Pseudo-acoustic modeling for tilted anisotropy with pseudo-source injection

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ABSTRACT

We provide a general framework for deriving fast finite-difference algorithms for the numerical modeling of acoustic wave propagation in anisotropic media. We specialize this framework to the case of tilted transversely isotropic media to implement a kinematically accurate fast finite-difference modeling method. This results in a significant reduction of the shear artifacts compared to similar kinematically accurate finite-difference methods.

INTRODUCTION

Transverse isotropy and orthorhombic media are of significant interest for industrial applications such as seismic imaging and inversion in complex, fractured rocks (Grechka, 2009). While full elastic data and models are needed to fully understand and invert for parameters in such media, the lesser task of imaging in the presence of such anisotropy can often be accomplished under a pseudo-acoustic assumption. In particular, the pseudo-acoustic method of Alkhalifah (1998) is the anisotropic counterpart of isotropic acoustic modeling. However, this and similar anisotropic finite-difference methods suffer from shear artifacts or rely on approximations that break down for strong anisotropy (Fowler et al., 2010; Zhan et al., 2012). We note that both references discuss transverse isotropy but similar challenges exist for finite-difference modeling in orthorhombic media.

In this work we develop a computationally efficient finite-difference wave propagation modeling method for tilted transversely isotropic (TTI) media that is largely free of shear artifacts. The concept extends the approach that Maharramov (2014, 2015) formulated for vertically transversal isotropic (VTI) media, but is not limited to polar anisotropy.

Our derivation of pseudo-acoustic (systems of) equations for a specific medium symmetry can be described as a three-step process:

1) Derive a phase velocity surface (Musgrave, 1970) as a function of the angle of propagation.

2) Derive a dispersion relation from 1) (Alkhalifah, 1998).

3) Interpret the dispersion relation as an evolutionary pseudo-differential equation, and transform it into a form suitable for numerical solution.

The cause of numerical artifacts is that the pressure and shear wave velocity surfaces remain coupled after deriving computationally feasible equations in step 3 (more specifically, the pressure mode and one of the shear modes remain coupled).

Our method can be summarized as follows:

2′) After step 1) above, extract the branch of the phase velocity surface corresponding to the pressure wave velocity.

3′) Approximate the resulting $V^2 = F(\mathbf{m}, \theta)$, where V is the pressure wave velocity, $\mathbf{m}$ stands for medium parameters, and θ is the propagation direction, with a computationally efficient numerical Fourier operator. This can be a trigonometric polynomial in θ (Iserles, 2008) with coefficients depending on $\mathbf{m}$, as practiced in some of the existing spectral pseudo-acoustic modeling methods (Etgen and Brandsberg-Dahl, 2009). We opt to use a pseudo-differential operator spatially constrained to a narrow depth range of sources and receivers.

4) Derive a coupled pseudo-pressure, pseudo-shear differential equation system analogous to Alkhalifah (2000).

5) At each time step apply the spatial component of the pseudo-differential operator derived in step 3′) to the injected source[1] using a spectral method with spatial interpolation. This results in a "pseudo-source".

6) Inject appropriate linear combinations of the pseudo-source and the source into the primary and secondary component of the system derived in 4).

When we assume a VTI anisotropy, and that the system described in step 4) is that of (Alkhalifah, 2000), step 6) reduces to injecting the pseudo-source into the secondary component and the true source into the primary component of the system derived in step 4).

THE PSEUDO-DIFFERENTIAL MODELING OPERATOR

In step 1) we start with the equation for $V(\theta)$ in a VTI medium (Tsvankin, 1996)

$$\begin{aligned} \frac{V^2(\theta)}{V_P^2} &= 1 + \epsilon \sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\sqrt{\left(1 + \frac{2\epsilon \sin^2\theta}{f}\right)^2 - \frac{2(\epsilon - \delta)\sin^2 2\theta}{f}}, \\ \text{with } f &= 1 - \frac{V_S^2}{V_P^2}, \text{ and} \\ \sin\theta &= \frac{V(\theta)\,[k_x]}{\left[\frac{\partial}{\partial t}\right]}, \quad \cos\theta = \frac{V(\theta)\,[k_z]}{\left[\frac{\partial}{\partial t}\right]} \end{aligned} \tag{1}$$

[1]This includes back-propagating receiver data in applications such as reverse time migration.

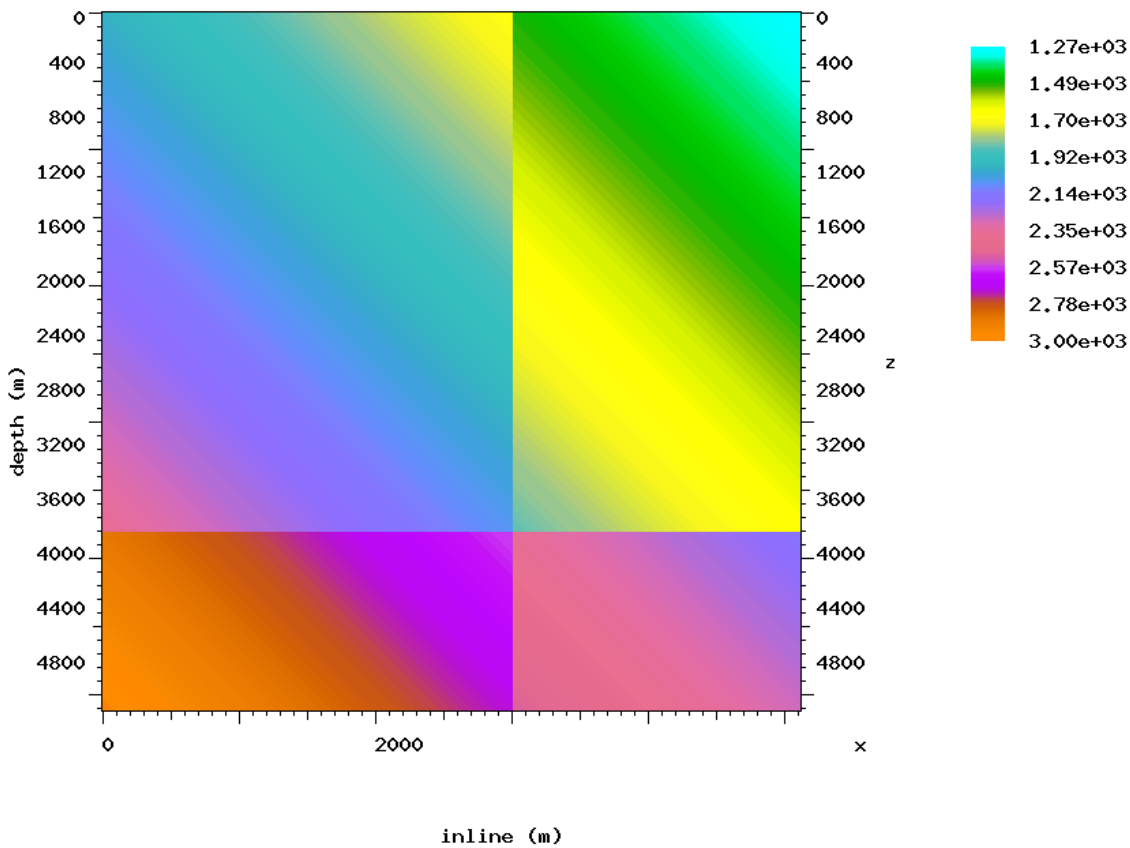

Figure 1: Test model with smooth and sharp V_P gradients and constant $\epsilon = 0.3$, $\delta = 0.1$, and tilt $\phi = 45°$. [**CR**]

