IMAGING WITH MULTIPLES BY LEAST-SQUARES REVERSE TIME MIGRATION

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Abstract

This dissertation presents a novel technique for using surface-related multiples to improve imaging in geologically complex areas. It overcomes a challenge of the migration-based approach where crosstalk artifacts appear in the image. In the case of reverse-time migration, these imaging artifacts are the result of cross-correlation between wrong pairs of incident and scattered wavefields. Joint least-squares reverse-time migration, also known as linearized inversion, can coherently focus the reflection energy of primary and surface-related multiples into one image. By posing the imaging problem as an inversion problem, spurious reflectors or noises in the image can be attenuated. In geologically complex areas that contain salt structures, the proposed method not only improves the imaging but also added additional angular coverage in poorly illuminated areas. With respect to improving subsurface illumination, it is particularly advantageous to apply this method to ocean-bottom node acquisition.

A modified modeling operator in the inversion process was introduced to model the surface-related multiples in the data. This modified modeling operator uses the data as an areal source, which removes the need to estimate the source wavelet. By using the data as a source, only the last down-going and up-going legs of the wavepath have dependency on the migration velocity model. As a result, this operator has the same sensitivity to the migration velocity model as the conventional primary modeling operator. From an imaging prospective, the robustness of the technique is improved when there are deviations between the migration velocity and the true velocity. The surface-related multiple operator can be combined with the conventional primary operator to form a joint operator that properly accounts for the physics of
both primary and surface-related multiple reflections.

Several methods are introduced to improve the convergence of least-squares reverse-time migration, particularly for areas with a complex salt structure. To emphasize the shadow zones in the image, a target-oriented data-reweighting scheme is incorporated in the inversion process. To extract the most information from the least-squares reverse-time migration algorithm, salt dimming data-weighting was introduced to down-weight the reflection energy coming from strong velocity contrasts in the migration velocity model. Such energy often dominates the inversion. When least-squares migration is extended to the angle domain, prestack extended-angle domain filtering can be incorporated into the modeling operator to remove unwanted noise in the image.

Applications to synthetic and field ocean-bottom node datasets show that, compared to migration or single-mode inversion, joint least-squares reverse-time migration provides the best overall subsurface image and angular coverage. In particular, areas near and underneath a complex salt structure are better illuminated when surface-related multiples are used as signal.
Preface

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Our testing is currently limited to LINUX 2.6 (using the Intel Fortran90 compiler) and the SEPlib-6.4.6 distribution, but the code should be portable to other architectures. Reader’s suggestions are welcome. For more information on reproducing SEP’s electronic documents, please visit [http://sepwww.stanford.edu/research/redoc/](http://sepwww.stanford.edu/research/redoc/).
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Chapter 1

Introduction

Traditionally, seismic imaging techniques have only accounted for primary reflections. When strong reflectors are present (e.g., air-water interface, hard water bottom or salt bodies), multiples can significantly degrade the images and make their interpretation ambiguous. Multiples have been considered as noise, therefore, much effort in the past few decades has been devoted to developing multiple suppression techniques.

Well-known demultiple tools such as deconvolution (in time, frequency, and slant-stack domains), Radon-transform demultiple, and frequency-wavenumber (f-k) de-multiple are limited to cases where the subsurface geology is simple. When the geology is complex multiples are not easily separated from primaries by criteria such as periodicity, moveout velocity, and spectra. Model-based techniques predict multiples with wavefield extrapolation (Morley, 1982; Berryhill and Kim, 1986; Wiggins, 1988; Lu et al., 1999). The accuracy of the predicted multiples strongly depends on the model used. Convolution-based techniques, such as surface-related multiple elimination (SRME) (Riley and Claerbout, 1976; Tsai, 1985; Verschuur et al., 1992), are in principle capable of suppressing multiples in complex geology. However, these methods require an overlap of source and receiver locations that is not realistic for many practical acquisition geometries and are limited to surface-related multiple suppression. Despite substantial progress in multiple elimination, complete removal of
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surface-related and internal multiples without distorting primary signals remains a challenge.

