Illumination compensation by L1 regularization and steering filters

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ABSTRACT

L1/L2-regularization techniques often generate better results than the conventional least-squares solutions for inverse problem in geophysics. We implement a method to combine L1 regularization with steering filters. We obtain steering filters iteratively from input data without using any prior information. The numerical examples show significant improvement in comparison with the standard least squares. We demonstrate our method is robust with respect to inaccurate steering filters.

INTRODUCTION

L1-regularized optimization often yields more robost results in comparison with the standard least-squares optimization and is useful in geophysics when data is contaminated by high-amplitude noise. It is also necessary, if we will apply a filter to the model after inversion to seperate data from a different physical process, like the primary-multiple separation. L1 regularization, as a sparsity-promoting technique, is especially useful for time-lapse inversion, because the area of production-induced change is bounded (Maharramov and Biondi, 2014a,b, 2015).

In our previous work (Ma et al., 2014), we solved L1-regularized linearized waveform inversion (Tang, 2008, 2011) using a number of solution techniques: least-squares with conjugate gradient (CGLS), iterative reweighted least squares (IRLS), alternating direction method of multipliers (ADMM) and Split-Bregman method (Goldstein and Osher, 2009; Boyd, 2010), hyperbolic penalty function (HPF) with conjugate directions (Claerbout, 2009; Zhang and Claerbout, 2010). While all the methods performed well, we demonstrated that L1 regularization delivered a significant improvement over standard least squares. However, the results still suffer from lingering effects of illumination gaps in the data. In this work, we address this issue by combining L1-regularized inversion with the concept of steering filters (Clapp et al., 2005; Prucha and Biondi, 2002).

Steering filters can improve the quality of inversion, especially in complex overburden. We style the L1 regularization term in a way that favors specified dip directions and sparsity in the direction orthogonal to the dip. We study the same test problem as Zhang and Claerbout (2010), and demonstrate that with suitable prior knowledge of the dip structure, we can effectively compensate insufficient illumination and greatly improve the inversion results.

In this work we demonstrate that even without prior geological information, and using only seismic data, we can iteratively construct L1-regularization steering filters, thereby achieving a quality of inversion comparable to that when good prior information is available.

We begin by solving a least squares problem with the zero-order Tikhonov regularization (minimal norm solution) that yields an estimate of the gradient field. We then use this estimate to construct an initial steering filter. This steering filter is incorporated into the L1 regularization term as described in Section 2.2, and we use the results of subsequent regularized inversions to update the steering filter iteratively. We discuss techniques of regularizing estimated gradient field for the purpose of constructing robust steering filters in the **appendix**.

OPTIMIZATION METHOD AND REGULARIZATION

We have implemented several L1/L2 solvers and an HPF solver in our Stanford Exploration Project (SEP)-Vector library. Previously we have tested solvers on a few geophysical examples (Ma et al., 2014). We have shown that the quality of inversion results and computational costs are comparable for all the solvers in the examples we considered. The ADMM/Split-Bregman methods are deemed to be better than IRLS for compressed sensing and denoising problems. However, because it is hard to select the proper parameters of the ADMM/Split-Bregman methods, in this work we conduct L1-regularized linearized waveform inversion using our IRLS solver.

Regularization plays an important role in geophysical inverse problems. Regularization prevents overfitting noisy measurements, and ameliorates ill-posedness due to lack of data. Our mathematical model is only an approximation of a real physical process, and a suitable regularization may mitigate the mismatch between model and observation.

Regularization can be considered as “model styling”, imparting on our model certain features considered desirable in the context of our problem (Claerbout, 2014). In this sense, L1-regularized steering filters can be considered a constraint on model inversion favoring the continuity of dip structures.

Adaptive regularization has been proposed before, e.g. based on Baysian analysis (Zamanian et al., 2014). This typically requires solving auxiliary inverse problems. In our deterministic approach, the L1-regularization term is updated between solution iterations.

LINEARLIZED WAVEFORM INVERSION

Description of the problem and challenges

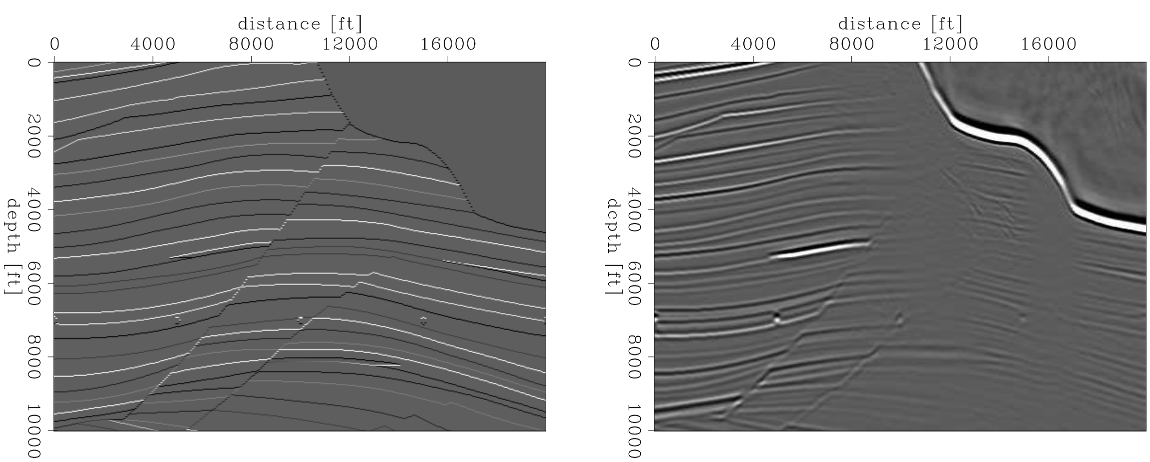
The target-oriented linearlized waveform inversion has been explored in several previous works (Clapp et al., 2005; Valenciano, 2006; Tang, 2008, 2011; Zhang and Claerbout, 2010; Ma et al., 2014). We are trying to solve:

