_{hx}, k_{hy}) \) domain: \( k_{hxy}^{(1)}, k_{hxy}^{(2)}, k_{hxy}^{(3)} \) and \( k_{hxy}^{(4)} \);

\[
\text{end for}
\]
\[
\text{end for}
\]

Example

Here, I use the previous Gaussian anomaly synthetic example to demonstrate the advantage of using this text-mapping scheme. I started with an initial subsurface offset domain CIG by migrating the data using the starting velocity model, as shown in Figure 3.5c. Then I applied a forward offset-angle transform to the offset domain CIG to get the angle domain CIG. As stated in the previous section, I generated the angle domain CIG along only three azimuth angles: \( 0^\circ, 30^\circ \) and \( 60^\circ \). I further used the backward transform to map the angle domain CIG back into the offset domain, of which the result is shown in Figure 3.5b. The backward transform used here implements the texture-mapping scheme described in algorithm 2. For comparison, a different result (Figure 3.5a) was also computed by applying a backward transform on the same angle domain CIG, but this time the adjoint of the forward mapping operator was used as the backward mapping operator, which simply does sample-to-sample mapping.
A good transform pair should make the resulting offset domain CIG resemble the original one as closely as possible, although we should note that there was no way we could retrieve a result that was exactly the same as the original gather because the information in the azimuth range \((60^\circ, 180^\circ)\) had already been lost in the angle domain CIG (only three azimuths \((\phi = 0^\circ, 30^\circ, 60^\circ)\) were computed in the angle domain CIG). Nonetheless, it is obvious that Figure 3.5b is less distorted than Figure 3.5a when compared with the original offset domain CIG (Figure 3.5c), especially at the far subsurface offset region. In other words, the discrepancy of gather curvature and amplitude pattern between Figure 3.5c (the original input) and Figure 3.5a is more prominent than that between Figure 3.5c and Figure 3.5b.

CONCLUSION

In this chapter, I have presented the work to extend the proposed RMO-based WEMVA method to the three-dimensional (3-D) case. I first augmented the theory to deal with multiple azimuths angle-domain CIG by assigning independent moveout parameters to each distinct azimuth. Then, I presented a synthetic example as a validation for my 3-D RMO-based WEMVA theory and implementation. Last, I described a practical challenge: the complexity of 3-D transforms between subsurface offset and angle domain CIG. Instead of using the adjoint of the forward (offset to angle) transform operator, I used a texture-mapping type algorithm in the backward (angle to offset) transform. Such modification significantly reduces artifacts introduced by the strong irregularity of the mapping relations between offset wavenumber domain \((k_{hz}, k_{hy})\) and angle domain \((\gamma, \phi)\).
Figure 3.5: (a) The result of applying a forward and backward offset-angle domain transform on an offset domain CIG, in which the sample-to-sample based backward mapping is used; (b) the same transformation as (a), except that the texture-mapping based backward mapping is used. The original offset domain CIG is shown in (c). All three gathers are chosen at location $x = 0.2 \text{ km}$, $y = -0.06 \text{ km}$. Compared with the ideal reconstruction result (c), the reconstruction in plot (b) is better than that in plot (a), especially at the far offset where $|h_x|, |h_y|$ is large. Note that the intermediate angle domain CIG we generate does not have full azimuth coverage ($60^\circ \sim 180^\circ$ excluded). Therefore, an exactly identical reconstruction of the original offset domain CIG is not possible. [CR]
Chapter 4

3-D field data test — a target-oriented approach

In this chapter, I apply my residual-moveout-based (RMO-based) wave-equation migration velocity analysis (WEMVA) method to an industry scale 3-D marine streamers wide-azimuth dataset — E-Octopus III in the Gulf of Mexico. This 3-D field dataset poses many challenges for my implementation, including irregular geometry, abnormal traces, complex 3-D salt geometry, and more importantly, huge data volume and large domain dimensions. To overcome these hurdles, I apply careful data regularization and preprocessing, and employ a target-oriented inversion scheme that focuses on the update of sediment velocities in specific regions of interest. Such target-oriented scheme significantly reduces the computational cost, allowing me to make my WEMVA method affordable on the CEES (Stanford Center for Computational Earth and Environmental Science) academic computing cluster, which I have access to. The imaging results on two subsalt sediments target regions show that the aperture angles of illumination on the subsalt sediments are very limited because of the complex salt overburden and the depth of the targets. However, although the lack of angular illumination in this region severely reduces the capability of any reflection tomography method that tries to resolve a better velocity model, my RMO-based WEMVA method is still able to detect the curvatures of the angle gathers and
produce good velocity model updates that further increases flattening of the angle
gathers and improve the quality of the structural image in the target region.

INTRODUCTION

In the previous chapters, I demonstrated the successful application of my proposed
RMO-based WEMVA on the 2-D Marmousi and BP synthetic models. I have also
shown that the theory of this method can be easily extended to three-dimensions
(3-D). Nonetheless, a realistic 3-D dataset application remains very challenging.

In this chapter, I first examine a marine streamers 3-D wide-azimuth (WAZ)
seismic dataset acquired from offshore Gulf of Mexico (GoM) by Schlumberger Mul-
ticlient, and then present the workflow involved in the application of the RMO-based
WEMVA to this dataset. I illustrate the practical problems encountered and de-
scribe the measures to overcome or mitigate these problems in order to obtain the
final WEMVA result in a timely manner. The remainder of this chapter is organized
as follows:

• In the first part, I give an overview of this WAZ dataset, as well as my data
  regularization and pre-processing procedures.

• In the second part, I analyze the initial 3-D migration images I obtained using
  the processed data.

• In the third part, I focus on two smaller subsalt target regions within the original
  imaging domain. To reduce the computational cost of applying my WEMVA
  method, for each target region, I synthesize a new target-oriented dataset that
  preserves all the velocity information of that target region.

• In the last part, I demonstrate how the application of my RMO-based WEMVA
  method to the synthesized datasets results in more accurate velocity models for
  both target regions. The velocity estimation results show major improvements
in the angle-domain common-image gathers (ADCIG) flatness and in the migrated image, which has better defined continuity and coherency in terms of sedimentary structure.