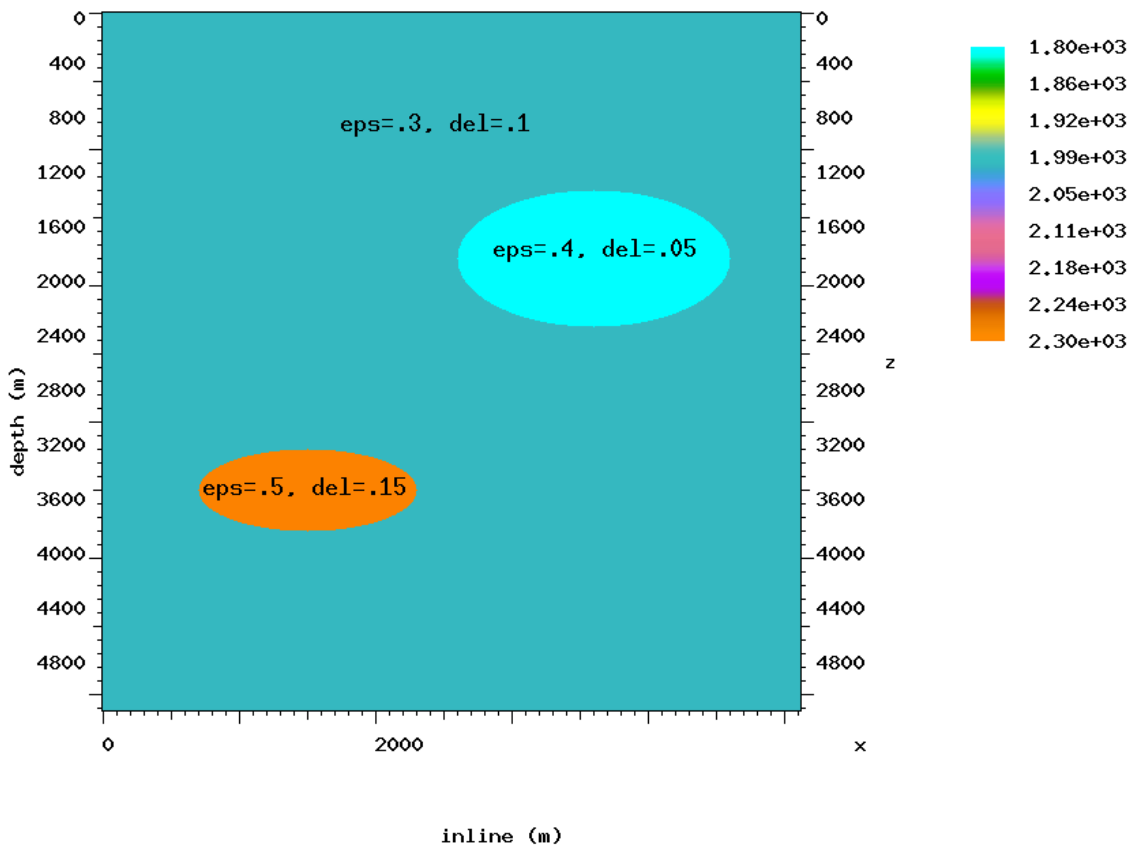

Figure 2: Test model with two anisotropic inclusions. The tilt angle is equal to 35° and 25° within the upper and lower inclusions, and is 30° in the background [**CR**]

where V_P and V_S are vertical pressure and shear wave velocities, θ is the propagation angle measured from the transverse isotropy symmetry axis, and ϵ and δ are the Thomsen parameters (Thomsen, 1986). We assume that $V_S = 0$, as we are not interested in propagating shear modes, thus $f = 1$. Note that here and in the subsequent analysis we consider two-dimensional TTI, however, the results naturally extend to three dimensions by identifying k_x with the radial wavenumber—see, e.g., Maharramov and Nolte (2011). We use the equivalence $k_u = -i\frac{\partial}{\partial u}$ in (1), where u is an arbitrary variable, to stress that the phase velocity equation can be interpreted as both a dispersion relation and a pseudo-differential operator. In step 2′), we extract the branch of the square root with the positive sign in (1), corresponding to the (higher) compressional wave velocity. The resulting dispersion relation can be interpreted as an evolutionary pseudo-differential operator

$$\begin{gathered}\frac{\partial^2}{\partial t^2} - V_P^2\left(\overset{2}{z},\overset{2}{x}\right)\frac{\Delta}{2} - \epsilon\left(\overset{2}{z},\overset{2}{x}\right)V_P^2\left(\overset{2}{z},\overset{2}{x}\right)\frac{\partial^2}{\partial x'^2} = \\ V_P^2\left(\overset{2}{z},\overset{2}{x}\right)\frac{\Delta}{2}\sqrt{\left[1+2\epsilon\left(\overset{2}{z},\overset{2}{z}\right)\frac{\partial^2}{\partial x'^2}\frac{1}{\Delta}\right]^2 - 8\left(\epsilon\left(\overset{2}{z},\overset{2}{x}\right)-\delta\left(\overset{2}{z},\overset{2}{x}\right)\right)\frac{\partial^2}{\partial x'^2}\frac{\partial^2}{\partial z'^2}\frac{1}{\Delta^2}}\ ,\end{gathered} \tag{2}$$

governing kinematically accurate propagation of the pressure wave, where x', z' are locally rotated coordinates with z' pointing along the tilted symmetry axis and x' pointing in the radial direction,

$$\Delta \;=\; \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial z'^2} \;=\; \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

is the Laplace operator (which is unaffected under rotation), and the "2" over x and z means that the multiplication by functions of spatial variables follows the application of differential operators in the pseudo-differential operator sense (Maslov, 1979). This is equivalent to "freezing" the operator coefficients, or assuming local homogeneity. Note that TTI in two dimensions requires one more parameter defined for each point of the subsurface: tilt angle ϕ from the vertical. This parameter implicitly enters (2) in the rotated coordinates x' and z'. Solving (2) for arbitrary heterogeneous media may be numerically challenging, because the Thomsen parameters $\epsilon(z, x)$ and $\delta(z, x)$ appear inside the square root of a pseudo-differential operator. However, operator (2) may simplify numerically if it is applied to a function with spatially bounded support – e.g., a source wavelet or receiver data. As noted earlier, an alternative to solving the full pseudo-differential operator equation (2) is to approximate, in step 3′), the extracted pressure velocity branch with a trigonometric polynomial:

$$V^2(\theta) \;\approx\; V_P^2\sum_{n=0}^{N} a_n \sin^{2n}(\theta), \tag{3}$$

where the coefficients $a_n,\ n = 0, \ldots, N$ depend on medium parameters. From the last line of (1) we can see that velocity surface (3) translates into the following pseudo-differential operator equation

$$\frac{\partial^2}{\partial t^2} \;=\; V_P^2\sum_{n=0}^{N} a_n \frac{\partial^{2n}}{\partial x'^{2n}}\Delta^{1-n}. \tag{4}$$