Motivations

The need to image areas with increasingly complex subsurface geology has driven the advancement of seismic acquisition geometry. Substantial improvement in seismic imaging is observed when the acquisition geometry goes from 2D to 3D, or from narrow azimuth to wide azimuth. Recently, coil shooting and ocean bottom node (OBN) acquisition provide rich- and full-azimuthal coverage.

Multiples can be treated as signal. One motivation to image with multiples is that they can supplement subsurface illumination that is not found in primary signals. Figure 1.1 illustrates the increase in illumination when surface-related multiples are used as signal. A figure extracted from Lu et al. (2013) compares two time slices at 120 ms two-way traveltime (TWT). The time slices were produced by a marine survey conducted in an area with a water depth of 70m. The left image shows a pronounced cross-line acquisition footprint. Using surface-related multiples yields an image (Figure 1.1 right-panel) that allows shallow geohazard interpretation.

I will discuss the benefit of imaging with higher-order surface-related multiples in terms of source and receiver arrangement. In ocean bottom node imaging, processing are sorted by common receiver gathers. Computationally, we treat each OBN receiver as a source and the physical sources as receivers. The first-order down-going reflection is also known as the mirror reflection. When we try to image a mirror reflection (Figure 1.2a), the sources are injected at location 1 and recorded at location 2. However, on top of the mirror reflection, we are also recording higher-order surface-related multiple reflections. The energy from the first-order reflection will then be reflected off the sea-surface and travel downward through the water column a second time (Figure 1.2b). In this sense, the higher-order reflection effectively has energy originating from the sea-surface (location 2) and is recorded again at the sea-surface (location 2). The physical sources (location 2) are well populated at the sea
Figure 1.1: This is a comparison between primary-only (left) and surface-related multiples (right) imaging. The comparison is of two time slices at 120 ms TWT. There is an increase in illumination when surface-related multiples are used. This figure is extracted from Lu et al. (2013).

Figure 1.2: (a) Computationally, the mirror reflection has a source at the sea-bottom (location 1) and receiver at the sea-surface (location 2). (b) A surface-related multiple reflection effectively has energy originating from the sea-surface (location 2) and is recorded again at the sea-surface (location 2). The surface-related multiple reflection has a much denser source and receiver grid.
surface. Using the surface-related multiples can provide information equivalent to a well-populated grid of sources and receivers. While the illustrations only highlight the differences in 2D, the benefit in 3D is greater with better azimuthal coverage.

We anticipate several types of illumination when including multiple events. In general, we expect to see a wider subsurface illumination. This is more apparent in shallower regions than in deeper regions. Figure 1.2 illustrates that using surface-related multiples simulates a well-populated source and receiver grid. Using the multiples will not provide much improvement when the subsurface is relatively simple. However, multiples can be useful in subsalt imaging. Due to the rugosity of the salt, energy that enters the salt is often bent at different angles. Frequently, that energy does not return to the receivers. Figure 1.3 shows a ray tracing plot of the Sigsbee model with rays shooting from the location within the shadow zone of the subsalt region. Rays at various incident angles are often bent and fail to return to the surface. Treating the multiples as signals provide more opportunities to record energy that has reached the poorly illuminated areas. In chapter 4, we will discuss more about the illuminate gain that multiples can provide.