E-OCTOPUS III WAZ DATASET OVERVIEW

The dataset I analyze in this chapter is a wide-azimuth towed streamer (WATS) survey acquired from offshore Gulf of Mexico (GOM) by Schlumberger. The data belongs to a portion of the “E-Octopus phase III” survey in the Green Canyon area. The corresponding survey area is about 35 km (inline) by 30 km (cross-line).

Figure 4.1 shows the geological structure map of the Northern Gulf of Mexico, with the survey area of my dataset marked. The processing report from Schlumberger describes the complex geological settings in this area:

“The E-Octopus Phase III survey lies primarily in the Green Canyon protraction areas of the MMS Central Planning Area in the Gulf of Mexico. The northern Gulf of Mexico is a geologically complex basin resulting from interaction and deformation of salt and overlying sediment layers over geologic time. The geology around the survey area is characterized by extensive salt sheets with intervening deep-water sediment-filled mini-basins. The salt canopy is characterized by simple to complex salt features, some of which have thicknesses up to 30,000 ft and some that are extremely shallow, i.e., just under the water bottom.”

Acquisition settings

This WATS survey uses 4 marine seismic vessels simultaneously to achieve a WAZ acquisition pattern. All 4 vessels act as sources but only 2 vessels tow receiver cables. The shooting/towing direction is from SW to NE (pass A) and vice versa (pass B). Figure 4.2 shows the nominal shooting configuration. The source positions are spaced
Figure 4.1: The structural summary map of the northern Gulf of Mexico, with the E-Octopus III survey area highlighted with yellow boxes. As can be seen from the map, there is strong presence of geological complexity manifested by the wide coverages of shallow salt and tabular salt structures. Courtesy of Schlumberger. [NR] fig1
every 150 m inline and 600 m crossline. The receiver streamers are \(~7.0\) km long; the receiver spacing is 12.5 m. The crossline spacing between neighboring streamers is 100 m. Figure 4.3 shows the total source and receiver coverages. From the shot locations map, it can be easily identified that there are two types of sail-lines (pass A and pass B) in opposite directions. As we can see from Figure 4.3a, the shotlines are almost perfectly straight, and their orientations are very well aligned with each other. The coverage of the shots and receivers are fairly uniform, except for only one small hole around \(x = 0.0\) km, \(y = 20.0\) km. The good quality of the field acquisition geometry eliminates the need for me to apply complex data regularization schemes to this data in later processing.

The full span of crossline offset is \([-4.2\) km, +4.2 km\], compared to the span of inline offset being \([-7.0\) km, 0.0 km\] (pass A) or \([0.0\) km, +7.0 km\] (pass B). Figure 4.4 illustrates the fold coverage map for a typical midpoint location (Rose diagram) under the nominal acquisition setting. We can see that the survey illuminates at least \(\pm 60^\circ\) (out of 180\(^\circ\)) of azimuth range. Given the highly complex geology in the area, the wide azimuth coverage is a significant advantage in terms of enhancing subsurface illumination coverage compared to the old conventional narrow azimuth acquisition setup.

**GEOMETRY PROCESSING AND DATA REGULARIZATION**

For the convenience of data processing, I rotate and translate my processing grid to align the \(X\)-axis with the inline direction and the \(Y\)-axis with the cross-line direction. Figure 4.5 shows the shot and receiver locations map after the geometric transform. Aligning the processing grid axes with the acquisition inline and cross-line directions makes it easy to bin the seismic shot gathers to the regularly sampled data grid I create.

The size of the original seismic data I receive is more than 11 TB, containing over 72,000 shots. The computational infrastructure in my department is not capable of
Figure 4.2: The nominal shooting configuration of this wide-azimuth towed streamer (WATS) survey. All 4 vessels act as sources, but only the outer 2 vessels tow streamer cables (drawn as pink blocks). Each recording vessel carries a total of 10 streamer cables, with 12.5 m inline (distance between hydrophones within the same cable) spacing and 100 m crossline (distance between cables) spacing. The shooting/towing direction is from SW to NE (pass A) and vice versa (pass B). Courtesy of Schlumberger.
Figure 4.3: The total (a) source and (b) receiver coverage in the survey. From the shot locations, we can clearly distinguish the two kinds of sail-lines in opposite directions.

[CR] [fig4,fig5]
handling a dataset on such a huge scale in its original form. To reduce the size of the data, as well as the computational cost required to process this data, I take several measures during the data regularization:

- I compute the inline offset and cross-line offset using the source and receiver locations, and my data grids contain 5 axes: time, inline offset, cross-line offset, shot location inline, and shot location cross-line. I regularize the shot gathers by binning them into two sets of regularly spaced data grids (one for pass A shotlines and one for pass B shotlines). The actual axis parameters in the data grids determine the final size of the regularized dataset. Because I create different grids for pass A and pass B shotlines, I can assign \([-7.0 \text{ km}, 0.0 \text{ km}]\)
Figure 4.5: (a): The shot locations map after applying the geometric transform. (b): the receiver locations map after applying the geometric transform.
range to inline offset axis for pass A, and assign \([0.0 \text{ km}, +7.0 \text{ km}]\) range to inline offset axis for pass B. If the same data grid was used for all shots, I would have had to assign \([-7.0 \text{ km}, +7.0 \text{ km}]\) to the inline offset, which would have generated a regularized dataset that takes twice as much space as my actual approach does. As I use the exact parameters from the nominal acquisition geometry described above (Figure 4.2), the regularized data size reduces to \(\sim 7.9 \text{ TB}\).  

- Given my target imaging resolution, I increase the inline receiver spacing from 12.5 m to 25 m. This reduces the data size by half, to \(\sim 3.9 \text{ TB}\).  

- I notice that the entire shot locations map covers an area of about 40 km by 42 km, but the given velocity model’s dimension is only about 34 km by 35 km. Therefore I discard the shot-receiver locations that are far away from the region defined by the available velocity model, and I further neglect the shots on the edge of the shot locations map that have very poor illumination folds (bin counts). After the shots decimation, the data size is further reduced to \(\sim 1.9 \text{ TB}\).  

- I group the shot gathers fired by all 4 source vessels at nearby locations (<75 m inline) into one supergather for each shot location. Although this grouping does not reduce the size of the data, it decreases the total number of shots by a factor of 4, which greatly speeds up the wave-equation migration and tomography by \(\sim 4\) times because a shot-profile type migration is used in my WEMVA application.  