Equation (4) can be solved by applying the operators

$$\frac{\partial^{2n}}{\partial x'^{2n}}\Delta^{1-n}$$

to the wave field in the spatial Fourier domain, then summing up the results with spatially-dependent coefficients a_n in the spatial domain. Important particular cases of approximation (3) are the weak anisotropy approximation (Grechka, 2009)

$$V^2(\theta) \;\approx\; V_P^2\left(1+\delta\sin^2\theta+\frac{\epsilon-\delta}{1+2\delta}\sin^4\theta\right), \tag{5}$$

and the VTI approximation due to Harlan and Lazear (Harlan, 1998) used by Etgen and Brandsberg-Dahl (2009)

$$V^2(\theta) \;=\; V_P^2\cos^2\theta+\left(V_{P\mathrm{NMO}}^2-V_{P\mathrm{Hor}}^2\right)\cos^2\theta\sin^2\theta+V_{P\mathrm{Hor}}^2\sin^2\theta, \tag{6}$$

where the subscripts PHor and PNMO denote the horizontal and NMO compressional wave velocities, respectively. Note that both (5) and (6) correspond to $N=2$ in (3) and are suitable for weakly anisotropic VTI but break down in strong anisotropy. The case of $N=3$ requires one additional inverse FFT for VTI but is accurate for a wide range of Thomsen parameters within (and beyond) practical requirements. Adapting (3) for TTI media would require the application at each time step of 5 additional inverse FFTs for $N=2$ and extra 16 inverse FFTs for $N=3$.

Solving (4) for $N=2,3$ using the described spectral method is an efficient modeling method in its own right, especially for VTI media where the number of FFTs at each time step is very low. However, in the next section we describe a finite-difference method that can outperform the spectral method for complex media and conceptually generalizes for other kinds of anisotropy.

THE FINITE-DIFFERENCE METHOD

In step 4) we square the pseudo-differential operator equation (2) so as to get rid of the square root, and obtain the following system of coupled second-order partial differential equations (Alkhalifah, 2000):

$$\begin{aligned}\frac{\partial^2 q}{\partial t^2} &= V_{P\mathrm{Hor}}^2\frac{\partial^2 q}{\partial x'^2}+V_P^2\frac{\partial^2 q}{\partial z'^2}-V_P^2\left(V_{P\mathrm{Hor}}^2-V_{P\mathrm{NMO}}^2\right)\frac{\partial^4 r}{\partial x'^2\partial z'^2},\\ \frac{\partial^2 r}{\partial t^2} &= q,\end{aligned} \tag{7}$$

where $r(z,x,t)$ and $q(z,x,t)$ are the pressure field and its second temporal derivative, and

$$V_{P\mathrm{Hor}}(z,x) \;=\; V_P(z,x)\sqrt{1+2\epsilon(z,x)},\; V_{P\mathrm{NMO}}(z,x) \;=\; V_P(z,x)\sqrt{1+2\delta(z,x)}.$$

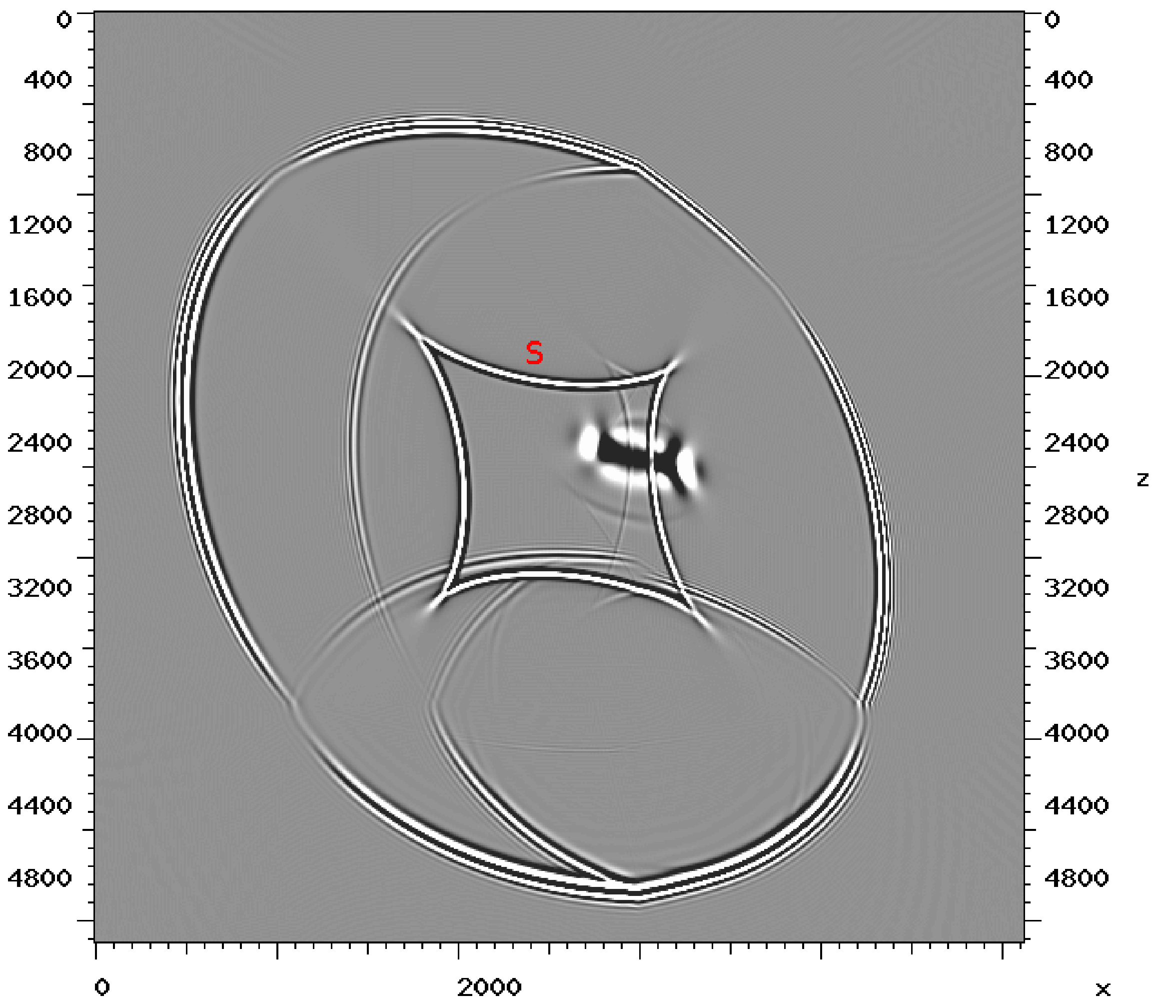

Figure 3: Shear artifact (marked with the “S”) in the solution of (7) for the model of Figure 1 with sources injected in component r. Note that the shear artifact is causing numerical instability. [**CR**]