Figure 1.3: Ray tracing of the Sigsbee velocity model with rays shooting out from location within the shadow zones. (a) rays shooting from the negative x-axis direction up to the vertical direction at 10-degree intervals; (b) rays shooting from the vertical direction to the positive horizontal axis at 10-degree intervals. The complexity of the salt has bend the rays at various angles. Only a select few returns to the surface.
Despite the advancement of multiple suppression techniques, complete removal of all multiples from the primaries still remains a challenge. Migrating such signals would result in crosstalk artifacts. Figure 1.4 and Figure 1.5 show a synthetic example when surface-related multiples are not completely removed from the data. A simplified version of the Sigsbee model is used to generate two sets of synthetic data. Figure 1.4a shows a shot gather with both primary- and surface-related multiple energy. Each order of surface-related multiple reflections arrives roughly one second apart. To simulate the situation of incomplete removal of all multiples, I also generated data with 20 percent of the surface-related multiple energy (Figure 1.4b). When migrating the data without multiple removal, multiples become noise in the migration image as highlighted by the circle in Figure 1.5a. Even with 80 percent of the surface-related multiple energy removed from the data, some unwanted noise remains in the migration image. In particular, the arrows in Figure 1.5 point to a bright spot region that is degraded by the noise. In this thesis, I explicitly tackle the problem of imperfect separation and aim to derive a solution that can remove such crosstalk artifacts in imaging. Instead of using migration, the imaging problem can be reformulated as an inversion problem. The basic idea is to estimate a seismic image by enforcing the consistency between the modeled data and the observed data. This consistency requirement can gradually push out unwanted noise such as crosstalk artifacts from the image. Such an inversion technique, commonly known as least-squares migration, will be discussed in the next chapter.

**History of multiple imaging**

Reiter et al. (1991) attempted to capitalize on the potential of multiples by formulating a prestack Kirchhoff time-migration method that includes the first-order water-layer reverberation in the migration operator. Because ocean bottom cable data could not be decomposed into up- and down-going components at the time, such work was limited to deep water datasets.
Figure 1.4: A shot gather displaying (a) both primary and multiple energy, and (b) primary data with 20 percent of surface-related multiple energy remaining to simulate incomplete multiple suppression.
Figure 1.5: Migrated image from the synthetic data in Figure 1.4 (a) using both primary and multiple signal and (b) using a primary signal with 20 percent of the surface-related multiple energy. The true image is displayed in (c). The crosstalk noise is highlighted by the circle. The arrow indicates a bright spot region that is being degraded by the noise. Even with 80 percent of the multiple removed, the image is compromised. [CR] [chap1/. chap1Zoom]
When surface-related multiples are explicitly separated from the primary reflections (e.g., using SRME), they can be imaged independently from the primary reflections by using shot-profile (Guitton, 2002) or source-receiver (Shan, 2003) depth migration. Muijs et al. (2007a) imaged primary and free-surface multiples for OBS data by decomposing data into up-going and down-going constituents followed by downward extrapolation and a 2D deconvolution-based imaging condition. All of these techniques image the surface-related multiples by transforming the primary signal into a pseudo-source for migration with the multiple signals using the one-way wave equation (Artman, 2007). Recently, Liu et al. (2011) extended the technique to the two-way wave equation. These methods image a certain mode of multiples by treating all other modes, including the primaries, as noise. While these techniques are computationally efficient, current migration algorithms would leave crosstalk artifacts in the image. Migration algorithms such as reverse-time migration use the cross-correlation imaging condition to produce an image $I(x)$.

$$I(x) = \sum_{x_s, t} S(x, t; x_s) R(x, t; x_s), \quad (1.1)$$

where $S(x, t; x_s)$ is the source wavefield and $R(x, t; x_s)$ is the receiver wavefield. When multiples are used in the imaging condition, the source and receiver wavefields can be broken down into different order of reflections.

$$S(x, t; x_s) = S_1(x, t; x_s) + S_2(x, t; x_s) + S_3(x, t; x_s) + ..., \quad (1.2)$$

$$R(x, t; x_s) = R_1(x, t; x_s) + R_2(x, t; x_s) + R_3(x, t; x_s) + ..., \quad (1.3)$$

where the subscript number refers to an order number that scales with an additional pass through the water-column. When cross-correlation imaging condition is applied, the final image can be classified into two parts: the signal image and the noise image. The signal image, $I_{\text{signal}}(x)$, is from the interference of wavefields corresponded to the same order of reflections. The noise image, $I_{\text{noise}}(x)$, are caused by the interference
of wavefields not associated with the same order of reflections.

\[
I(x) = I_{\text{signal}}(x) + I_{\text{noise}}(x), \tag{1.4}
\]

\[
I_{\text{signal}}(x) = \sum_{x_s, t}(S_1R_1 + S_2R_2 + S_3R_3 + S_4R_4 + \ldots), \tag{1.5}
\]

\[
I_{\text{noise}}(x) = \sum_{x_s, t}(S_2R_2 + S_1R_1 + S_3R_3 + S_4R_4 + \ldots), \tag{1.6}
\]

A robust technique is needed to gain the benefit of multiple imaging without compromising its quality.