After applying the procedures above, the final regularized dataset is reduced to \(\sim 1.9 \text{ TB}\), containing \(\sim 8,500\) shots.

**DATA PREPROCESSING**

The original seismic recordings I received from Schlumberger has undergone many data processing steps. Some noteworthy steps include low-cut noise filtering, air
gun source de-bubble and source/receiver deghosting, as well as surface multiples removal. Figure 4.6 shows the time-domain recordings of a typical shot gather. Note that the crossline receiver sampling (120 m) is much coarser than the inline sampling (25 m). Figure 4.7 shows the frequency spectrum of the same shot gather, in which the frequency range containing prominent energy is about [5 Hz, 65 Hz]. To better inspect the signal patterns in the time-domain data, a zoom-in view of a 2-D inline slice (with constant crossline offset 0.0 km) within the same shot gather is shown in Figure 4.8.

As highlighted in Figure 4.8, several noticeable artifacts and noise patterns are present. Therefore I apply the following preprocessing steps to address these related issues:

- First, I apply a $t$-power gain (power=2.4) to the time-domain data to compensate for geometric spreading and medium attenuation.

- Second, I mute all energy above the water-bottom reflection in the shot gather data because there are some residual energy above the highlighted water-bottom reflection (yellow solid line), which might be refracted waves or processing artifacts. Moreover, muting these early arrivals would not cause loss of useful information in my application because I aim to image deep targets using the reflection energy. The $t$-power gain and water-bottom mute clean up the unhelpful energy from the direct arrivals and the refracted waves from the shallow area, as shown by Figure 4.9.

- Next, I limit the high end of the spectrum to be less than 48 Hz to remove the high frequency noise observed in the data. Note the vertical checker-board pattern noise at some traces (framed in blue) in Figure 4.8; this is likely caused by the corruption of high frequency components in the data. By examining the time-domain data at different frequency bands, I find that the highest usable frequency is about 48 Hz. Capping the maximum frequency in the data at 48 Hz filters out the coherent checker-board pattern noise.
Figure 4.6: A three-panel view of the time-domain recordings of a typical shot gather, of which the shot location is \(x = 15.9\) km, \(y = -0.3\) km. Note that the crossline receiver sampling (120 m) is much coarser than the inline sampling (25 m). A \(t\)-power gain (power=2.4) is applied to the data before plotting. [CR].
Furthermore, I decimate the inline offset sampling from 12.5 m to 25 m. As can be seen Figure 4.8, there are intermittent dead (all-zeros) traces (framed in orange), most likely caused by sensor dysfunction or cable feathering. Fortunately, the number of missing traces is very small, and the artifacts they cause are expected to be largely stacked out in the final image. My decimation further mitigates this issue since the majority of missing traces will be filled because of a larger bin.

Last, but not least, I truncate the data up to 12.0 seconds because there are some staircase-type horizontal noise patterns at very late arrival time (marked by red arrows). Although I do not have a good explanation on the cause of these artifact, since it comes at very late arrival time, I can simply cut short the time axis. Given that the maximum depth I want to image is 12.0 km, and that the inline and cross-line offset ranges are relative short, 12.0 seconds long recordings are sufficient to contain all reflection events up to that depth.
Figure 4.8: A zoom-in view of a 2-D inline slice within the shot gather data shown in Figure 4.6. Note that there are several artifacts and noise patterns highlighted in this figure. A $t$-power gain (power=2.4) is applied to the data before plotting. [CR] data-1shot
The application of these preprocessing steps greatly facilitates my later imaging and tomography tasks in deeper regions.

Figure 4.9: The same inline slice in a shot gather (Figure 4.8): (a) before applying the \( t \)-power gain and the water-bottom muting; (b) after applying the \( t \)-power gain and the water-bottom muting. For display purposes, the data is truncated at 7.0 secs.

INITIAL MIGRATION IMAGES

Figures 4.10 and 4.11 show the velocity model I received from Schlumberger. According to the processing report, this model is the final result of a comprehensive velocity
analysis workflow, which includes:

- one round of multi-azimuth tomography on the sediment velocity above the salt body;
- one sediment flooding and migration;
- two rounds of salt flooding and migration for the salt interpretation;
- one final “slow gradient” revision in subsalt areas.

The velocity plots show the 2-D sections of the same model at different slicing co-ordinates. As can be seen from Figure 4.10 and Figure 4.11, significant crossline structural dips are present in this model, as well as strong lateral velocity variations along both the $X$ and $Y$ directions. Therefore the 3-D wave propagation effects are prominent, rendering any effort to analyze any individual 2-D portion of the data ineffective (illustrated in the next subsection).

I use the one-way wave-equation migration (WEM) method to migrate the dataset; therefore, I convert the data from the time domain to the frequency domain. To further reduce the amount of computation required, I use only $\sim 200$ frequency slices in the range of [5 Hz to 20 Hz] after considering the following factors: 1) the usable frequency band; 2) the desired spatial resolution of the image; and 3) the total length of time record required to image the target depth. The spacing of the imaging grid used is 25 m ($X$) by 30 m ($Y$) by 20 m ($Z$).

Figure 4.12 shows one section of the full 3-D migrated image. As we can see, the image quality of the sediments above the salt is very good, with continuous and coherent reflector events, which is an indication of accurate velocity. However, the subsalt areas are not as well imaged. There are many discontinuities in the reflectors, as well as conflicting dips. While the poor image quality can be partially attributed to the lack of wave energy illumination in the subsalt region, it is also an indication of a less accurate velocity model. I therefore focus my efforts in these regions.
Figure 4.10: Three-panel display of the velocity model used for migration, the model view is sliced at $x = 10.0$ km, $y = -9.0$ km. Notice the strong model variations along both $X$ and $Y$ directions. The color map ranges from 1450 m/s (deep blue) to 4480 m/s (deep red).
Figure 4.11: Three-panel display of the velocity model used for migration, the model view is sliced at $x = 20.0 \text{ km}$, $y = -0.99 \text{ km}$. Notice the strong model variations along both $X$ and $Y$ directions. The color map ranges from 1450 m/s (deep blue) to 4480 m/s (deep red). Combined with Figure 4.10, it demonstrates the significant geometric complexity of the velocity model (especially the salt body), thus the model is “truly 3-D”.