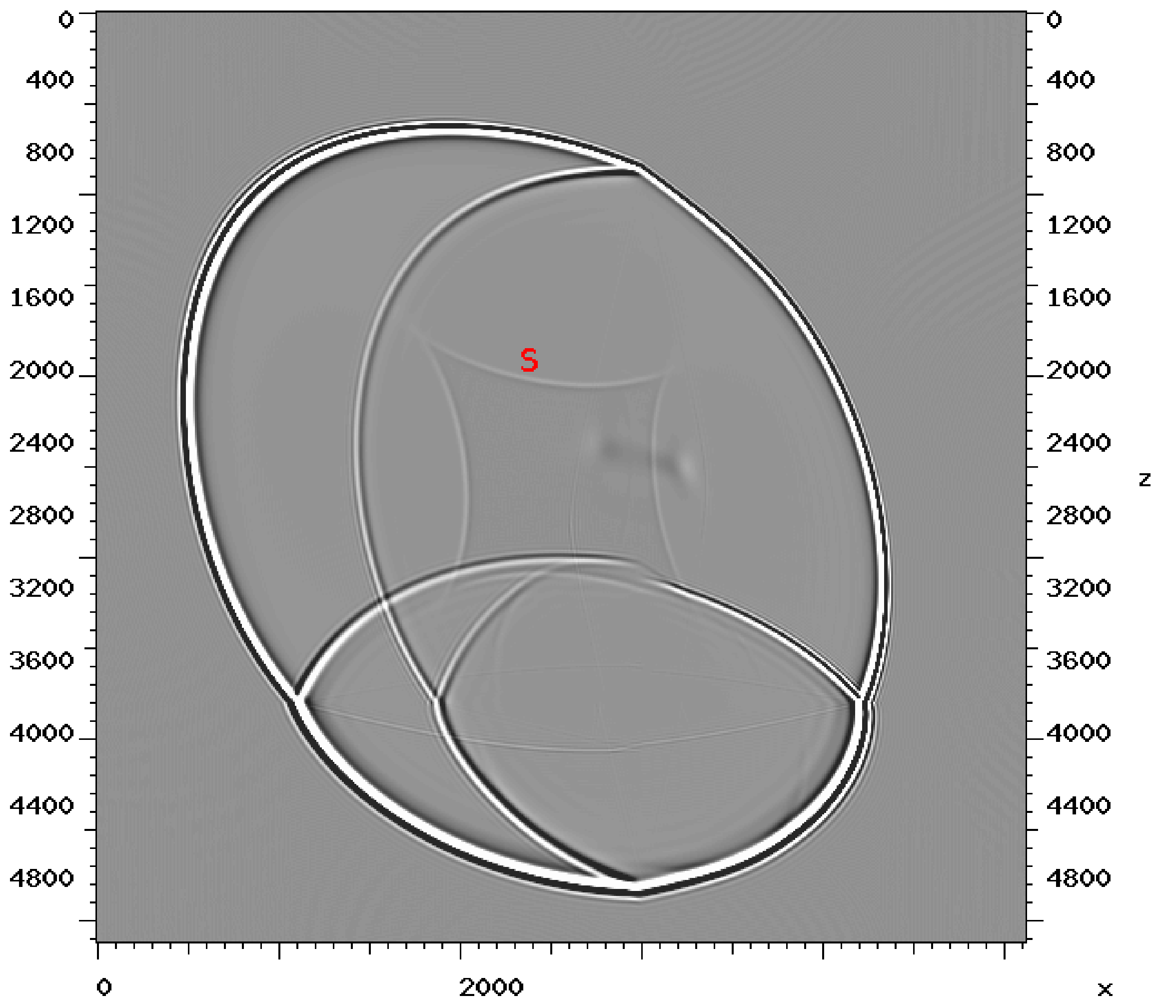

Figure 4: Shear artifact (marked with the "S") in the solution of (7) with sources injected in component q. Although significantly reduced, the shear artifact is still sufficiently strong to cause imaging cross-talk. [**CR**]

Since the resulting system now includes the branch with the negative square root in (1), solution of this system may suffer from shear artifacts as shown in Figure 3. The artifacts can be reduced by injecting sources into the second component q (Fowler et al., 2010); however, they are still present—see Figure 4. However, the pseudo-differential operator equation (2) can be used to reduce the unwanted artifacts (appearing as the "diamond"-shaped inverted wavefront in the figure). Equation (1) and the corresponding pseudo-differential equation do not describe any pressure to shear conversion but rather govern the independent propagation of the pressure and shear waves. The same is true of the "coupled" system of differential equations. Consequently, any shear artifacts that appear in a solution to the coupled system of differential equations we attribute to the pseudo-shear modes present in the wave field. We can use the fact that the system of two coupled equations requires injecting two sources, to manufacture a pseudo-source to be injected into one of the components so as to suppress the shear modes. More specifically, if $\phi(z,x,t)$ is a time-dependent source function, then at each time step component r is injected with ϕ, and component q is injected with the result of applying the spatial part of the pseudo-differential operator (2) to $\phi(z,x,t)$:

$$\begin{aligned}
r(z,x,t_n) &= r(z,x,t_n) + \phi(z,x,t_n),\\
q(z,x,t_n) &= q(z,x,t_n) + V_P^2 \left\{ \left(\overset{2}{z},\overset{2}{x}\right)\frac{\Delta}{2} + \epsilon\left(\overset{2}{z},\overset{2}{x}\right)\frac{\partial^2}{\partial x'^2} + \right.\\
&\left. +\frac{\Delta}{2}\sqrt{\left[1+2\epsilon\left(\overset{2}{z},\overset{2}{z}\right)\frac{\partial^2}{\partial x'^2}\frac{1}{\Delta}\right]^2 - 8\left(\epsilon\left(\overset{2}{z},\overset{2}{x}\right)-\delta\left(\overset{2}{z},\overset{2}{x}\right)\right)\frac{\partial^2}{\partial x'^2}\frac{\partial^2}{\partial z'^2}\frac{1}{\Delta^2}}\right\}\phi,
\end{aligned} \tag{8}$$

followed by a finite-difference time propagation step of system (7). This procedure ensures that the two-component source in the right-hand side of (8) satisfies equation (2). Since solutions of (2) are shear-free, the injected sources will not give rise to shear modes because the solution of (7) is effectively projected on to the space of solutions of (2).