$$\mathbf{Hm} \approx \mathbf{m_{mig}}, \tag{1}$$

where $\mathbf{H}$ is the Hessian operator,$\mathbf{m_{mig}}$ is called migrated data which is known, and $\mathbf{m}$ is the model we want to compute.

A standard method to solve the problem is least squares inversion. Regularization terms are usually included in the objective function for least squares inversion when the modeling operators are singular and data contains noise.

In this specific example, we have several challenges, as can be seen from the true reflectivity model and data used for inversion in Figure 1. The forward modeling is a linear approximation of the true operator. We have poor illumination under the salt. The existence of faults prevent us from getting a high quality model considering that Hessian operator acts like "convolution" operator.



(a) (b)

Figure 1: (a) Reflectivity model. (b) input migrated image. [**NR**]

Power of L1 regularization and steering filters

Previous results (Zhang and Claerbout, 2010; Ma et al., 2014) using objective function:

$$\mathbf{J}(\mathbf{m}) = \frac{1}{2}||\mathbf{Hm} - \mathbf{m_{mig}}||_2^2 + \varepsilon||\mathbf{m}||_1, \tag{2}$$

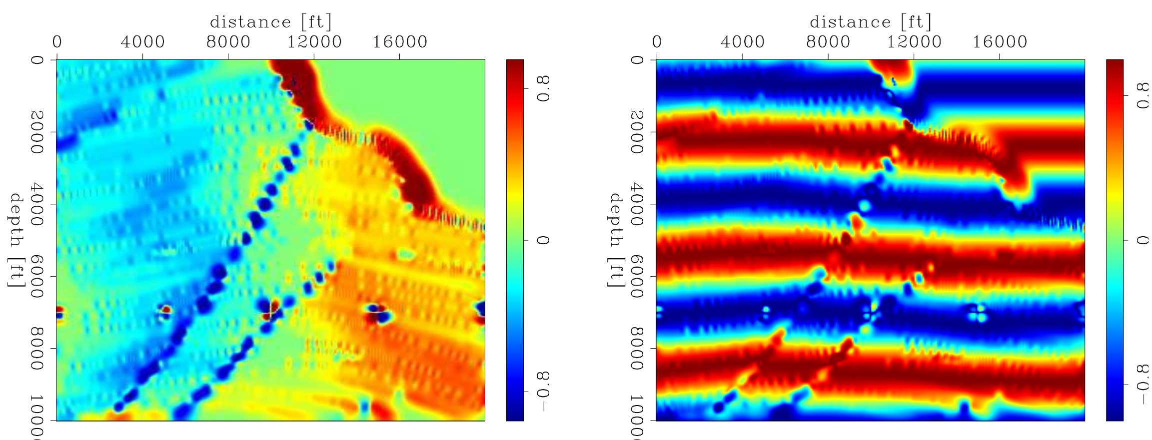
suggest that we can get more sparse reflectors in the well-illuminated zone (the left part of the model) using L1/L2 solvers. However, we do not get significant improvement under the salt. In addition, the data residues from L1/L2 solvers are more correlated comparing with CG method, suggesting our results are not convincing.

To further improve the results, the regularization term is replaced by more geologically plausible constraints, namely steering filters,

$$\mathbf{J}(\mathbf{m}) = \frac{1}{2}||\mathbf{Hm} - \mathbf{m_{mig}}||_2^2 + \varepsilon_{\mathbf{a}}||\mathbf{W_a}\nabla_{\mathbf{r}}\mathbf{m}||_1 + \varepsilon_{\mathbf{b}}||\mathbf{W_b m}||_1, \tag{3}$$

where $\mathbf{W_a}$ and $\mathbf{W_b}$ control the strength of regularization at each point, because we do not have equal illumination. The derivative $\nabla_{\mathbf{r}}$ is taken along the gradient prior. The geological information is contained in $\mathbf{W_a}$, $\mathbf{W_b}$ and $\mathbf{r}$.

To test the power of L1 regularization and steering filters, we assume we already have a good prior for $\mathbf{W_a}$, $\mathbf{W_b}$ and $\mathbf{r}$ from external knowledge. $\mathbf{r}$ in Figure 2(a) is used as a prior gradient field. The choice is natural, because previous constraints $\nabla_{\mathbf{x}}\mathbf{m}$ would penalize dipping reflectors. Next, we set $\mathbf{W_b} = 1.$ for $\mathbf{x} < 10000ft$ and $\mathbf{W_b} = 0.$ for $\mathbf{x} > 10000ft$, and $\mathbf{W_a} = 1.$. We made this choice based on knowing the area under the salt is poorly illuminated, and we need more compensation from steering filters.