[fig9]
Figure 4.12: 3-D seismic image migrated with the preprocessed dataset using the given velocity model (shown in Figure 4.10 and 4.11). An AGC (Automatic Gain Control) of 1.2 km window size in $Z$ is applied. [CR] [fig10]
2-D vs. 3-D image comparison

The strong lateral variations of the subsurface model along both inline and crossline directions underline the importance of using truly three-dimensional physical modeling to analyze this dataset. The comparison between the results from using 2-D migration and using 3-D migration on this dataset, as shown in Figure 4.13, demonstrates this claim. The 3-D image is significantly better than the 2-D image, especially in the subsalt region, where many layering structures observed in the 3-D image cannot even be identified from the 2-D image. This comparison allows us to conclude that it is not worthwhile to apply the less expensive 2-D processing and analysis to this dataset because the result we obtain would not be informative.

FOCUSING ON SUBSALT SEDIMENTS WITH A TARGET-ORIENTED APPROACH

One of the major problems in applying WEMVA to such a big dataset is the high computational cost. Even with all the precautions I made in the data preparation stage, on the CEES (Stanford Center for Computational Earth and Environmental Science) computing cluster of 120 Intel Xeon nodes (E5520, 2.26 GHz, quad-core), a full migration on the entire domain (like Figure 4.12) costs $\sim 5000 \text{ node} \times \text{hours}$, which amounts to $\sim 40$ consecutive hours at 100% cluster usage in an optimal case. In practice it takes at least 5 days to complete a job of this kind.

This turn-around time would be too long for practical applications, given that the wave-equation tomographic operator is even more expensive than the imaging operator, and that tens of iterations have to be performed in a typical WEMVA inversion. The previous section showed how I reduce computation by reducing the size of data to be imaged. Fortunately, further reduction can be achieved in the model domain, i.e., I can apply my WEMVA inversion only to the part of model that is of most interest. In my example, I choose two subsalt regions as the region of interest (ROI) because subsalt areas on the one hand are very challenging for model
Figure 4.13: Comparison between an inline section of (a) 2-D migration image and (b) 3-D migration image at the same crossline location \( y = -0.3 \text{ km} \). The 2-D migration uses the single shotline data at \( y = -0.3 \text{ km} \). Note the significant differences in the subsalt image quality between (a) and (b). [CR] fig11
estimation due to the complexity of wave propagation through salt bodies; on the other hand, subsalt regions often contain host structures for oil and gas reservoirs; therefore, enhancing subsalt images has potential economic benefits.

The first target area (which is named ROI 1) I choose is a 7.5 km by 5 km by 5 km subsalt sediment region in the shallower part (depth from 1.5 km to 6.5 km) of the model, as shown in Figure 4.14. This region is populated by salt body protrusions with complex geometry. Combined with the effect of steeply dipping salt flanks, “shadow zones” are formed in the sediments next to the salt, in which the image quality is severely compromised because the seismic energy illumination is weak and the energy is not well focused by the imaging algorithm due to a less accurate velocity. This target area is a good candidate for WEMVA-based velocity improvements because it is very difficult for ray-based methods to model wave-propagation through salt bodies with such complexity. Moreover, my WEMVA method is robust with respect to the illumination variations in the model, and thus will concentrate on the shadow zones, which have weak amplitudes but more kinematic errors; and because improving these shadow zones is my primary goal, my WEMVA method serves this purpose well. Figure 4.15 shows the uneven illumination in the target area with the diagonal imaging Hessian computed using random-phased encoding approximation (Tang, 2008). The diagonal imaging Hessian is equivalent to a source-receiver illumination indicator, and we can see the strong contrast in the illumination level between the shadow zones and the sediments away from the salt.

The second area (which is named ROI 2) is a 9 km by 6 km by 6 km subsalt sediment region in the center of my imaging domain, as shown in Figure 4.16. I consider this area a good target for WEMVA-based velocity improvement because there are many discontinuities among the imaged sediment layer interfaces, indicating an inaccurate migration velocity. Furthermore, although the target area is deep (depth from 6 km to 12 km), the salt overburden above this region is relatively well imaged; therefore, I have confidence about the correctness of the salt body model, which allows me to conclude that the velocity errors mainly exist within the subsalt sediments; and my WEMVA method is good at resolving this type of velocity error. Last, the relatively
simple salt overhang enables enough seismic energy to pass through and illuminate the target area, as demonstrated by the diagonal imaging Hessian shown in Figure 4.17. From the figure we can see that there is reasonable amount of illumination (although quite uneven) for the target region.

In the remaining chapter, I will present my target-oriented RMO-based WEMVA application on the two regions of interest (ROI 1 and 2) respectively.

WEMVA APPLICATION ON ROI 1

Initial 3-D common image gathers in the target region

To show the existence of velocity error in the target region, I compute the subsurface-offset CIG (both in $h_x$ and $h_y$) for the target region. With regard to the ranges of the subsurface offset axes, I need to use 21 points in $h_x$ with 50 m spacing and 19 points in $h_y$ with 60 m spacing in order to capture most of the unfocused energy. Computing these gathers is very I/O intensive because my gathers are almost 400 times the size of my image (which contains only zero subsurface offset). The actual timing measurements show that computing these gathers takes twice the amount of time needed to compute the image without gathers. Fortunately the cost is still manageable because I compute only gathers within the target area.

Figure 4.18 shows the subsurface offset CIG within an inline image section ($y = -10.11$ km). I only show a $h_x$ section (while $h_y=0$) and a $h_y$ section (while $h_x=0$). As we can see from the figure, there is a great deal of unfocused energy in the subsurface offset CIG due to the complex salt overburden. Figure 4.19 shows the subsurface angle-domain CIG (ADCIG) within the same inline section ($y = -10.11$ km). Note the strong variations of the ADCIG along different azimuths. Also, we can see that in the right half of the model where the salt becomes thicker, the angular illumination becomes narrower and the shape of the moveout is more complex. Nonetheless, there are many upward-curving or downward-curving events in the ADCIG, which my RMO-based WEMVA method is able to utilize for velocity improvement.
Figure 4.14: A three-panel view of the initial full 3-D image, in which the ROI 1 (region of interest) for my target-oriented inversion is marked by the yellow box. For display purposes, a $z$-power (similar to $t$-power) gain is applied to the image to boost the amplitudes of deeper reflectors. The dimension of ROI 1 is 7.5 km by 5.0 km by 5.0 km.
Figure 4.15: A three-panel view of the 3-D imaging Hessian (diagonal) under the original acquisition setting, in which the ROI 1 for my target-oriented inversion is marked by the yellow box. For display purposes, a $z$-power (similar to $t$-power) gain is applied to the image to boost the amplitudes in deeper regions. [CR] hess-all-roi7-mark
Figure 4.16: A three-panel view of the initial full 3-D image, in which the ROI 2 (region of interest) for my target-oriented inversion is marked by the yellow box. For display purposes, a $z$-power (similar to $t$-power) gain is applied to the image to boost the amplitudes of deeper reflectors. The dimension of ROI 2 is 9 km by 6 km by 6 km.
Figure 4.17: A three-panel view of the 3-D imaging Hessian (diagonal) under the original acquisition setting, in which the ROI 2 for my target-oriented inversion is marked by the yellow box. For display purposes, a $z$-power (similar to $t$-power) gain is applied to the image to boost the amplitudes in deeper regions.
Target-oriented tomography using generalized Born-modeling data