NUMERICAL EXAMPLES

Figure 6 shows the result of applying the pseudo-source finite-difference method to the propagation in a heterogeneous VTI medium described by the model of Figure 1, with a Ricker source. The corresponding result obtained by solving the full pseudo-differential operator equation (2) is shown in Figure 5. Note the significant reduction of the shear artifacts and that although we use the full pseudo-differential operator for generating the pseudo-source in (8), the fact that the source is localized makes this computationally efficient, obviating the need for approximations like (4).

The model of Figure 1, while featuring both sharp and smooth vertical velocity variation, assumes constant $\epsilon = 0.3$, $\delta = 0.1$, and tilt of 45°. While adding the pseudo-source (8) ensures that the solution of the coupled system (7) stays within

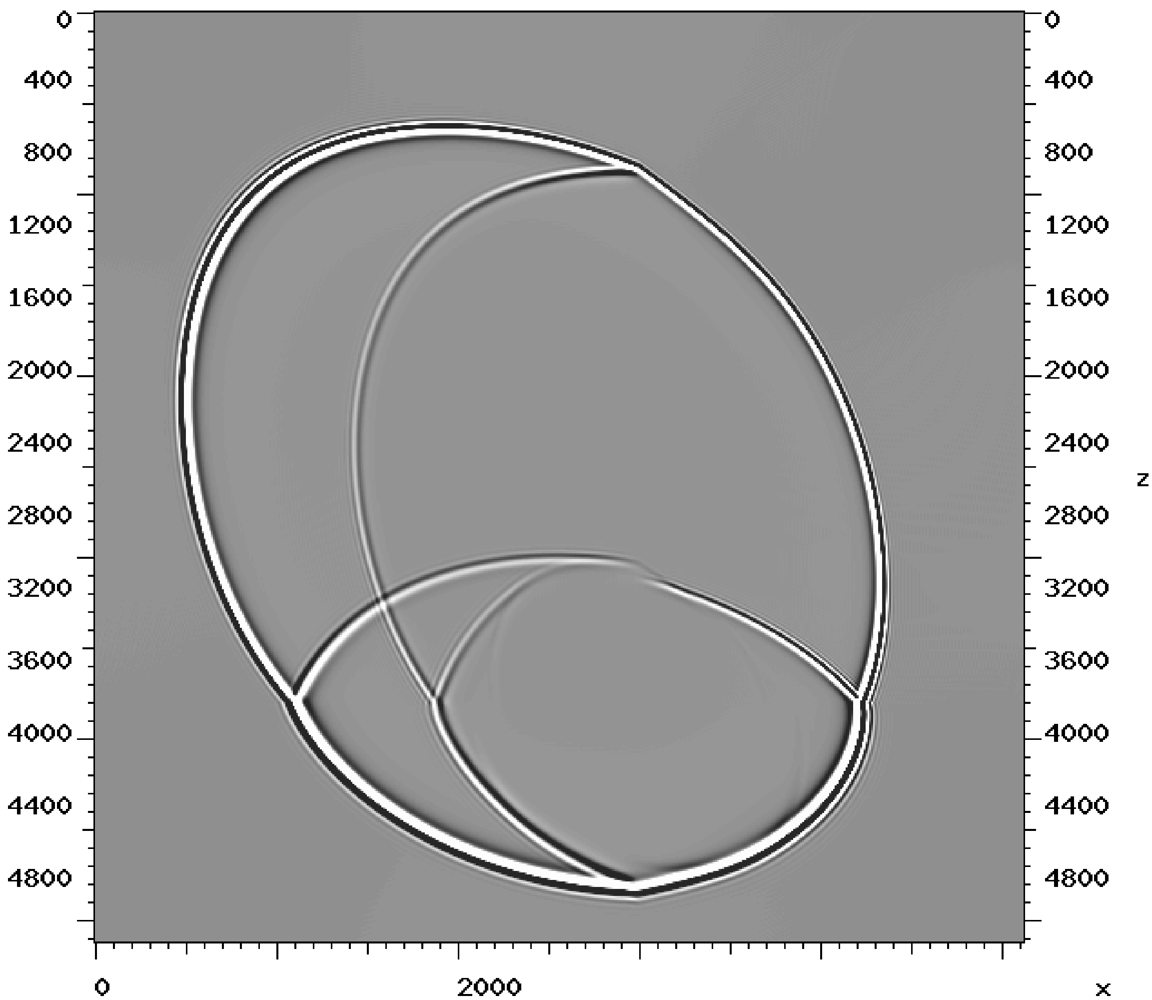

Figure 5: Solution of the full pseudo-differential operator equation (2) for the model of Figure 1. Note the good agreement with the result of finite-difference modelling using shear-reducing pseudo-sources shown in Figure 6. [**CR**]