(a) (b)

Figure 2: (a) x component of correct gradient prior. (b) x cpomponent of wrong gradient prior. [**ER**]

The final results in Figure 3(e) are better comparing with CG method (Figure 3(a)) and previous L1/L2 results (Figure 4(a)). We have obtained sparse reflectors as expected from hybrid L1/L2 solvers. The faults are correctly recovered and the reflectors under the salts are properly inverted. The data residue in Figure 3(f) looks more random comparing with previous results. We have run 30 by 20 CG steps to obtain the results in Figure 3(e), which is ~ 10 times more expensive than the CG method. The cost is acceptable considering the improvement of inversion results.

Obtaining steering filters from the data iteratively

In the previous subsection, we have demonstrated L1 regularization and steering filters can help in the inverse problems. However, it is not trivial to obtain proper geological constraints. We can add external knowledge that is independent of seismic data: geophysical, geological, and geomechanical information. Before we have enough knowledge, we can squeeze the data and construct reasonable steering filters from the seismic data.

Numerically, assume a prior gradient field is not available, though a little "external information" is known: the strongly dipping events at $z \sim 4000ft$ and $x \sim 12000ft$ are artifacts. We use the following workflow. First, we do inversion with uniform constraints as in equation 2. Next, we extract the gradient field $\mathbf{r}$ from the inversion results and mute the dipping reflectors near $z \sim 4000ft$ and $x \sim 12000ft$. Then we use the gradient field as prior to construct steering filters and do the inversion again with equation 3 and repeat the process until the results converge. $\mathbf{W_a}$, $\mathbf{W_b}$ should also be changed iteratively based on previous inversion results; however, we adjust them manually for simplicity.

We run the IRLS algorithm 6 times, and we can see the results in Figure 4. Significant improvement can be observed which means we can really get more information from the data if we go beyond simple least squares inversion. However, we could not match the results in the previous section (where we assume we know good prior).

Do the wrong steering filters ruin the inversion results?

We put our knowledge into the objective function as prior and we want to know how our choice affects the results. Firstly, we are geophysicists, not data scientists. When we solve an inverse problem, we rarely obtain our results exclusively from the data. Even in the simplest case of geophysical inverse problem with regularization, the extra terms $\nabla_{\mathbf{r}}\mathbf{m}$ or $\mathbf{m}$ come from our geophysical knowledge of the previous subsurface studies. Secondly, we iteratively construct steering filters and there is inevitable inaccuracy.

Ideally, we want proper prior to promote correct reflectors in the poorly illuminated area, and wrong prior has negligible effects. We can justify our assumption qualitatively. Because the true model lies close to the null spaces of forward modeling operator $\mathbf{H}$ and correct regularization operator $\nabla_{\mathbf{r}}$, the correct objective yields minimum norm solution within the intersection of the two null space. If the null space is not large, we would expect the inversion results to be close to the true model.

