Double-difference time-lapse FWI with a total-variation regularization

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ABSTRACT

In this paper we study double-difference FWI with a total-variation (TV) model-difference regularization (Maharramov and Biondi, 2014c). This data-space technique reduces the sensitivity of time-lapse FWI to inaccurate velocity model reconstruction. We describe a computational framework for conducting a TV-regularized double-difference FWI both as a simultaneous inversion and as an extension of single-model inversion. The method is demonstrated on linearized time-lapse waveform inversion of production effects for a synthetic example of compacting sub-salt reservoirs. We demonstrate the resolution of production effects and discuss stability of the results with respect to inaccuracies in the background velocity model.

INTRODUCTION

Simultaneous time-lapse (4D) full-waveform inversion (FWI) with a total-variation (TV) regularization achieved a considerable success in resolving production-induced changes both in synthetic and field-data tests (Maharramov and Biondi (2015b, 2014c, 2015a), and Maharramov and Biondi (2014a), supplementary material). As a model-space technique, the simultaneous inversion is stable with respect to repeatability issues such as different acquisition parameters, while the conventional double-difference method (Watanabe et al., 2004; Denli and Huang, 2009; Zheng et al., 2011; Asnaashari et al., 2012; Raknes et al., 2013) is quite sensitive to survey repeatability and may require a significant preprocessing effort to achieve data equalization (Maharramov and Biondi, 2014b,a). However, one important potential advantage of the double-difference FWI is that once the baseline and monitor data sets are equalized and measurable production effects can be observed and isolated in the data difference, the double-difference FWI seeks to resolve the model difference by matching the data difference only, rather than matching the separate acquisition data sets. This suggests the double-difference FWI as a potentially useful technique in situations where resolution of the baseline model is still subject to considerable uncertainty but the production-induced effects in the data are significant enough to estimate model perturbations that are causing them. Maharramov and Biondi (2014c) proposed a formulation of the double-difference method that allows matching the observed data difference by simultaneously inverting subsurface models of different vintage while imposing a blockiness-promoting total-variation regularization on the model difference:

$$\alpha\|\mathbf{F}(\mathbf{m}_b)-\mathbf{d}_b\|_2^2 \ +\beta\|\mathbf{F}(\mathbf{m}_m)-\mathbf{d}_m\|_2^2 \ + \tag{1}$$

$$\gamma\|\left(\mathbf{F}(\mathbf{m}_m)-\mathbf{F}(\mathbf{m}_b)\right)-(\mathbf{d}_m-\mathbf{d}_b)\|^2 \ + \tag{2}$$

$$\alpha_1\|\mathbf{W}_b\mathbf{R}_b(\mathbf{m}_b-\mathbf{m}_b^{\mathrm{PRIOR}})\|_2^2 \ + \tag{3}$$

$$\beta_1\|\mathbf{W}_m\mathbf{R}_m(\mathbf{m}_m-\mathbf{m}_m^{\mathrm{PRIOR}})\|_2^2 \ + \tag{4}$$

$$\delta\|\mathbf{W}\mathbf{R}(\mathbf{m}_m-\mathbf{m}_b-\Delta\mathbf{m}^{\mathrm{PRIOR}})\|_1 \ \rightarrow \ \min, \tag{5}$$

In the above equations, subscripts b and m denote baseline and monitor acquisitions, $\mathbf{d}_{b,m}$ is a vector of observations (survey data), $\mathbf{m}_{b,m}$ are the baseline an monitor models, $\mathbf{F}$ is the forward-modeling operator, $\mathbf{W}_{b,m}, \mathbf{W}$ and $\mathbf{R}_{b,m}, \mathbf{R}$ are weighting and regularization operators for the baseline, monitor, and the model difference. Model and model-difference priors can be explicitly specified in the objective function as shown in (3,4,5), however, in this work we assume no prior information. For a TV-regularized model-difference inversion, we use the model-difference regularization operator $\mathbf{R}$ such that