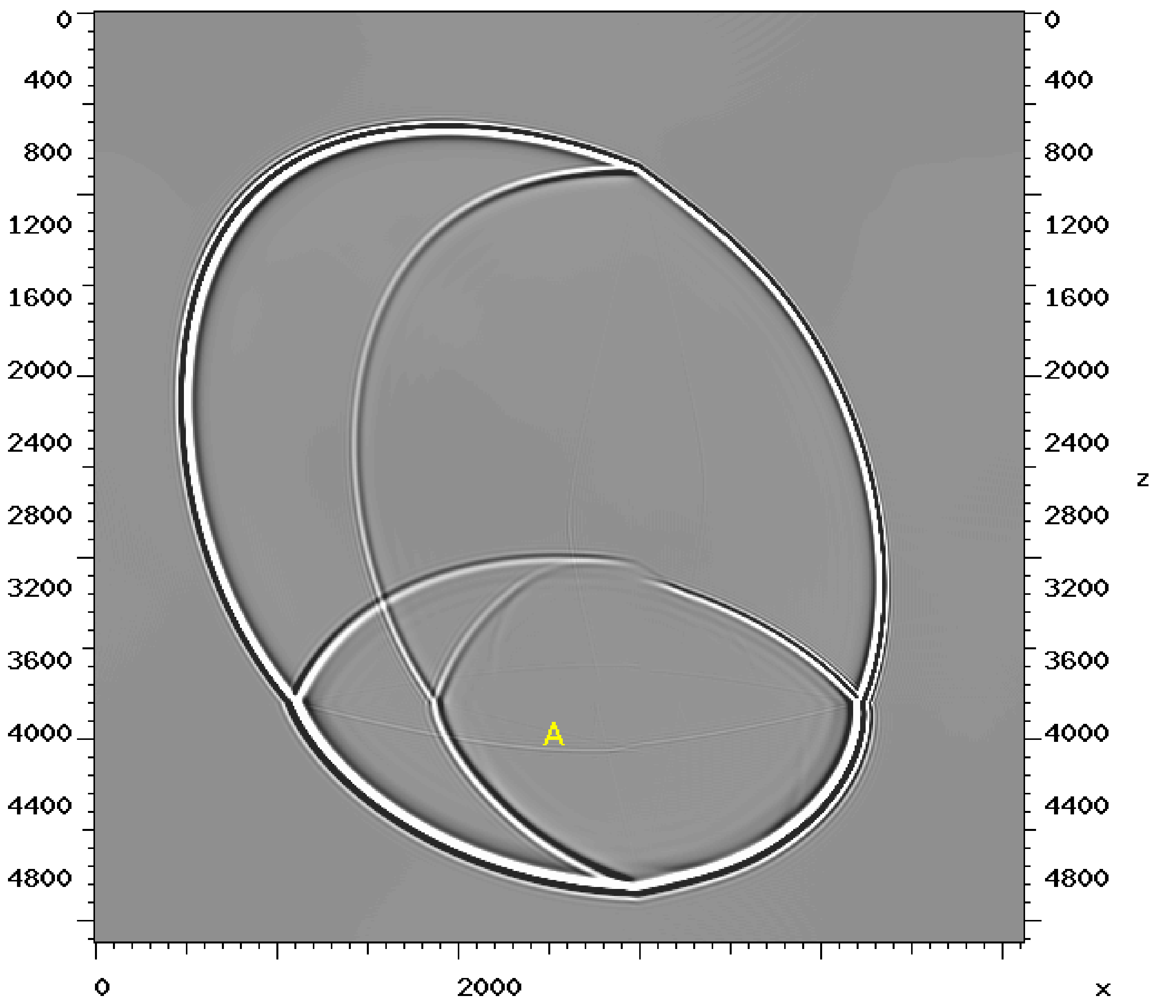

Figure 6: Solution of (7) for the model of Figure 1 with shear-reducing pseudo-sources is in kinematic agreement with Figure 5. A high-wavenumber computational artifact (marked with the "A") is caused by sharp model contrasts. [**CR**]

the space of solutions of (2) in the continuous limit $\Delta t \to 0$, sharp contrasts in medium parameters may introduce numerical approximation errors that may contain a non-negligible shear component. Indeed, applying the method to the model of Figure 2, featuring two inclusions with significantly different Thomsen parameters, we can see weak artifacts (single lines) within the inclusion detail in Figure 10 for the finite-difference method that are absent from the result in Figure 9 obtained by solving the full pseudo-differential operator (2). Figure 11 shows the result of using the finite-difference method with pseudo-sources after smoothing the vertical velocity but keeping ϵ, δ and tilt contrasts unchanged. The result shows that the artifacts within the inclusions were almost completely removed.

CONCLUSIONS AND PERSPECTIVES

The proposed pseudo-source finite-difference method allows us to take advantage of computationally efficient finite-difference solvers for the traditional pseudo-acoustic (fourth-order) systems while achieving a significant reduction in shear artifacts. The method is kinematically accurate for VTI media, and can be extended in principle to other kinds of anisotropy. While this implementation is based on using the coupled system (7) of Alkhalifah (2000), the method can be adapted to use equivalent systems such as that of Fowler et al. (2010). In that case the two-component source becomes a linear combination of the true source and the pseudo-source terms, with the coefficients of the linear combination determined by the relationship between the solution of the equivalent system and that of system (7).

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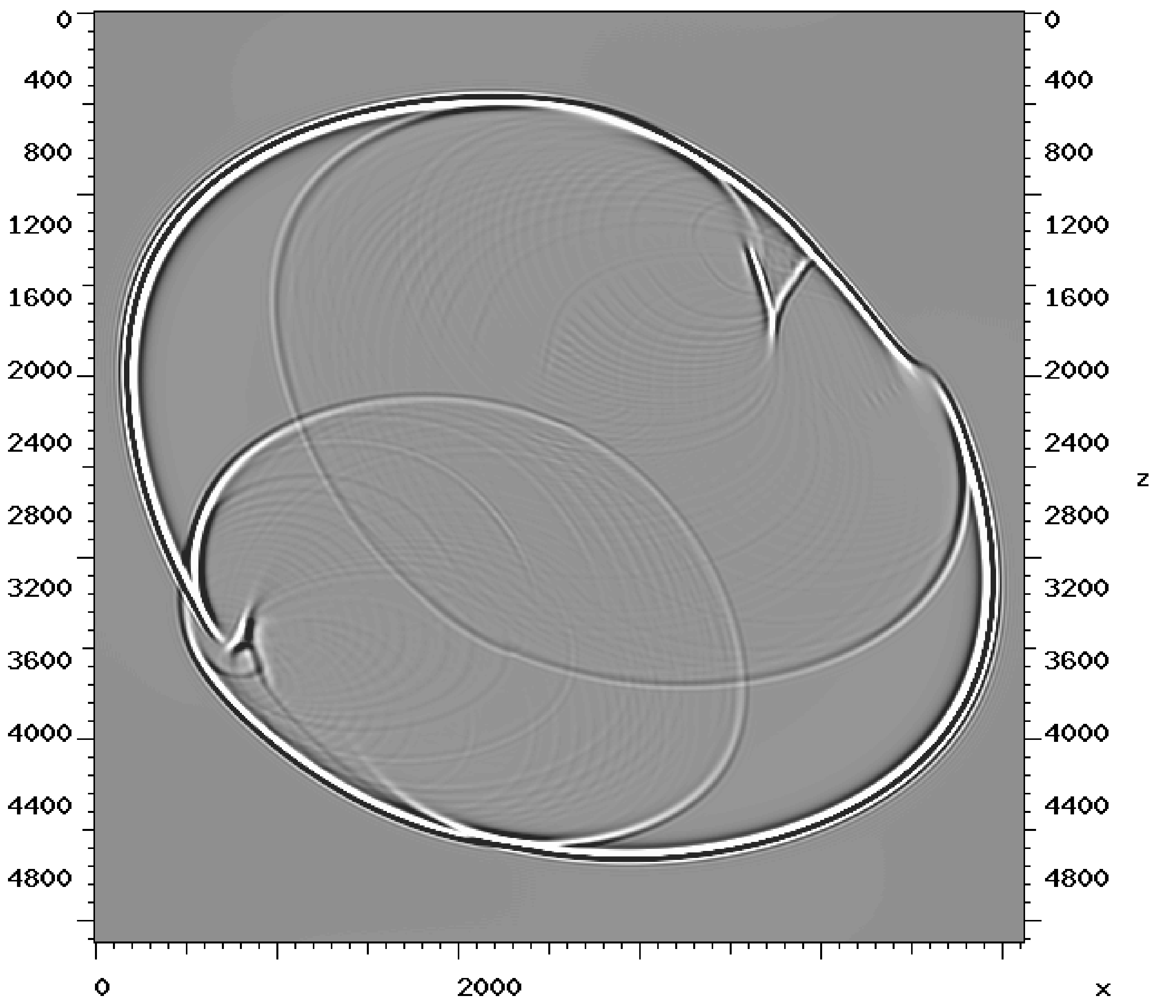

Figure 7: Solution of the full pseudo-differential operator equation (2) for the model of Figure 2. [**CR**]