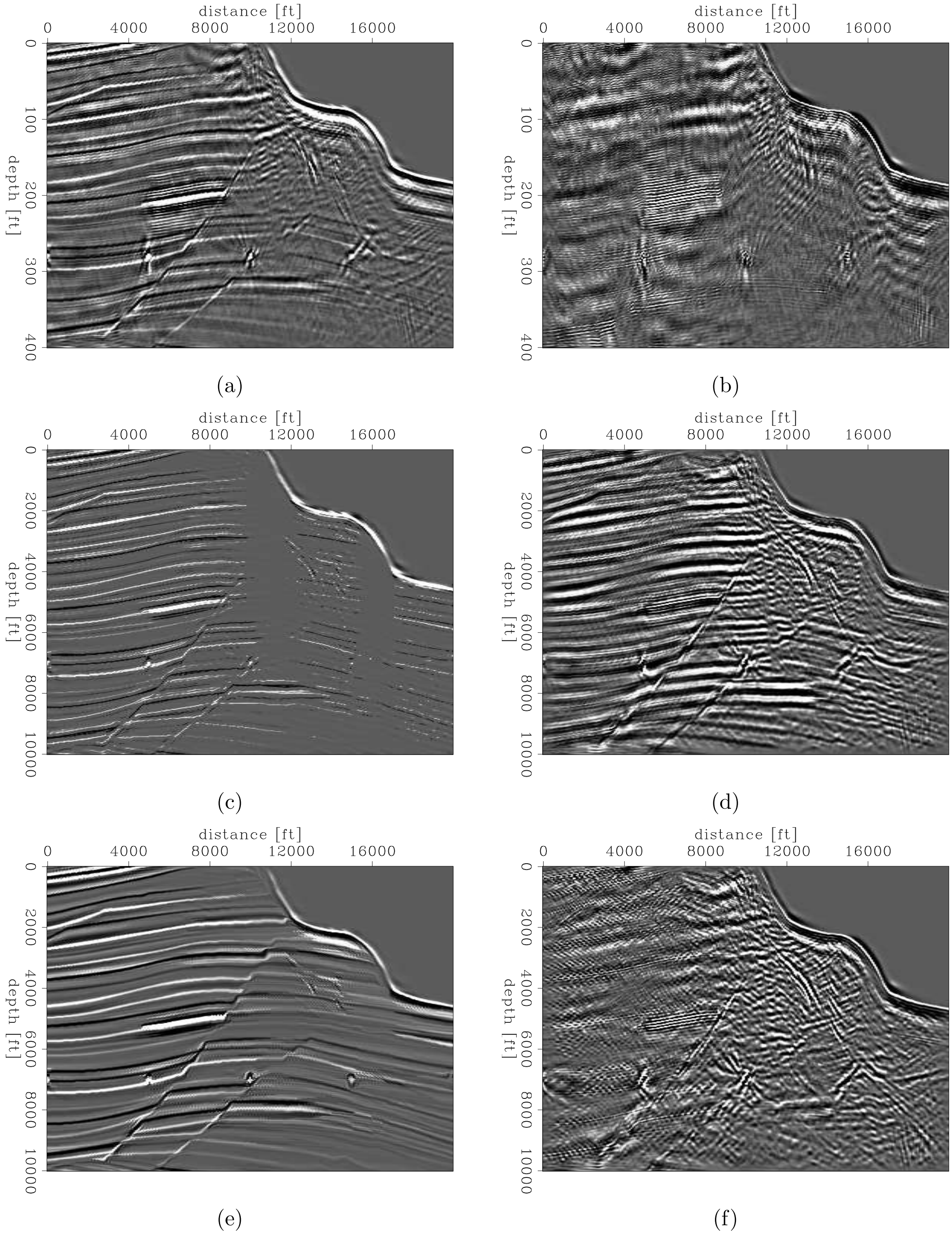


Figure 3: Comparison of linearized waveform inversion results and residues: (a) CGLS inversion result. (b)CGLS inversion residue. (c) IRLS inversion result . (d) IRLS inversion residue. (e) IRLS inversion result with gradient prior. (f) IRLS inversion residue with gradient prior. All figures are clipped at the same level. [**CR**]