Brown (2004) approached the multiple imaging problem with linearized inversions. Using the NMO-based operator and assuming a relatively simple subsurface, Brown was able to construct operators that image with all types of peg-leg multiples. I propose using least-squares reverse time migration to show that multiples can add values, even for a complex subsurface.

**DISSERTATION OVERVIEW AND CONTRIBUTIONS**

The remaining chapters in this dissertation are organized according to the following outline:

**Chapter 2 Joint imaging of up- and down-going signal:** In this chapter, I discuss joint imaging of up- and down-going signal for an ocean bottom dataset. The benefits and characteristics of imaging with either type of signal will be highlighted. I apply a data-domain linearized inversion technique, known as least-squares reverse-time migration, to optimally combine the information from both the up- and down-going signals. I demonstrate how high quality images can be obtained with results from a 2D synthetic and the 2D Cascadia field datasets.

**Chapter 3 3D field data examples - Deimos Dataset:** In this chapter, I apply the inversion method in Chapter 2 onto the 3D Deimos ocean bottom field
dataset from the Gulf of Mexico. Several challenges arose when I applied this method to a 3D field dataset. One challenge was imperfect separation between up-going and down-going signals in the dataset. I will discuss how this affected regular migration images. I propose a way to handle this problem by manipulating the properties in the extended image space. I show that the joint imaging result can provide a better image than migrating either of the signals alone. Using least-squares migration also gives fewer migration artifacts and better amplitude information underneath complex overburdens.

Chapter 4 Imaging with multiples using linearized inversion: In this chapter, I discuss the theory for imaging beyond the first-order of surface-related multiples using least-squares reverse-time migration. The procedure for constructing the modeling and migration operator for the higher-order surface-related multiples involves using the data as an areal source. I demonstrate the improvement in illumination and noise suppression with results from a 2D synthetic dataset.

Chapter 5 3D Gulf of Mexico data example: In this chapter, I apply the method developed in Chapter 4 onto a 3D Gulf of Mexico ocean bottom node dataset to image with the mirror and double-mirror reflection energy. I will incorporate some of the techniques discussed in Chapter 3 to improve the robustness of the joint LSRTM method. The result from the 3D Gulf of Mexico example shows that there are crosstalk reduction and illumination improvement in the image. In particular, I will highlight improvements in areas near and underneath a complex salt structure.

Chapter 6 Conclusions: In this chapter, I first summarize the most important results in this dissertation. I then discuss some possible directions for future research.
Chapter 2

Joint least-squares inversion of up- and down-going signal for ocean bottom data sets

This chapter presents a joint least-squares inversion method for imaging the acoustic primary (up-going) and mirror (down-going) signals for ocean-bottom seismic (OBS) processing. Joint inversion combines into one image the benefits of wider illumination from the mirror signal and wider angular illumination from the primary signal into one image. Results from a synthetic SEAM model and the 2D Cascadia ocean-bottom node field dataset show better subsurface illumination and improved resolution in geologically complex areas.

Ocean-bottom seismic acquisition is an established technology in which seismometers are placed at the sea-bottom and shots are fired at the sea surface. In areas congested by platforms or other obstacles, OBS acquisition is advantageous because it is operated by small boats without cumbersome towed streamers. Such a geometry enables OBS acquisition to provide full-azimuth illumination, shear-wave recording, a quiet recording environment, high-resolution data and repeatability. Applications of OBS acquisition includes imaging in obstructed oilfields and time-lapse monitoring.
CHAPTER 2. JOINT LSRTM ON SYNTHETIC OBN DATA

of hydrocarbon reservoirs.