As explained previously, applying expensive wave-equation tomography methods to the target region suggests that I use a target-oriented approach which greatly shrinks the problem domain, thus significantly reducing the computational cost. The exact approach I use is the target-oriented tomography scheme proposed by Tang (2011). This scheme synthesizes a new dataset that concentrates on the region of interest (ROI), and the synthesized dataset is capable of preserving all kinematic information that is related to the ROI in the original dataset. Furthermore, this scheme offers great flexibility on the acquisition geometry of the synthesized dataset.

The scheme proposed by Tang (2011) first computes the initial subsurface ODCIG in the target region using a starting velocity model. Then, the amplitudes of the initial ODCIG are balanced using the diagonal values of the imaging Hessian — which can be efficiently computed using the phase-encoding method (Tang, 2011) — to optimally compensate for the uneven subsurface illumination. Tang (2011) demonstrated that the velocity information contained in the original seismic data is fully transformed into the ODCIG. Therefore, this scheme uses the generalized Born modeling method — which includes the ODCIG instead of merely the zero subsurface offset image in the modeling process — to simulate a new dataset immediately on top of the target region. With this scheme, the user can design a substantially different acquisition geometry for the synthesized new dataset. Once the new dataset is generated, the wave-equation tomography is carried out exclusively on the new dataset.

The synthesized new dataset

The first step in my target-oriented tomography workflow is to synthesize a dataset that is recorded immediately on top of the target region while preserving all velocity information around the target region from the original dataset. I choose plane-wave acquisition geometry for the new dataset because it results in fewer total number of
Figure 4.18: A close-up view of the images and subsurface offset domain common image gathers at the target ROI 1 (4.14). (a): An inline image section at $Y = -10.11\text{ km}$; (b): The inline subsurface-offset ($h_x$ range spans $-0.50\text{ km} \sim +0.5\text{ km}$) CIG for different lateral locations in (a); (c): The crossline subsurface-offset ($h_y$ range spans $-0.54\text{ km} \sim +0.54\text{ km}$) CIG for different lateral locations in (a).
Figure 4.19: A close-up view of the subsurface angle-domain CIG at the target ROI 1 (Figure 4.14). The gathers shown are extracted from the same inline section as shown in Figure 4.18a. Each subplot shows the angle gathers at a certain azimuth (as noted in the caption) respectively. The range of the reflection angle ($\gamma$) spans $-40^\circ \sim +40^\circ$. [CR] \textit{adcigs-7-3pcs}
shots that need to be simulated, and the cross-talk effect usually caused by composite
sources are kept at a minimum in the plane-wave data. I determine the acquisition
parameters of the plane-wave survey based on the maximum subsurface illumination
angle revealed in the ADCIG and the average velocity at the top of the target region.
The new acquisition samples 21 inlines and 21 crosslines (441 in total) plane-wave
directions, with the ray parameters ranging between $\pm 300 \mu s/m$, which corresponds
to at least $\pm 35^\circ$ subsurface angle illumination given the average velocity at the new
acquisition surface ($\sim 1900 \text{ m/s}$). Because of the reduction of the physical propagation
domain, I reduce the recording time from 14.0 secs (in the original dataset) to 6.1
seconds (in the synthesized dataset). This reduction in time-domain translates to
coarser sampling in the frequency-domain. Therefore, I sample 99 frequencies between
4 Hz and 20 Hz for the new dataset, compared to $\sim 200$ frequencies for the original
dataset.

The mapping from the original dataset to this target-oriented plane-wave dataset
yields huge savings in computation. Specifically, the size of the dataset is reduced by
90%. This, combined with the shrinkage of the modeling physical domain, reduces
the required computation time for each migration from 5000 node hours to 130 node
hours, a more than 35 times improvement, which in turn makes overnight calculation
per iteration feasible.

To verify that the newly synthesized dataset contains the same velocity informa-
tion as the original data, I recompute the subsurface ODCIG (both in $h_x$ and $h_y$)
using the new dataset. The offset dimension of the recomputed ODCIG is the same
as the initial subsurface ODCIG I used to generate the new dataset. Figure 4.21 and
Figure 4.22 show the comparison between the ODCIG migrated using the original
dataset and the synthesized new dataset at cross-line location $y = -10.11 \text{ km}$, re-
spectively. As we can see, except for a few disparities near the boundary and some
differences in illumination distribution, the kinematic behaviors of the two ODCIG
agree very well.
Figure 4.20: A close-up view at the target ROI 1. (a): The starting velocity model, which is used for new data synthesis as well as the starting velocity model in later WEMVA inversion; (b): the zero subsurface-offset image of the ODCIG that I use for new data synthesis. The inverse of diagonal Hessian matrix is applied to the ODCIG for amplitude balancing.
Figure 4.21: 3-D subsurface ODCIG in the target region, used as the input ODCIG for modeling the new dataset; the section shown is extracted at $y = -10.11$ km. 
(a): the inline image section; (b): the inline subsurface-offset ($h_y$ is fixed at 0.0 km while $h_x$ range spans $-0.50 \text{ km}, +0.50 \text{ km}$) CIG for different lateral locations in (a). (c): the cross-line subsurface-offset ($h_x$ is fixed at 0km while $h_y$ range spans $-0.54 \text{ km}, +0.54 \text{ km}$) CIG for different lateral locations in (a). [CR] odcigs-initmig1-roi7
Figure 4.22: 3-D subsurface ODCIG in the target region, migrated using the generalized Born-modeling dataset; for each subplot, refer to the descriptions in Figure 4.21. Compare the image and gather kinematics in this figure with those in Figure 4.21.