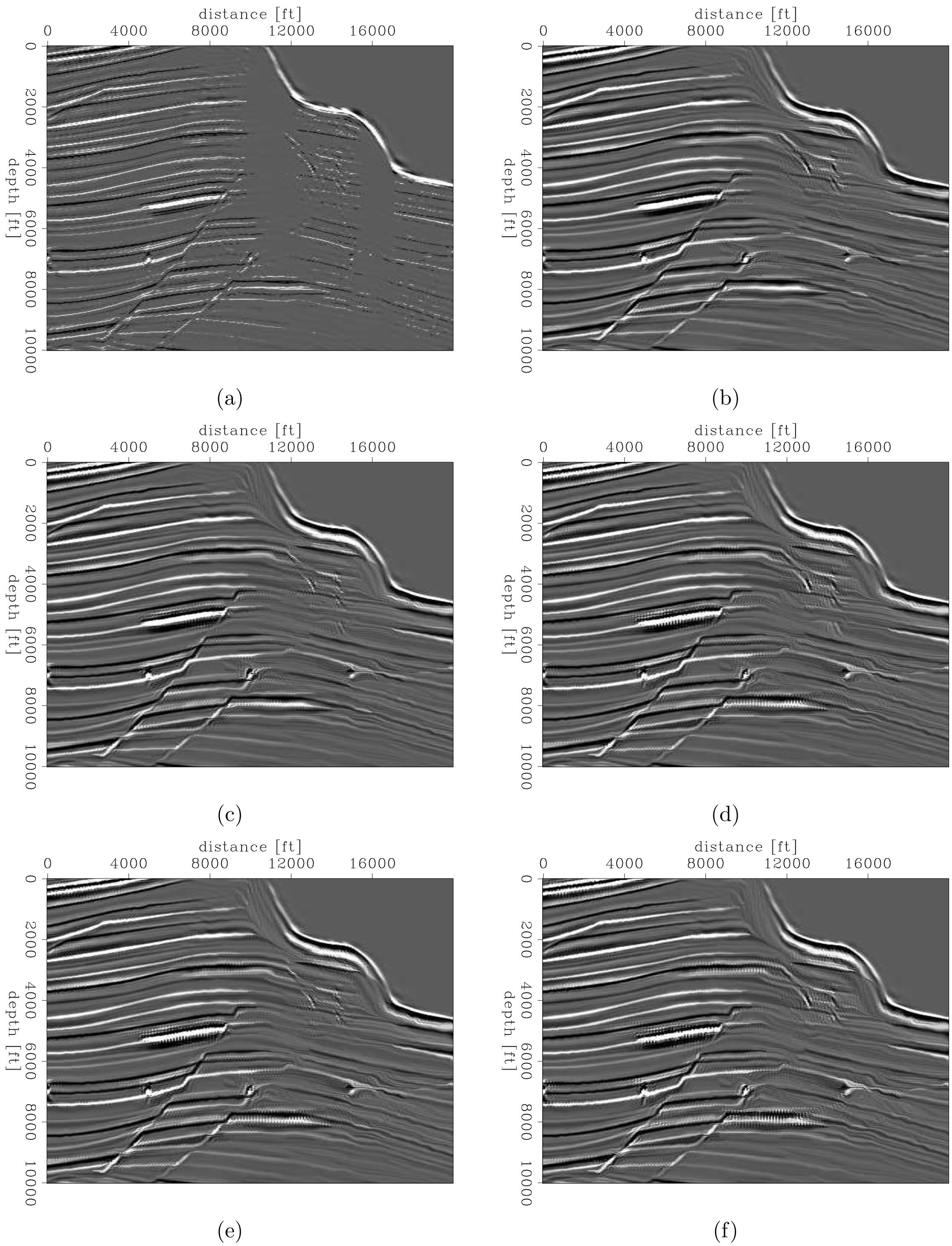


Figure 4: Comparison of linearized waveform inversion results and residues: (a) 1st iteration of inversion with no prior. (b) 2nd iteration of inversion with prior. (c) 3rd iteration of inversion with prior. (d) 4th iteration of inversion with prior. (e) 5th iteration of inversion with prior. (f) 6th iteration of inversion with prior. [**NR**]

However, if we provide wrong constraints $\nabla_{\tilde{\mathbf{r}}}$, then $Null(\mathbf{H})$ would be far away from $Null(\nabla_{\tilde{\mathbf{r}}})$, and there is no way to fit them at the same time. Considering the weight on the regularization term is small comparing with the weight of the data fitting term, the wrong constraints will be negligible.

We use the same objective function as described in the previous subsection with the wrong gradient prior as in Figure 2(b). We set $\mathbf{W_b} = 1$ everywhere, because in this case as we assume we do not know whether the gradient prior is good or bad. We also twist the gradient prior as in Figure 2(b), and we can see the inversion result in Figure 5(c). The correct gradient prior yields better results at the gaps under the salt, while the wrong gradient prior does not introduce too many artifacts at the gaps, which proves our method is robust against inaccurate steering filters.

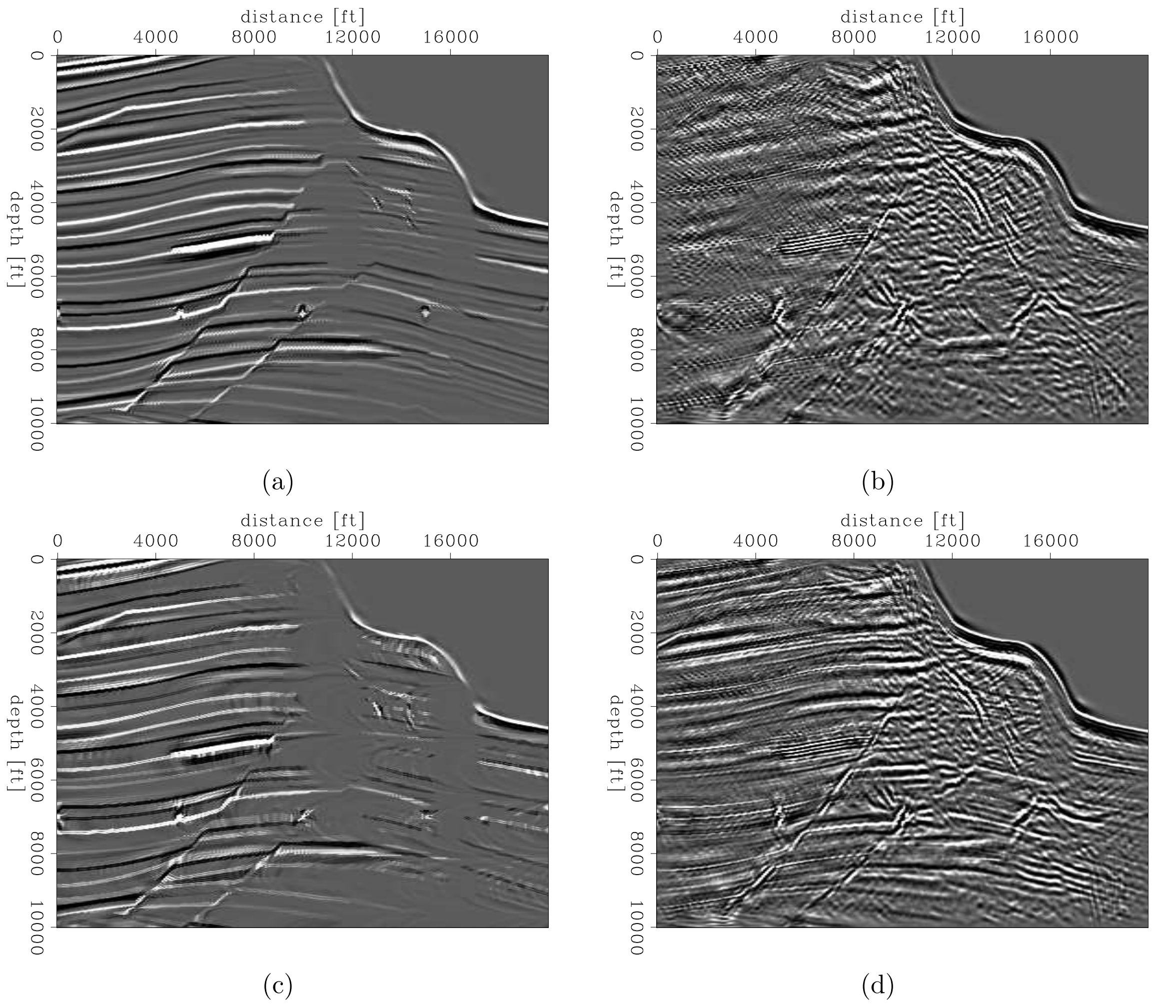


Figure 5: Comparison of linearized waveform inversion with different gradient prior: (a), (b) Inversion result and residue with gradient prior in Figure 2(a). (c), (d) Inversion result and residue with gradient prior in Figure 2(b). All figures are clipped at the same level. [**CR**]