$$\mathbf{R}f(x,y,z) \ = \ |\nabla f|, \tag{6}$$

i.e., $\mathbf{R}$ computes the spatial gradient of its argument function at each point of the subsurface. This means that we seek a monitor model $\mathbf{m}_m$ that differs from the inverted baseline by a spatially bounded or *blocky* component (Rudin et al., 1992). The assumption of blockiness and, more generally, spatial boundedness of production-induced model perturbations is consistent with the physical effects of fluid substitution, reservoir compaction and overburden dilation (Johnston, 2013). An example of a TV-regularized simultaneous FWI applied to estimating spatially localized production-induced overburden dilation from Gulf of Mexico time-lapse data is provided by Maharramov and Biondi (2015a) in this report. In this work, we assess the feasibility of TV-regularized double differencing with the terms (1,3,4) omitted, i.e., TV-regularized double-differencing without simultaneously fitting data of different vintage. In our initial tests we consider a linearized formulation with adaptive sparsity-promoting steering-filter regularization (Ma et al., 2015a), and compare the results with the simultaneous linearized inversion of Ma et al. (2015b). Of particular interest to us is the effect on the two methods of inaccuracies in the baseline model, and whether matching the data difference only can achieve a greater robustness with respect to the uncertainty in the background model.

METHOD

We consider the following optimization problem

$$\|\left(\mathbf{F}(\mathbf{m}_m)-\mathbf{F}(\mathbf{m}_b)\right)-(\mathbf{d}_m-\mathbf{d}_b)\|^2 \ + \tag{7}$$

$$\delta\|\mathbf{W}\mathbf{R}(\mathbf{m}_m-\mathbf{m}_b)\|_1 \ \rightarrow \ \min, \tag{8}$$

with operator $\mathbf{R}$ given by (6). The regularized optimization problem (7,8) can be solved with respect to the combined baseline/monitor model vector $(\mathbf{m}_b, \mathbf{m}_m)$ (i.e., by inverting the two models simultaneously) or using the traditional double-difference approach by e.g. fixing the baseline model $\mathbf{m}_b$ and minimizing with respect to the monitor model $\mathbf{m}_m$ (Watanabe et al., 2004; Denli and Huang, 2009; Asnaashari et al., 2012; Zheng et al., 2011; Maharramov and Biondi, 2014b). The latter approach reduces the size of optimization problem and alleviates the null-space issues associated with resolving two subsurface models from the data difference alone. Note that the latter issue can be remedied by adding back the terms (1)—i.e., effectively combining the double-differencing with a simultaneous baseline and monitor model inversion. However this combined approach is outside the scope of this work.

Our method can be summarized as follows:

1) Invert the baseline model $\mathbf{m}_b$:

$$\|\mathbf{F}(\mathbf{m}_b) - \mathbf{d}_b\|_2^2 \;\to\; \min . \tag{9}$$

2) Generate new synthetic monitor survey data $\mathbf{d}_2$ by adding the observed data difference $\mathbf{d}_m - \mathbf{d}_b$ to the forward-modeled baseline data $\mathbf{F}(\mathbf{m}_b)$:

$$\mathbf{d}_2 \;=\; \mathbf{F}(\mathbf{m}_b) \;+\; (\mathbf{d}_m \;-\mathbf{d}_b)\,, \tag{10}$$

3) Invert the monitor model $\mathbf{m}_m$ from the new synthetic data $\mathbf{d}_2$:

$$\|\,(\mathbf{F}(\mathbf{m}_m) - \mathbf{d}_2)\,\|_2^2 + \tag{11}$$

$$\delta \|\mathbf{W}\mathbf{R}(\mathbf{m}_m - \mathbf{m}_b)\|_2^2 \;\to\; \min . \tag{12}$$

Method (9,10,11,12) can be used with both 4D FWI and the linearized waveform inversion. Note that in the latter case, because the forward-modeling operator $\mathbf{F}$ is linear, the above procedure is the only correct approach to solving (7,8) in the absence of extra constraints, as the linearized inversion problem has a null space dimension of (at least) the subsurface model: adding the same perturbation to both the baseline and monitor models does not affect the data difference.

Because for linearized inversion the inverted models are qualitatively interpreted as reflectivity, the total-variation regularization operator (6) should be replaced in this case with an operator that promotes blockiness of the reflectivity only along reflector dips while enforcing sparsity in the orthogonal direction (Ma et al., 2015a). This is achieved by replacing the full gradient in (6) with a directional gradient