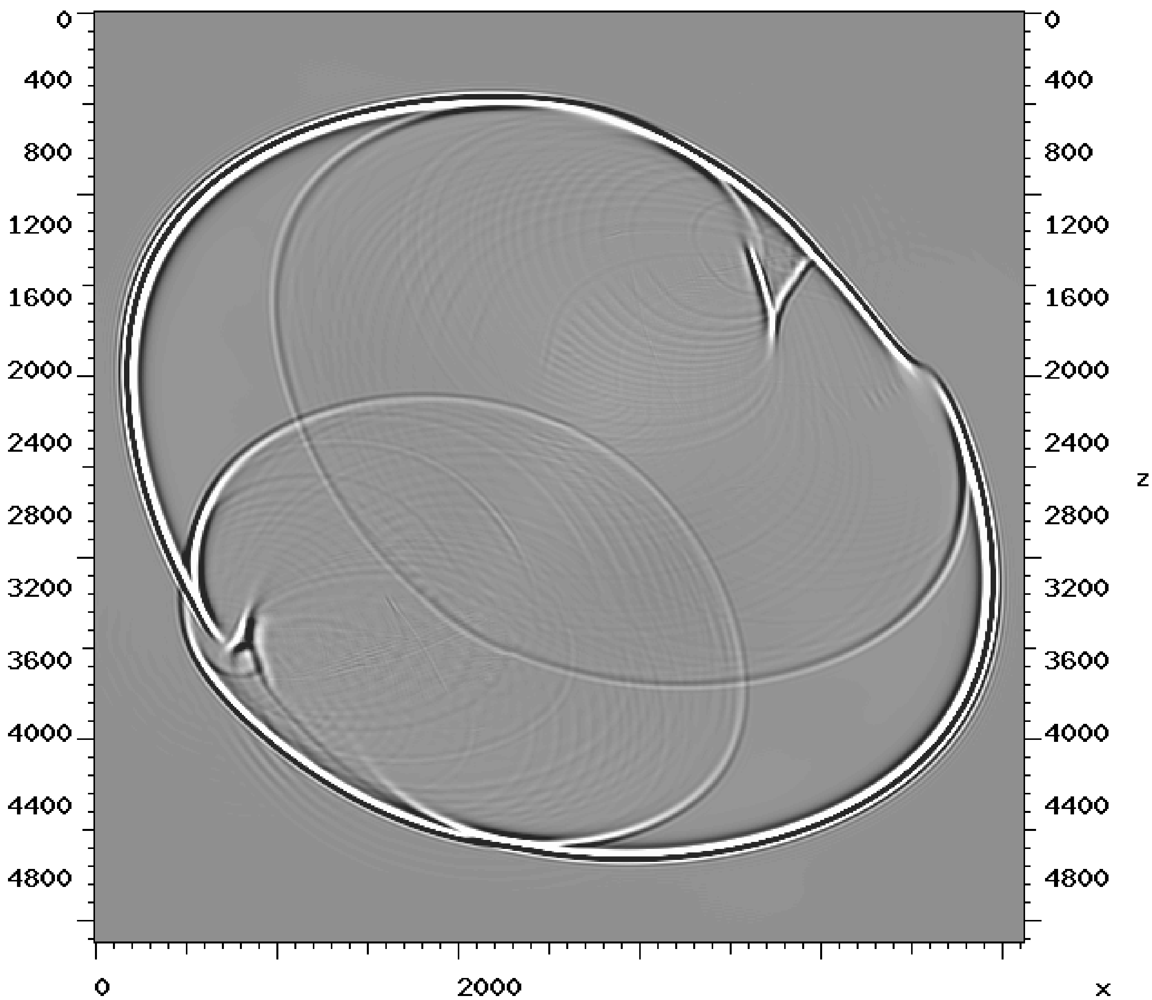

Figure 8: Solution of (7) for the model of Figure 2 with shear-reducing pseudo-sources. Note the good agreement with Figure 7. [**CR**]

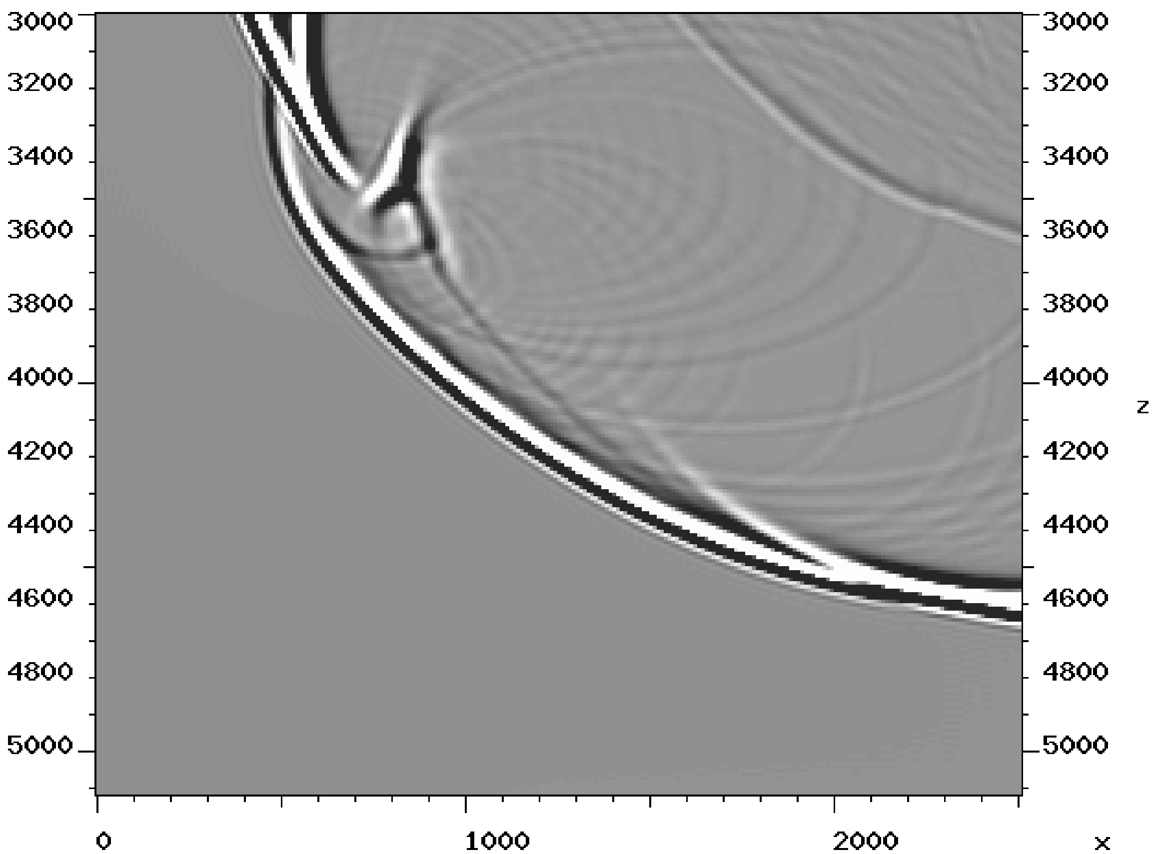

Figure 9: Artifact-free solution of the full pseudo-differential operator equation (2) showing multiple reflections within the lower inclusion of the model in Figure 2. [**CR**]