CONCLUSIONS

In conclusion, we present a workflow to obtain better results from geophysical inverse problems by combining L1 regularization with steering filters. We prove improvement is possible when no prior knowledge is available. When applying the methods to a real problem, we can use external knowledge like geomechanics to construct more reasonable steering filters.

ACKNOWLEDGMENT

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APPENDIX A: CONSTRUCTION OF STEERING FILTERS BASED ON LEAST SQUARES

In this appendix, a method to construct steering filters from images is presented. The filters are used as prior information for the next iteration, before we understand how to incorporate geomechanics into regularization.

We are interested in obtaining a smoothly varying gradient field. Correct gradient direction boosts the desired events in the area of poor illumination. Incorrect prior of gradient direction is ignored by the optimization process thus does not create many artifacts, as proved previously by (Prucha and Biondi, 2002). In order to test the strength and weakness of our method, we apply it to the example in Figure 6, following the idea from Hale (2007).

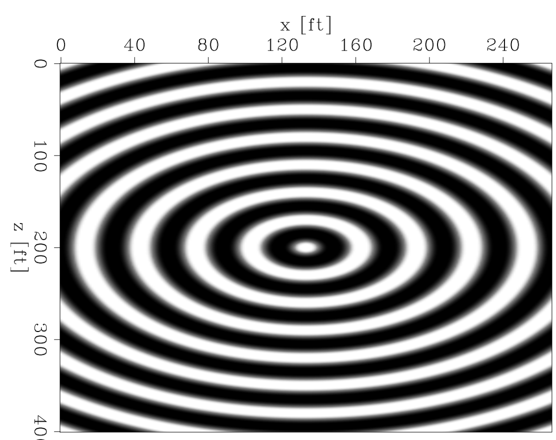


Figure 6: Test model with all dipping directions. [**ER**]

Suppose we have a 2D image $\mathbf{m}_0(x, z)$ from which we want to extract the gradient field. We first use the Sobel filter to estimate $\partial_x \mathbf{m}_0$ and $\partial_z \mathbf{m}_0$, from which we can compute the norm of gradient $\|\nabla \mathbf{m}_0(x, z)\|_2$ and dipping angle $\theta_0(x, z)$. It is obvious that $\theta_0(x, z)$ cannot be directly used as prior gradient field for the next iteration. θ_0 is not smooth because of the noise in the image and crossing events, etc. θ_0 is not reliable when $\|\nabla \mathbf{m}_0(x, z)\|_2 \approx 0$.

To fix those problems, we need to construct a weighting function $\mathbf{W}(x, z)$ to suppress unreliable θ_0. We choose

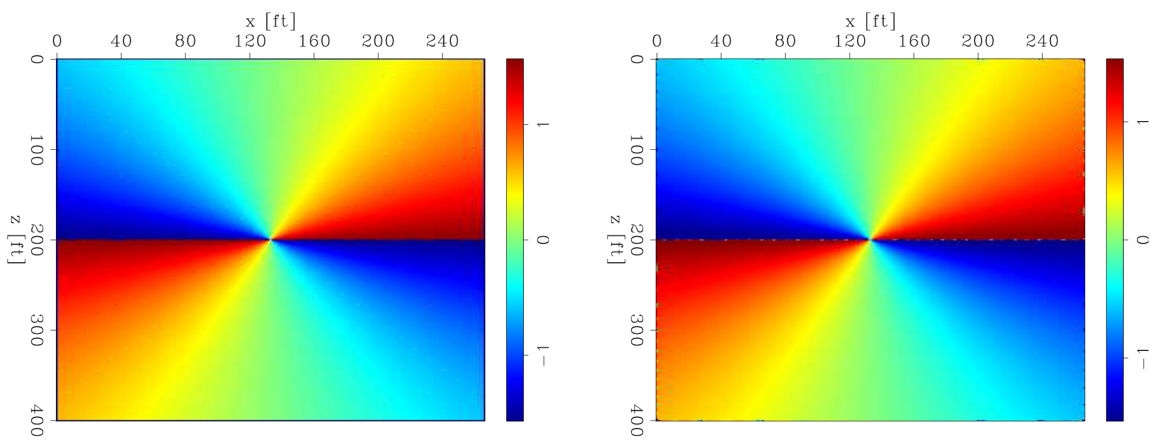
$$\mathbf{W}(x, z) = \|\nabla \mathbf{m}_0(x, z)\|_2 \times (\lambda + |\mathbf{m}_0(x, z)|), \tag{4}$$

where we have a smaller weight when $\|\nabla \mathbf{m}_0(x, z)\|_2 \approx 0$ or $|\mathbf{m}_0| \approx 0$. With this weight function we can set up a linear inverse problem with an objective function,

$$\mathbf{J}(\theta) = \frac{1}{2}\|\mathbf{W}(\theta - \theta_0)\|_2 + \frac{\varepsilon}{2}\|\nabla^2 \theta\|_2. \tag{5}$$

The first term is data fitting with larger weight on the reliable estimation of gradient direction. The second term is regularization which promotes a smooth solution.