$$\mathbf{R} f(x,y,z) \;=\; |\nabla_{\boldsymbol{\xi}} f|, \tag{13}$$

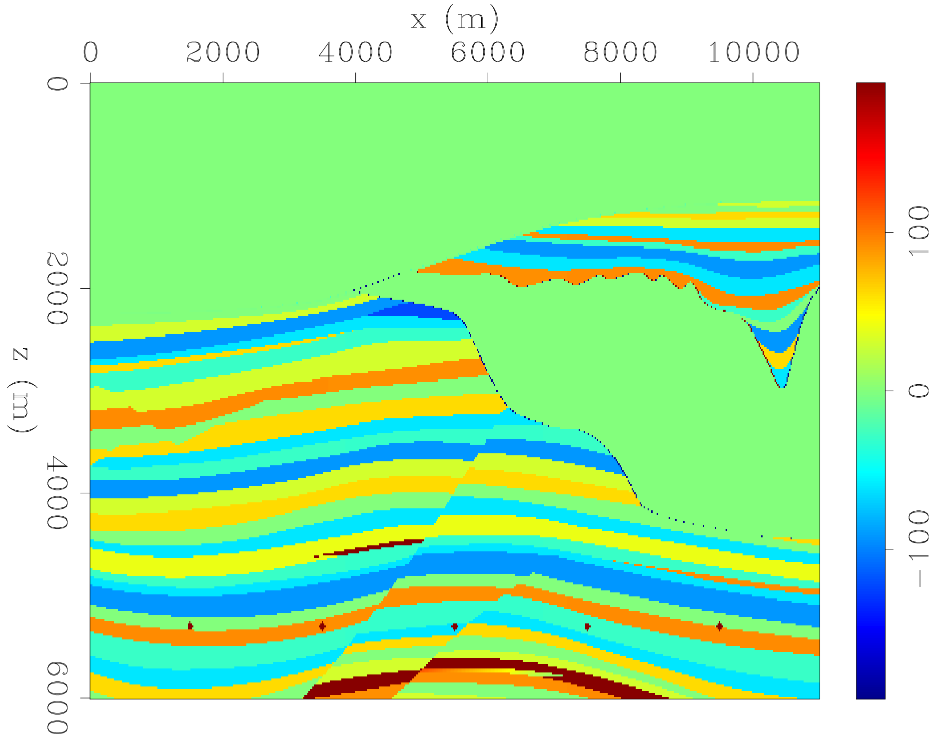
where $\boldsymbol{\xi} = \boldsymbol{\xi}(x,y,z)$ is the dip direction at (x,y,z) that is updated at each iteration of (11,12). This approach is effectively equivalent to promoting blockiness only along reflector surfaces, and therefore we will call this approach *Steering TV* (STV) regularization.