There are different processing schemes for ocean bottom data. The traditional way, inherited from surface seismic processing, is to remove all free-surface multiples and to migrate only with the primary signal (Wang et al., 2009). Therefore, initial work on OBS data processing has been dedicated to the removal of free-surface multiples. One way to attenuate strong free-surface multiples is to combine the geophone and hydrophone recordings to eliminate the receiver ghost and the water column reverberations, a technique known as PZ summation (Barr and Sander, 1989; Soubaras, 1996; Schalkwijk et al., 2003). The vertical component of the geophone (Z) records up- and down-going signal at opposite polarity. The hydrophone recording (P) is not sensitive to whether the energy is coming from above or below. In this way, we can separate out the up- and down-going signal by adding and subtracting the P and Z recording in OBN data. As an alternative to PZ summation, Sonneland and Berg (1987) and Amundsen (2001) addressed free-surface multiples with the theory of up-down deconvolution in both layered and complex media. In this approach, not only are all free-surface multiples attenuated, but also de-ghosting and signature deconvolution are conducted in a single step.

Although multiples are often treated as noise, they are formed by the same source signal as primaries but travel along different paths in the medium. The receiver ghost, also known as the mirror signal, is the next order of reflection beyond the primaries. The receiver ghost has an additional reflection off the sea surface. The source grid in a deep water OBS survey has a much wider lateral extent than the receiver grid. Therefore, the subsurface reflection point of the receiver ghost is located at greater distances from the receiver station than the primaries (Figure 2.1). The mirror signal can provide wider subsurface illumination than the primaries if the energy is properly migrated. Several authors have used the mirror signal in the migration of OBS data (Godfrey et al., 1998; Ronen et al., 2005; Grion et al., 2007; Dash et al., 2009).

While most authors conclude that the mirror image produces a better result than the conventional primary image, the information in the primary image is also valuable. The primary reflection can illuminate the subsurface at a different reflection angle
Figure 2.1: The subsurface reflection point of the receiver ghost, also known as the mirror signal (in black), is located at a greater distance from the receiver station than the primary signal (in red). For a deep water OBS survey, the source grid has a much wider lateral extent than the receiver grid. This translates to a wider subsurface illumination for the mirror signal than the primaries, and it provides more potential for velocity and AVO analysis.
CHAPTER 2. JOINT LSRTM ON SYNTHETIC OBN DATA

than the mirror reflection (Figure 2.2). Figure 2.2 shows the illumination angle by two of the ocean bottom receivers with primary and mirror reflection. Theoretically, if there is an acquisition geometry with a dense sampling of sources and receivers and at a very large lateral extend, we can expect the subsurface image point to be illuminated by a wide and continuous range of angles. However, in the ocean bottom node acquisition, the receiver grid is often sparse. This means that the subsurface angular illumination can be very different between different order of reflections.

The Kirchhoff migration result from Dash et al. (2009) highlights the difference between the two types of signals. The illumination area from the mirror signal is much wider than the illumination area from the primary signal (Figure 2.3). However, as annotated by the circled region (Figure 2.3), the primary contains more details than the mirror image. Figure 2.4 shows six rays with the same source and receiver pairs illuminating the subsurface with the primary reflection (Figure 2.4a) and the mirror reflection (Figure 2.4b). The image point of each ray is shown in the yellow region. The primary reflection image point is much closer together than the mirror reflection image point. Although the mirror signal can image a bigger area, the primary signal has a higher image space sampling. Instead of treating the primary image and the mirror image separately, I can combine the information from the two sets of data coherently by joint least-squares reverse-time migration (LSRTM). The LSRTM imaging method can improve the structure and aperture of the seismic images by using both the up-going primary and the down-going mirror signals.

Muijs et al. (2007b) made an early attempt to image primary and free-surface multiples together. It requires the data to be decomposed into up-going and down-going constituents at the seabed level. It is then followed by downward extrapolation of the up- and down-going data and a 2D deconvolution-based imaging condition. While this technique is computationally efficient, the produced image would contains crosstalk artifacts. Different orders of surface-related multiples reflection are not separated in the up-going and down-going data. As a results, when the up-going data is downward extrapolated into the up-going wavefield, there are energy within the wavefield that is associated the primary and higher-order surface-related reflections.
Figure 2.2: (a) shows the reflection angle ($\theta_1$) by the primary reflection and (b) shows the reflection angle ($\theta_2$) by the mirror reflection at the same sub-surface image location for ocean bottom node acquisition geometry. Notice that the reflection angles are different between the primary and the mirror signals.