We can solve the optimization problem to obtain θ^*, we can see θ_0 and θ^* in Figure 7. There are some artifacts in θ_0 caused by the numerical issue with 3 by 3 Sobel filter, and the artifacts are removed in θ^*.



(a) (b)

Figure 7: Left: $\theta_0 - \pi/2$. Right: $\theta^* - \pi/2$. [**ER**]

Once we obtain a smooth gradient field characterized by dipping angle, we can construct local steering filter,

$$\mathbf{L}_{\theta^*} = \nabla_{r^*_\perp}, \tag{6}$$

where $r^*_\perp$ is perpendicular to the gradient direction associated with θ^*. We apply the local filter to the original image, and obtain the result in Figure 8(a). We can see that our method works very well except for the vertical-dipping direction. The reason is that our objective function is built on angle and suffers from branch cut problem. Removing the branch cut will lead to a nonlinear optimization problem that is beyond the scope of this report.

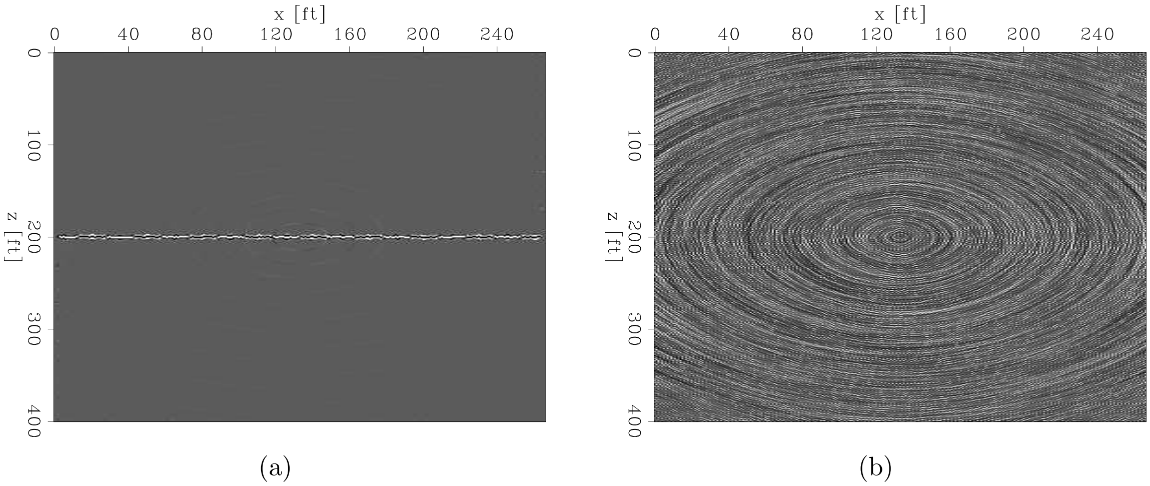


Figure 8: (a) Apply local filter to original image.(b) Reconstructed image from noise. [**ER**]

We also test the local filter by solving an inverse problem with

$$\mathbf{F}(\mathbf{m}) = \frac{1}{2}\|\mathbf{L}_{\theta^*}\mathbf{m} - \mathbf{d}\|_2^2 + \frac{\alpha}{2}\|\mathbf{m}\|_2^2, \tag{7}$$

where $\mathbf{d}$ is uniform random noise. We would expect $\mathbf{m}^*$ obtained by minimizing $\mathbf{F}(\mathbf{m})$ should have the same curvature as the original image. We can see $\mathbf{m}^*$ in Figure 8(b).

Finally, in this paper, we apply our method to the example from the Sigbee2A model and the results can be seen in Figure 9(c). It is interesting to see that after applying the local filter, we are able to see the fault and multiple more clearly.

REFERENCES

Boyd, S., 2010, Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers: Foundations and Trends in Machine Learning, **3**, 1–122.

Claerbout, J. F., 2009, Blocky models via the L1/L2 hybrid norm: SEP Report, **139**, 1–10.

——, 2014, GEOPHYSICAL IMAGE ESTIMATION BY EXAMPLE.

Clapp, M. L., R. G. Clapp, and B. L. Biondi, 2005, Regularized least-squares inversion for 3-D subsalt imaging: SEG Technical Program Expanded Abstracts 2005, 1814–1817.

Goldstein, T. and S. Osher, 2009, The Split Bregman Method for L1-Regularized Problems: SIAM Journal on Imaging Sciences, **2**, 323–343.

Hale, D., 2007, Local dip filtering with directional Laplacians: CWP Report-567, 91–102.