[CR] odcigs-remig1-roi7
**Inversion result**

I use the same Polak-Riberie nonlinear conjugate-gradient (CG) solver described in Chapter 2 to perform the WEMVA iterations. Based on my empirical observation, I use only half of the data frequencies when applying the wave-equation tomographic operator to further save computation time. In addition, I re-parameterize the model space using coarsely sampled B-spline nodes to control the resolution of the inversion among iterations. By gradually increasing the number of spline nodes as the number of iterations increases, I constrain the inversion to resolve the low-wavenumber part of the model first, then gradually move up to a higher wavenumber to retrieve more model details (Biondi, 1990). The initial spline nodes spacing is set as 1.2 km in X, 1.2 km in Y, and 0.3 km in Z; and the spacing reduces by ∼0.05 km in X, Y and 0.02 km in Z for each iteration.

I build a binary mask from the starting velocity model to mark the salt body in the target region. Because the smooth tomography gradient is not suitable to modify the salt model, when computing the model gradient, I use the salt area mask to zero out any updates inside the salt. For the ADCIG sampling, I use 31 points along the reflection angle axis ranging from −30.0° to +30.0°, and I compute 7 azimuths from −60.0° to +60.0°.

Figure 4.23 shows the final velocity inversion result obtained using this method after 15 iterations. Compared with the starting model (Figure 4.20(a)), the updated model has more lateral variations, especially in the area below salt. Figure 4.24 shows the history of step sizes and normalized objective function values by iteration.

To illustrate the imaging improvements brought by the inversion, the comparison between the initial image and the image migrated using the inverted model is shown in Figure 4.25, 4.26 and 4.27. To compare the image coherency, I use the same clip value when displaying the initial image and the updated image. As we can see from these figures, the events of the updated image become more coherent and have higher amplitudes, especially in the subsalt shadow zones where the amplitudes are very weak. Specifically, in Figure 4.25, the continuity of the reflectors in the circled
areas are significantly improved; in Figure 4.26, the subsalt reflectors in the inline section are much better imaged after velocity update; in Figure 4.27, the subsalt reflectors are more continuous and have higher resolution because of the velocity improvement. In addition, we can see (in the cross-line section of all three figures) that the sediments near the salt boundary get better imaged and tilt upward against the salt flank, which is geologically more plausible. Figure 4.28 shows the comparison between the ADCIG before and after velocity update. The locations of these ADCIG are the slicing locations shown in Figure 4.25. (In this example, it is the Z-X slice at Y = −11.55 km.) Similarly, Figure 4.29 shows the ADCIG comparison at another location corresponding to Figure 4.27. We can see that in the highlighted areas where the gathers moveout is better defined and relatively easy to characterize, the ADCIG become more flat after velocity update.
Figure 4.24: Iterative velocity inversion statistics for ROI 1: (a) History of step sizes by iteration; the step sizes are quantified by the maximum velocity value perturbation within the model space caused by that step size. (b) History of normalized objective function values by iteration.
In addition, I plot the derivative of the normalized semblance over the moveout parameter (the $\partial S_m/\partial \rho$ term) at every image location before and after the inversion in Figure 4.30. Note that I have averaged the derivative over all azimuths so that the output shrinks to three dimensions and can be plotted more conveniently. These derivatives carry similar meanings as the residuals in a data-fitting inversion scenario, and their amplitudes indicate how far the angle gathers deviate from being flat; therefore, we can examine them as an extra QC for the inversion. From Figure 4.30, we can see that the deviations of angle gathers flatness are reduced after the inversion. To examine the distribution of the derivative values, I further plot their corresponding histograms in Figure 4.31. As can be seen from the figure, the derivative values are more concentrated around zero and are distributed more evenly between positives and negatives. These observations indicate the model improvement as a result of inversion.
Figure 4.25: Three-panel view showing the comparison between the zero subsurface-offset images migrated with (a) the starting velocity model (Figure 4.20a) and (b) the inverted velocity model (Figure 4.23). The markers highlight areas of improvements. The coordinates of each section are annotated in the figure. Both figures are plotted using the same clip value.
Figure 4.26: Same as Figure 4.25 but at a different location. [CR]

img4-roi7-beginv, img4-roi7-endinv
Figure 4.27: Same as Figure 4.25 but at a different location.  

[CR] 

img5-roi7-beginv, img5-roi7-endinv
Figure 4.28: A comparison of the 3-D ADCIG migrated using (a) the starting velocity model (Figure 4.20a) and (b) the inverted velocity model (Figure 4.23). The blue circles highlight areas of improvements, while the red boxes indicate degradation of gather flatness. The angle gathers are extracted from the same location where the X-Z image section in Figure 4.25 is located. For each subplot, each of the three panels (left,middle,right) shows the angle gathers at a certain azimuth for different lateral locations in X. Specifically, the angle gathers are at 0°, −40.0° and 40.0° azimuth, respectively. [CR] adcig-beg1-roi7-azims,adcig-end1-roi7-azims
Figure 4.29: Same descriptions as in Figure 4.28, except that the angle gathers are extracted from the locations shown in Figure 4.27. [CR] adcig-beg5-roi7-azims,adcig-end5-roi7-azims
CHAPTER 4. 3-D FIELD DATA TEST — A TARGET-ORIENTED APPROACH

WEMVA APPLICATION ON ROI 2

Initial 3-D common image gathers in the target region

To show the existence of velocity error in the target region, I compute the subsurface-offset CIG (both in $h_x$ and $h_y$) for the target region. Again, I use 21 points in $h_x$ with 50 m spacing and 19 points in $h_y$ with 60 m spacing in order to capture most of the unfocused energy.