RESULTS

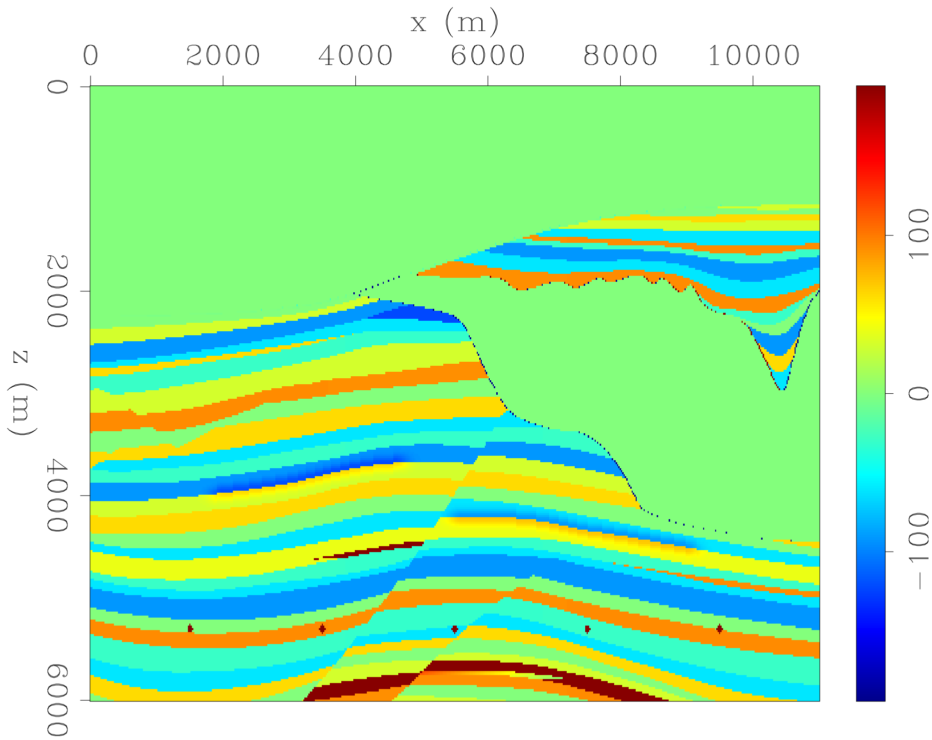
We applied the proposed method to a sub-salt time-lapse reflectivity inversion problem studied by Ma et al. (2015b) in this report. Synthetic baseline and monitor acoustic velocity models are shown in Figures 1(a) and 1(b), respectively. The monitor model has been chosen to simulate the effects of gas substitution with water (a higher velocity below the reservoir top) and overburden dilation due to reservoir compaction (lower velocity above the reservoir top) at two locations below and on the left side of the salt body. The time-lapse linearized waveform inversion seeks to recover image difference between monitor and baseline migrations, typically using the same (baseline) background velocity model for both baseline and monitor images. The result of applying the simultaneous STV-regularized linearized inversion (Ma et al., 2015b) is shown in Figure 2(a). The corresponding STV-regularized double-difference inversion is shown in Figure 2(b). In both cases a target-oriented inversion was conducted within a target window shown in the figures, using the exact background velocity model matching the baseline velocity model. Both methods are expected to achieve similar results because under the assumption that problem (9) is solved exactly, the two methods are mathematically equivalent. Production-induced reflectivity changes for both reservoirs stand out prominently in both images. The double-difference result was obtained using identical acquisition geometries to match the effect of data equalization that is part of standard time-lapse processing (Maharramov and Biondi, 2014b). However, when the background model is inaccurate, the simultaneous inversion and double-difference solve two different problems: the first method seeks to match two different sets of reflection data using the wrong background velocity, while the regularized double difference seeks to match the observed *relative* data difference by perturbing the predicted (and inaccurate) baseline reflectivity model. Inversion results for the two methods using a 10% overestimated velocity model in the target zone are shown in Figures 3(a) and 3(b). While the wrong background velocity still results in a slight mispositioning of the target reflectors, the double-difference reflectivity inversion appears to be more robust with respect to inaccurate velocity. The double-difference inversion is still able to resolve the reflectivity change along two isolated reflectors corresponding to the two reservoirs, but the simultaneous inversion result using the wrong background velocity is contaminated with artifacts that can be misinterpreted as production effects (e.g. the artifacts marked with red circles in Figure 3(a))

CONCLUSIONS AND PERSPECTIVES

Double differencing with steering TV (STV) regularization may yield more robust inversion of reflectivity changes in the presence of velocity uncertainty. Better imaging of reflectivity changes due to fluid-substitution effects leads to improved infill strategies and reservoir monitoring, and therefore is of paramount importance for reservoir production management. While linearized waveform inversion presents a useful initial application of the regularized double-differencing technique, application