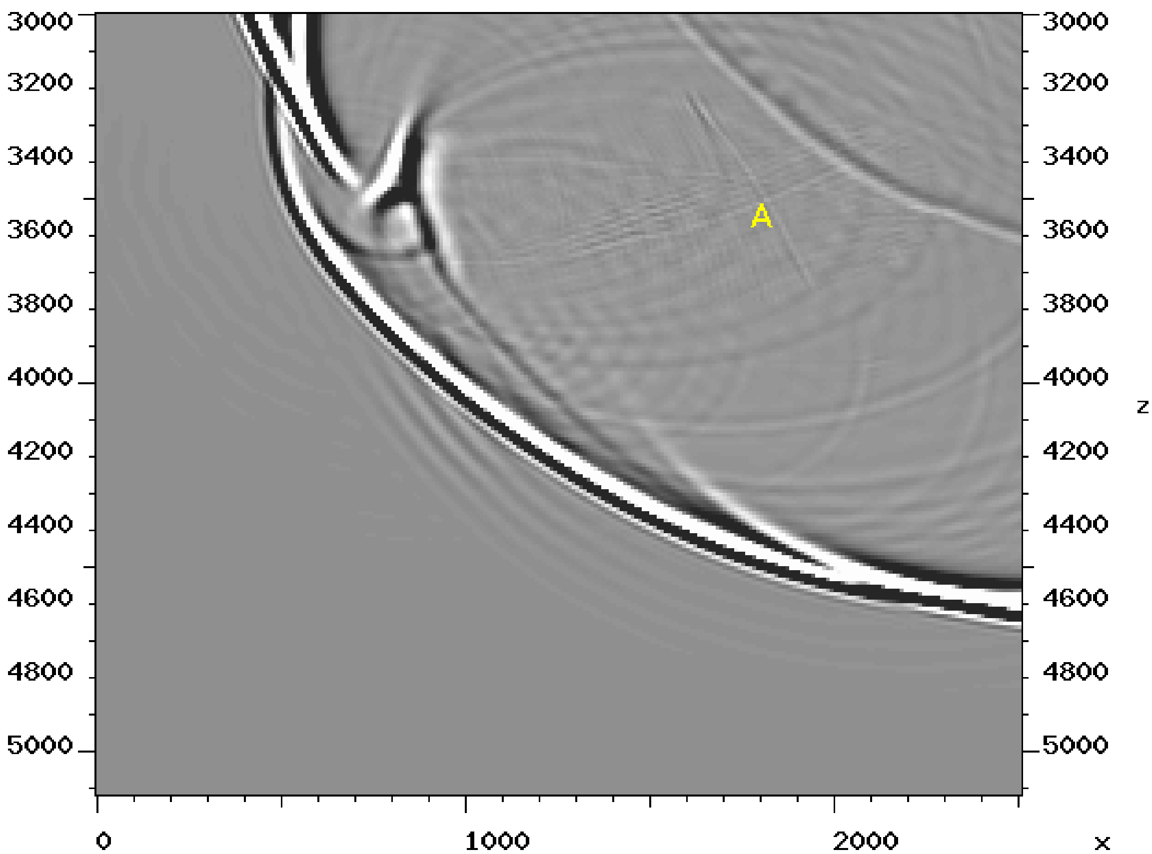

Figure 10: Solution of (7) for the model of Figure 2 with shear-reducing pseudo-sources. A sharp velocity contrast causes weak artifacts ("A"). [**CR**]

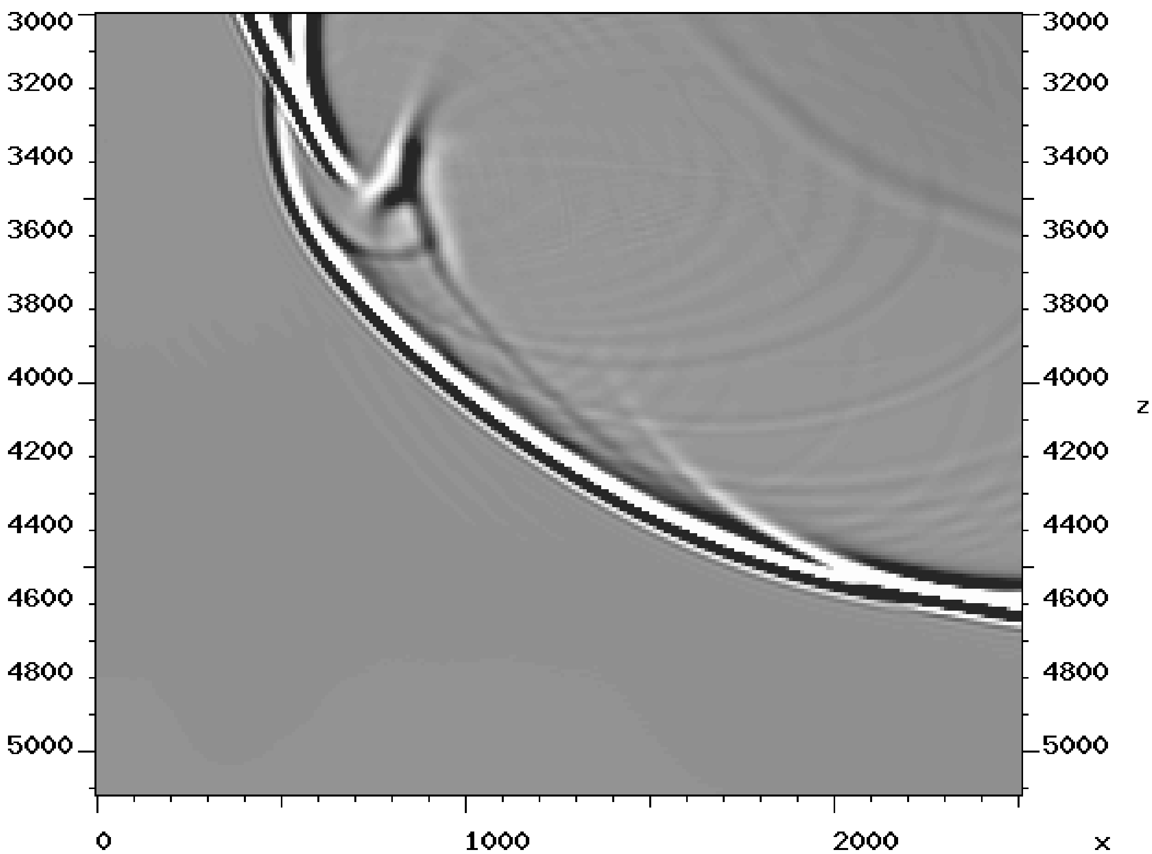

Figure 11: Moderate smoothing of the velocity contrasts remove the high-wavenumber artifacts. [**CR**]