Figure 4.32 shows the subsurface offset CIG within an inline image section at $y = -3.51$ km. I only show a $h_x$ section (while $h_y=0$) and a $h_y$ section (while $h_x=0$). As can be seen from the figure, there is a great deal of unfocused energy in the subsurface offset CIG. This can be partially attributed to the very limited range of illumination angles; nonetheless, many of the curved events in the offset CIG clearly indicate the inaccuracy of the velocity model. Figure 4.33 shows the subsurface angle-domain CIG (ADCIG) within the same inline section ($y = -3.51$ km). Note the very limited range of angular illumination (less than 25$^\circ$) in the angle gathers, even along the best illuminated inline azimuth direction (azim=0$^\circ$). This is a combined result of a complex salt overburden, the relatively short shot-receiver offset (both inline and cross-line) compared to the target depth. Nonetheless, there are still many curved events in the ADCIG that can be used for the RMO-based WEMVA inversion.

Synthesized born-modeling dataset for target-oriented tomography

Same as I have done for ROI 1, I start my target-oriented tomography workflow by synthesizing a dataset that is recorded immediately on top of the target region while preserving all velocity information around the target region from the original dataset. I determine the acquisition parameters of the new plane-wave survey based on the fact that the maximum subsurface illumination angle depicted in the ADCIG, as shown in Zhang and Biondi (2014), is no more than 25$^\circ$, and the average velocity at the
Figure 4.30: Three-panel view showing the comparison between the derivatives of the normalized semblance over the moveout parameter (the $\partial S_m/\partial \rho$ term) computed using (a) the starting velocity model (Figure 4.20a) and (b) the inverted velocity model (Figure 4.23). Both figures are normalized with the same scaling factor and plotted with the same clip value.
top of the target region is $\sim 3000 \text{ m/s}$. The new acquisition samples 17 inlines and 17 crosslines (289 in total) plane-wave directions, with the ray parameters ranging between $\pm 160 \mu\text{s/m}$, which corresponds to $\pm 30^\circ$ subsurface angle illumination with $\sim 3.75^\circ$ sampling along the angle axis. Because of the reduction of the physical propagation domain, I reduce the recording time from 14.0 secs (in the original dataset) to 6.1 seconds (in the synthesized dataset), which allows me to sample only 99 frequencies between 4 Hz and 20 Hz for the new dataset, rather than $\sim 200$ frequencies for the original dataset.

I take extra precautions dealing with the salt body when synthesizing the new dataset. First, I build a binary mask from the starting velocity model to mark the area occupied by salt in the target region. Then I use this salt mask to erase the subsurface ODCIG that will be used for the generalized Born modeling because the salt reflection does not provide any velocity information for the sediments below. In addition, as we can see from the velocity model in Figure 4.34a, over half of the source and receivers in the synthesized survey would be placed in salt. The strong lateral variation in the velocity model at the recording depth would have caused extra distortion to the plane-wave source wavefields, and it would also have forced
me to use more reference velocities in my one-way propagator, which increases the computational cost. To address this potential issue, I create a sediments-flooded velocity model from the original model in which the salt velocity is replaced with the velocity values of its surrounding sediments. And the salt-excluded subsurface ODCIG combined with the sediments-flooded velocity model is used to synthesize the new dataset. Figure 4.35 shows the velocity model and zero subsurface offset section of the ODCIG that I use for the new data synthesis, respectively.

The reason that salt could be replaced with sediments in the velocity model is that the actual salt area is considered unsuitable for tomography updates and thus remains fixed throughout my velocity estimation workflow. Therefore, an ideal way to place the recording locations of the new survey would be to put sources and receivers immediately at the salt/sediment boundary because preserving the sources/receivers wavefields at this boundary would preserve all velocity information about the underlying sediments. However, this would cause extra complexity for my one-way propagator implementation. Using the sediments-flooded velocity model allows me to avoid such extra complexity because I am able to place sources/receivers at the same depth; more importantly, this method still preserves the source/receiver wavefields at the salt/sediment boundary since the part of the velocity model that is originally occupied by salt is not changed throughout the tomography workflow.

Similar to my previous example on ROI 1, The mapping from the original dataset to the target-oriented plane-wave dataset reduces the computational cost of WEMVA iterations by more than 90%.

To verify that the newly synthesized dataset contains the same velocity information as the original data, I recompute the subsurface ODCIG (both in $h_x$ and $h_y$) using the new dataset. Figure 4.36 and Figure 4.37 show the comparison between the ODCIG migrated using the original dataset and the synthesized new dataset at cross-line location $y = -4.77$ km, respectively. As we can see, except for a few disparities close to the boundary, the kinematic behaviors of the two ODCIG agree very well. In particular, the focusness and the shapes of the ODCIG are almost identical. Although the long-wavelength amplitude trends are slightly different due to different
Figure 4.32: A close-up view of the images and subsurface offset domain common image gathers at the target region of interest (4.16). (a): An inline image section at \( Y = -3.51 \) km; (b): The inline subsurface-offset (\( h_x \) range spans \(-0.50 \) km \( \sim +0.5 \) km) CIG for different lateral locations in (a); (c): The crossline subsurface-offset (\( h_y \) range spans \(-0.54 \) km \( \sim +0.54 \) km) CIG for different lateral locations in (a).
Figure 4.33: A close-up view of the subsurface angle-domain CIG at the target region (Figure 4.16). The gathers shown are extracted from the same inline section as shown in Figure 4.32a. Each subplot shows the angle gathers at a certain azimuth (as noted in the caption) respectively. The range of the reflection angle ($\gamma$) spans $-35^\circ \sim +35^\circ$. [CR] adcigs-3d-3510-3pcs
Figure 4.34: A close-up view of (a) the starting velocity model and (b) the zero subsurface-offset image at the target region (Figure 4.16).
Figure 4.35: A close-up view at the target region. (a): The sediments-flooded starting velocity model, which is used for new data synthesis as well as the starting velocity model in later WEMVA inversion; (b): the zero subsurface-offset image of the ODCIG that I use for new data synthesis. The salt reflectors from the original ODCIG are erased and the inverse of diagonal Hessian matrix is applied to the ODCIG for amplitude balancing. Compare the velocity model and image in this Figure with those in Figure 4.34.
acquisition geometries, my WEMVA inversion algorithm balances the amplitude of the reflectors so that the amplitude variations will not adversely affect the inversion.

**Inversion result**

I use the same inversion setup described in the previous ROI 1 example. For the spline nodes parameters used in gradient preconditioning, the initial spline nodes spacing is set as 1.5 km in X, 1.5 km in Y, and 0.3 km in Z, and the spacing reduces by ~ 0.05 km in X,Y and 0.02 km in Z for each iteration. With regard to the ADCIG sampling, I use 21 points along the reflection angle axis ranging from −25.0° to +25.0°, and I compute 7 azimuths from −60.0° to +60.0°. Again, I use the aforementioned salt area mask to prevent any updates inside the initial salt area by the WEMVA gradient.