[NR] cha2/primir
Figure 2.3: Kirchhoff migration of ocean bottom dataset using (a) the primary data and (b) the multiple data. This figure is extracted from Dash et al. (2009). [NR/\texttt{chap2/. DashRTM}]
Figure 2.4: The relative illumination area (highlighted in yellow) of the (a) primary and (b) mirror events for ocean-bottom node geometry. Six rays with the same source and receiver pairs are shown. The ray diagram suggests that for the same set of rays, the primary reflection has a higher image space sampling. [NR]

[chap2/. illumupdown]
Likewise for the down-going wavefield. In the traditional cross-correlation imaging condition, the up-going wavefield and the down-going wavefield are correlated. There will be energy from the up-going wavefield and energy from the down-going wavefield that are not associated with the same order of reflections, which results in crosstalk noise in the image. Although deconvolution-based imaging condition can alleviate this problem, the mechanism that generates these artifacts still exists in the imaging condition. In contrast to Muijs’ method, joint inversion can optimally combine structural information provided by two types of reflections that are free from crosstalk.

In this chapter, I will focus on the joint inversion of the acoustic (P) wave signal. First, I will discuss the data-domain least-squares reverse-time migration method. Next, I will apply the inversion scheme to a 2D synthetic SEAM model and the 2D Cascadia ocean-bottom field dataset and show the overall improvement of the joint inversion result. In chapters 3, I will show the application of these methods to a 3D field data example.

**SEISMIC IMAGING BY LEAST-SQUARES REVERSE-TIME MIGRATION (LSRTM)**

Least-squares migration (LSM) (Cole and Karrenbach, 1992; Ji, 1992a; Lambare et al., 1992; Nemeth et al., 1999) poses the imaging problem as an linearized inversion problem. The Born forward modeling operator in LSM is obtained by linearizing the wave equation with respect to the image model. Many authors (Ji, 1992b; Wong et al., 2011) has pointed out that the Born modeling operator is the adjoint of the migration operator used in imaging. If we think of the adjoint operation as an approximation to the inverse operation, then migration provide an approximate solution to the LSM inverse problem. The goal of LSM is to obtain a more accurate solution than traditional imaging by enforcing the consistency between the synthetic and observed data. Least-squares reverse-time migration (LSRTM) is a class of least-squares migration that uses two-way wave propagation. I will explain LSRTM by first introducing the reverse-time migration (RTM) operator. Afterward, I will introduce the Born forward
modeling as the adjoint to RTM and how they are represented in the LSM objective function.

In reverse-time migration, the image is produced by multiplying the forward modeled source wavefield, \( U_o(x, x_s, t) \), with the reverse-time extrapolated receiver wavefield, \( U_r(x, x_r, t) \), at every time-step,

\[
\mathbf{m}_{\text{mig}}(x) = \sum_{x_r, x_s, t} U_o(x, x_s, t)U_r(x, x_r, t; x_s),
\]

where \( t \) is time, \( m(x) \) represents the structural image at subsurface location \( x \). \( x_s \) represents the source location and \( x_r \) represents the receiver location. \( U_o(x, x_s, t) \) and \( U_r(x, x_r, t; x_s) \) are solutions to the two-way acoustic constant density equation. The source and receiver wavefields satisfy,

\[
\left(s_o^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2\right) U_o(x, x_s, t) = f_s(t)\delta(x - x_s),
\]

\[
\left(s_o^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2\right) U_r(x, x_r, t; x_s) = d_{\text{obs}}(x_r, t; x_s),
\]

where \( s_o(x) \) is the slowness, \( f_s(t) \) is the source signature, and \( d_{\text{obs}}(x_r, t) \) is the observed data. The source and receiver wavefields are calculated using a finite-difference time domain method. For the receiver wavefield, the solution is calculated backward in time. To obtain a better image, we can go beyond migration by posing the imaging problem as an inversion problem. The solution \( m_{\text{inv}}(x) \) is obtained by minimizing the objective function, \( S(m) \).