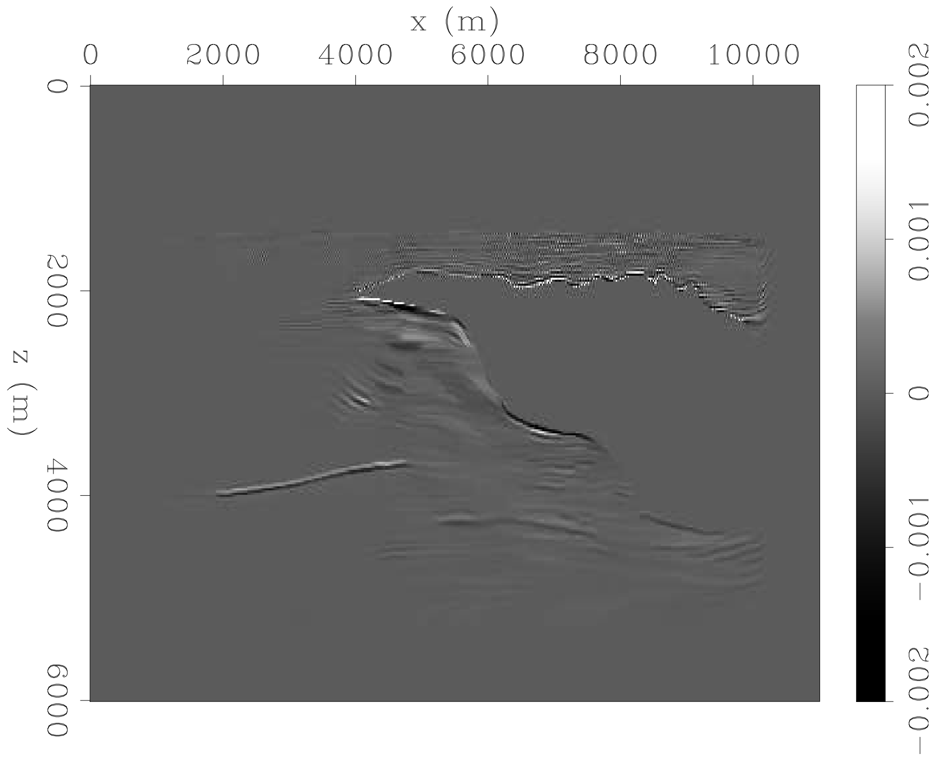


(a)

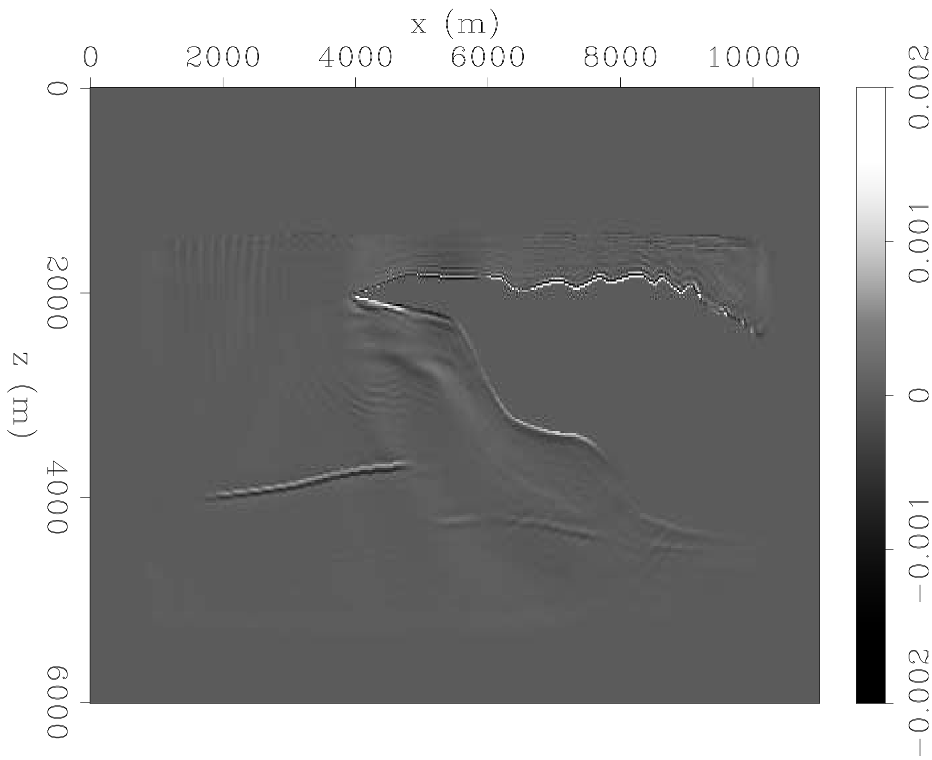


(b)

Figure 1: (a) Baseline velocity model. (b) Monitor velocity model simulating production-induced fluid substitution effects (positive change) and overburden dilation effects (negative change) for two reservoirs. [**ER**]