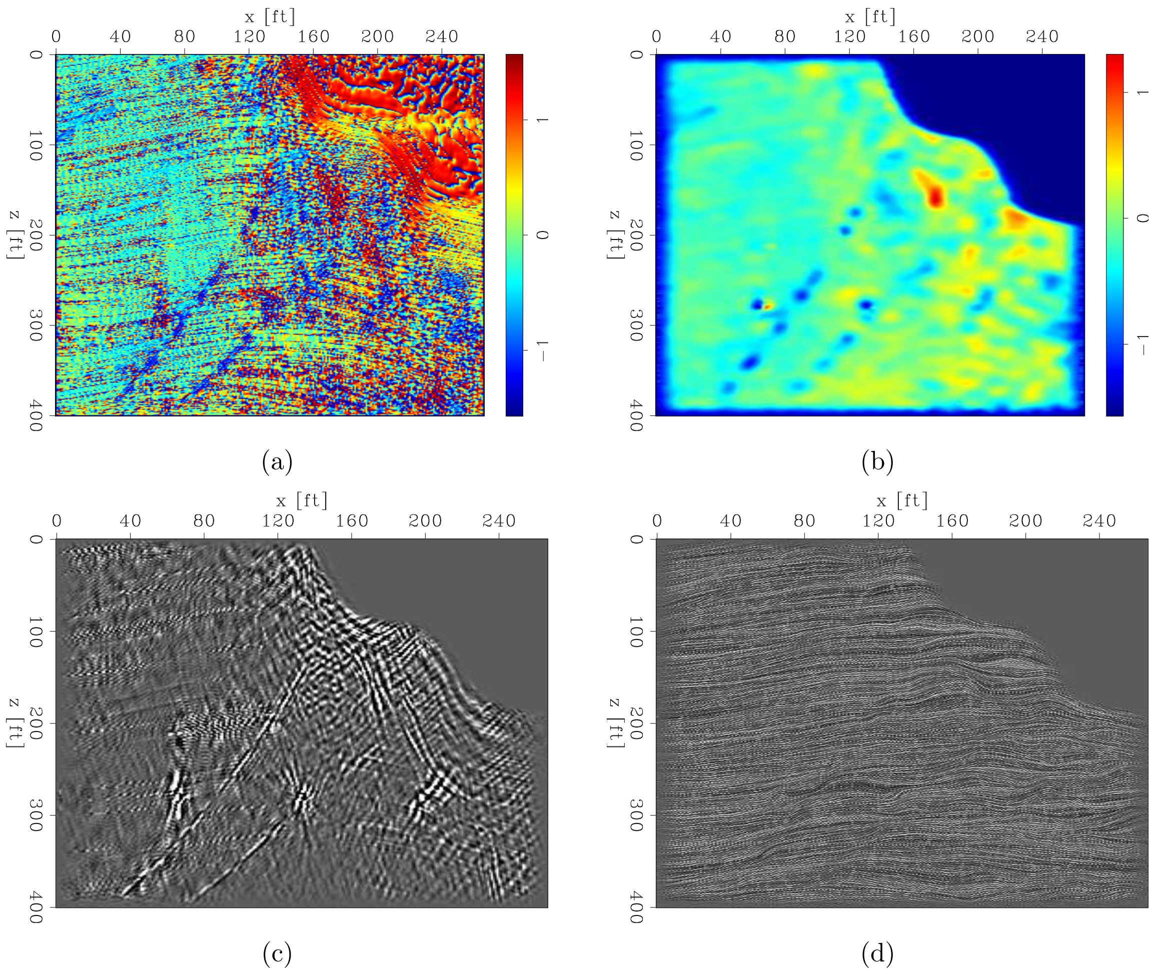


Figure 9: (a) $\theta_0 - \pi/2$. (b) $\theta^* - \pi/2$. (c) Apply steering filter to original image. (d) reconstructed image from noise. [**ER**]

Ma, Y., M. Maharramov, and B. Biondi, 2014, Geophysical applications of hyperbolic and hybrid L1/L2 optimization: SEP Report, **155**, 133–144.

Maharramov, M. and B. Biondi, 2014a, Joint full-waveform inversion of time-lapse seismic data sets: SEG Technical Program Expanded Abstracts 2014, 954–959.

———, 2014b, Robust joint full-waveform inversion of time-lapse seismic datasets with total-variation regularization: SEP Report, **155**, 199–208.

———, 2015, Robust Simultaneous Time-lapse Full-waveform Inversion with Total-variation Regularization of Model Difference: 77th EAGE Conference and Exhibition, Extended Abstracts (accepted).

Prucha, M. L. and B. L. Biondi, 2002, Subsalt event regularization with steering filters: SEG Technical Program Expanded Abstracts 2002, 171–174.

Tang, Y., 2008, Wave-equation Hessian by phase encoding: SEG Technical Program Expanded Abstracts 2008, **27**, 2201–2205.

———, 2011, Imaging and velocity analysis by target-oriented wavefield inversion: PhD thesis, Stanford University.

Valenciano, A., 2006, Target-oriented wave-equation inversion: Geophysics, **71**, A35–38.

Zamanian, S. A., W. L. Rodi, M. I. T. Earth, J. A. Kane, and S. I. E, 2014, Hierarchical and Variational Bayesian Approaches to Least-Squares Migration: SEG Technical Program Expanded Abstracts 2008, 3731–3736.

Zhang, Y. and J. F. Claerbout, 2010, Least-squares imaging and deconvolution using the hybrid norm conjugate-direction solver: SEP Report, **140**, 129–142.