Figure 4.38 shows the final velocity inversion result obtained using the RMO-based WEMVA method. An interesting point about the inverted velocity model is that the tomography is suggesting a low velocity zone beneath the salt body, and the lowest value in that zone is ~ 2650 km/s, which is ~ 10% slower than the initial velocity model. Figure 4.39 shows the history of step sizes and normalized objective function values by iteration.

To further verify that my inverted model does in fact improve the subsalt images, the comparison between the initial image and the image migrated using the inverted model is shown in Figure 4.40, 4.41 and 4.42. To compare the image focusness, I use the same clip value when displaying the initial image and the updated image. As we can see from these figures, the events of the updated image become more coherent, thus have higher amplitudes. Specifically, in Figure 4.41 an anticline structure is formed because of the low velocity zone, and the continuity of those reflectors are also significantly improved. Figure 4.43 shows the comparison between the ADCIG before and after update. The locations of these ADCIG are the slicing locations shown in Figure 4.40. (For this plot, it is the Z-X slice at Y = −3.99 km.) Similarly, Figure 4.44 shows the ADCIG comparison at a different location corresponding to Figure 4.42. From these ADCIG comparisons, we can see that the majority of the
Figure 4.36: 3-D subsurface ODCIG in the target region, used as the input ODCIG for modeling the new dataset; the section shown is extracted at $y = -4.77$ km. (a): the inline image section; (b): the inline subsurface-offset ($h_y$ is fixed at 0.0 km while $h_x$ range spans $-0.50$ km, $+0.5$ km) CIG for different lateral locations in (a). (c): the cross-line subsurface-offset ($h_x$ is fixed at 0 km while $h_y$ range spans $-0.54$ km, $+0.54$ km) CIG for different lateral locations in (a). [CR] odcigs-initmig1
Figure 4.37: 3-D subsurface ODCIG in the target region, migrated using the generalized Born-modeling dataset; for each subplot, refer to the descriptions in Figure 4.36. Compare the image and gather kinematics in this figure with those in Figure 4.36.
Figure 4.38: The inverted target region velocity model using the propose RMO-based WEMVA. The original salt area is restored before plotting. Note the low velocity values immediately beneath the salt. [CR] velinv-roi

ADCIG become more flat after velocity update.

Similarly as in the inversion example of ROI 1, I also plot the derivative of the normalized semblance over the moveout parameter (the $\partial S_m/\partial \rho$ term) at every image location before and after the inversion in Figure 4.45, as additional QC. From the figure, we can see that the deviations of angle gathers flatness are reduced after the inversion. I then plot their corresponding histograms in Figure 4.46. As can be seen from the figure, the derivative values are more concentrated around zero and are distributed more evenly between positives and negatives. These observations again indicate the model improvement as a result of inversion.
Figure 4.39: Iterative velocity inversion statistics for ROI 2: (a) History of step sizes by iteration; the step sizes are quantified by the maximum velocity value perturbation within the model space caused by that step size. (b) History of normalized objective function values by iteration.

[CR] steps, objs
CONCLUSION

This chapter presents a field data application of my target-oriented, RMO-based WEMVA tomography workflow using the E-Octopus III WAZ dataset. By doing careful data regularization and preprocessing, and focusing on relatively small target areas containing mainly subsalt sediment layers, I am able to generate images and gathers of satisfactory quality with limited academic computing facility that I have access to. The generalized Born-modeling based target-oriented approach I use enables further significant computational savings, while preserving the velocity information in the original data almost losslessly. Although subsalt areas are very challenging for velocity estimation because of the very limited angular illumination and complex salt overburden, nonetheless, the application of my RMO-based WEMVA on two individual target subsalt regions producing promising results. In the first target region, my inversion improves significantly the image quality beneath the salt and near the salt flank, which leads to better geological interpretation in these challenging areas. In the second target region, the inverted velocity model from my RMO-based WEMVA demonstrates convincing imaging improvement and uncovers an interesting low velocity zone beneath the salt, which might be worthy to investigate from a geological perspective.
Figure 4.40: Three-panel view showing the comparison between the zero subsurface-offset images migrated with (a) the starting velocity model (Figure 4.35a) and (b) the inverted velocity model (Figure 4.38). The blue circles highlight areas of improvements, while the red boxes indicate some image degradation. The coordinates of each section are annotated in the figure. Both figures are plotted using the same clip value. [CR] img4-roi-beginv, img4-roi-endinv
Figure 4.41: Same as Figure 4.40 but at a different location. [CR]

[img5-roi-beginv, img5-roi-endinv]
Figure 4.42: Same as Figure 4.40 but at a different location. [CR]

```plaintext
img2-roi-beginv,img2-roi-endinv
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Figure 4.43: A comparison of the 3-D ADCIG migrated using (a) the starting velocity model (Figure 4.35a) and (b) the inverted velocity model (Figure 4.38). The blue circles highlight areas of improvements. The angle gathers are extracted from the same location where the X-Z image section in Figure 4.40 is located. For each subplot, each of the three panels (left, middle, right) shows the angle gathers at a certain azimuth for different lateral locations in X. Specifically, the angle gathers are at $0^\circ$, $-40.0^\circ$ and $40.0^\circ$ azimuth, respectively. [CR] adcig-beg4-roi-azims,adcig-end4-roi-azims
Figure 4.44: Same descriptions as in Figure 4.43, except that the angle gathers are extracted from the locations shown in Figure 4.42. [CR] adcig-beg2-roi-azims,adcig-end2-roi-azims
Figure 4.45: Three-panel view showing the comparison between the derivatives of the normalized semblance over the moveout parameter (the $\partial S_m/\partial \rho$ term) computed using (a) the starting velocity model (Figure 4.34(a)) and (b) the inverted velocity model (Figure 4.38). Both figures are normalized with the same scaling factor and plotted with the same clip value.
Figure 4.46: The histograms of the quantities shown in Figure 4.45 (derivatives of the normalized semblance over the moveout parameter); (a) before inversion and (b) after inversion. [CR] dSdrho-hist-beg-roi2,dSdrho-hist-end-roi2
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