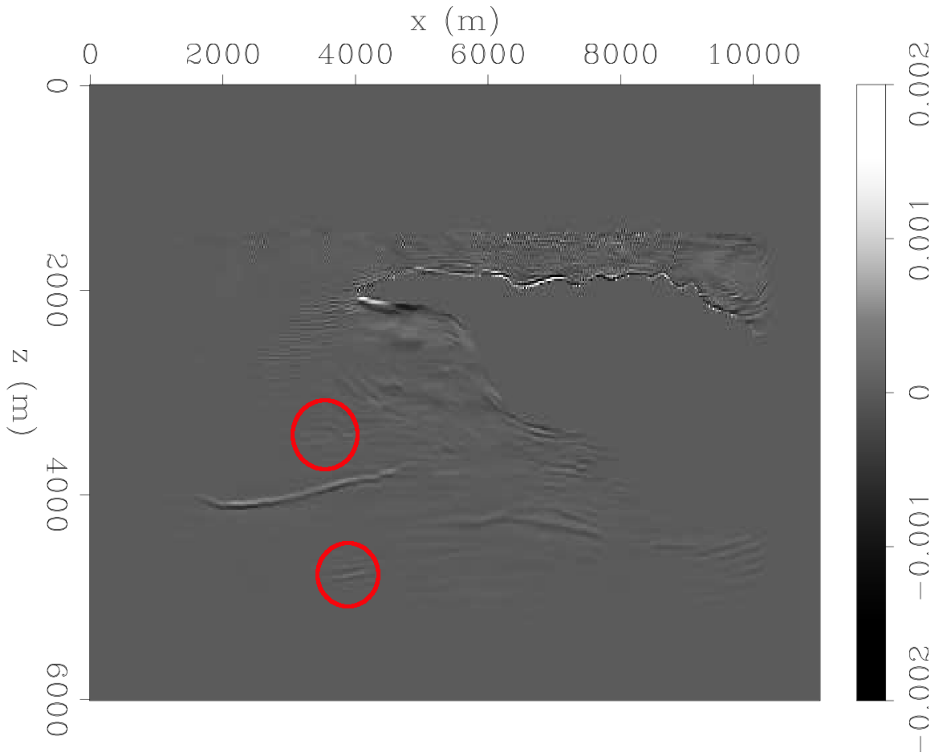


(a)

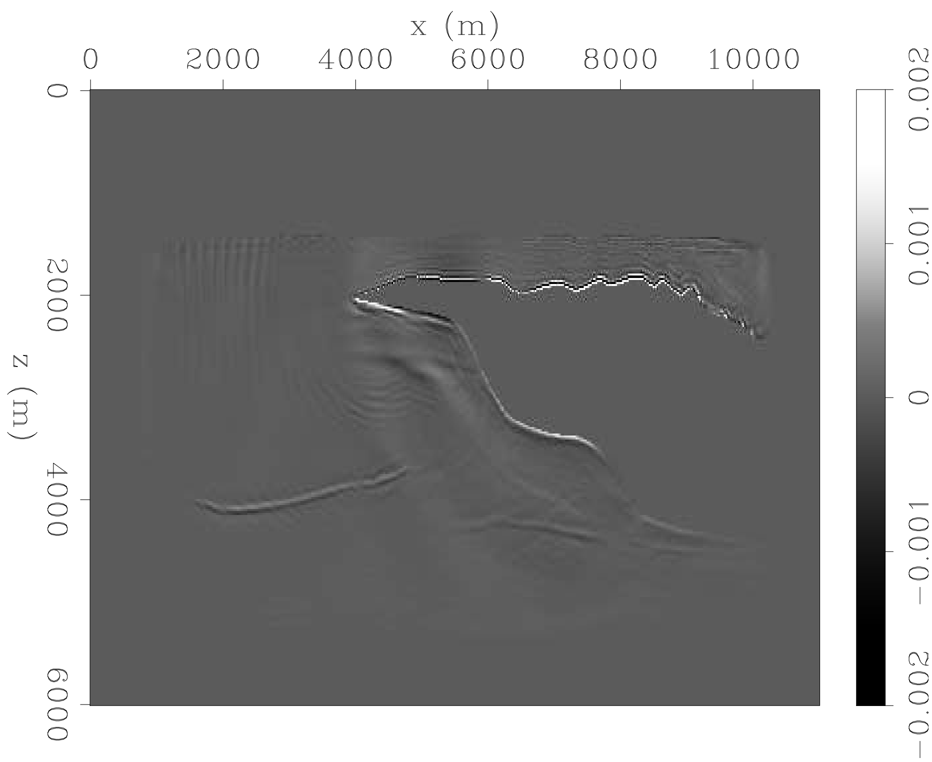


(b)

Figure 2: (a) Steering TV-regularized simultaneous linearized inversion (Ma et al., 2015b). (b) Steering TV-regularized double-difference method (9-12). In both cases the exact baseline velocity and reflectivity models were used. Both methods resolved reflectivity changes for the two reservoirs. [**CR**]



(a)



(b)

Figure 3: (a) Steering TV-regularized simultaneous linearized inversion using a 10% overestimated velocity model. (b) Steering TV-regularized double-difference method (9-12). The simultaneous inversion result is now contaminated with artifacts (e.g. marked with red circles) that are absent from the double-difference result. [**CR**]

to full-waveform inversion, especially in a hybrid approach involving simultaneous inversion of the baseline and monitor data-fitting terms (1,2), requires further analysis and will be subject of our future work.

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