\[
S(m) = \| d_{\text{mod}} - d_{\text{obs}} \|^2 = \| d_o + d_{\text{born}} - d_{\text{obs}} \|^2.
\]

\( S(m) \) measures the least-squares norm of the difference between the forward modeled data, \( d_{\text{mod}} \), and the recorded data, \( d_{\text{obs}} \). Notice that in linearizing the wave-equation, the forward modeled data, \( d_{\text{mod}} \), is broken down into two terms. \( d_o \) is the forward modeled data using the full acoustic wave equation and the background slowness, \( s_o \), as shown in equation 2.2. It can be represented as \( d_o = d_o(x_r, t; x_s) = U_o(x, x_s, t)\delta(x-\)

If the background slowness is slowly varying, the $d_o$ term will be insignificant and it can be ignored in the objective function (equation 2.4). In the next chapter, I will discuss the situation when $d_o$ is significant and how to handle it in a field dataset. The second term, $d_{born}$, is the Born approximation of the linearized acoustic wave equation. The derivations for the linearization of the acoustic wave equation and the Born modeling operator are shown in Appendix A. The expression for $d_{born}$ is,

$$d_{born}(x_r, x_s, \omega) = \delta U(x, t; x_s) \delta(x - x_r), \quad (2.5)$$

where $\delta U(x, t; x_s)$ represents the perturbed wavefield that is the solution to the following equation,

$$\left(s_o^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \delta U(x, x_s, t) = -m(x) \frac{\partial^2}{\partial t^2} U_o(x, x_s, t). \quad (2.6)$$

Notice that the force term in equation 2.6, $(-m(x) \frac{\partial^2}{\partial t^2} U_o(x, x_s, t))$, is a product of the source-side wavefield and the image model defined earlier. A common way to represents the Born modeling transformation is by introducing a linear operator $L$.

$$d_{born} = Lm \quad (2.7)$$

Note that the Born modeling operator, $L$, is the adjoint of the reverse-time migration operator, $L^T$.

$$m_{mig} = L^T d_{obs} \quad (2.8)$$

The goal of least-squares migration is to invert for the model indirectly using the conjugate gradient method.

**The meaning of the least-squares migration model parameter**

The output of least-squares migration can be used in reflectivity analysis. For the derivation of the equations in this section, I will direct the reader to Appendix B and
its references. At normal incidence, the LSM model parameter is,

\[ m(x) = \frac{2}{v_o^2(x)} \left( \frac{\delta v(x)}{v_o(x)} + \frac{\delta \rho(x)}{\rho_o(x)} \right), \]  

(2.9)

where \( v_o \) and \( \rho_o \) are the migration velocity and density, respectively. \( \delta v \) and \( \delta \rho \) are the perturbation velocity and density. We can think of the perturbation velocity as deviation between the migration velocity and the true velocity. Likewise for the perturbation density. When compared to the reflectivity expression, the LSM model parameter and the reflectivity, \( R(x) \) are related as follow,

\[ R(x) = \frac{1}{4v_o^2(x)m(x)}, \]

\[ = \left( \frac{v_o(x)}{2} \right)^2 m(x) \]  

(2.10)

Equation 2.10 suggests that we can scale the LSM output by the square of half of the background velocity to obtain the reflectivity at normal incidence.

**Joint inversion of up/down-going P wave**

Joint inversion of up- and down-going signals for ocean-bottom data can potentially be a better imaging technique than migrating either signal alone, because it combines information from both sets of signals. Ocean-bottom data are first separated into acoustic up- and down-going components above the seabed. The decomposed signals are then inverted to yield one optimally combined reflectivity image. The fitting goal for such an inversion is:

\[ 0 \approx \begin{bmatrix} L_{\uparrow} & L_{\downarrow} \end{bmatrix} \mathbf{m} - \begin{bmatrix} \mathbf{d}_{\uparrow} \\ \mathbf{d}_{\downarrow} \end{bmatrix}, \]  

(2.11)

where \( L_{\uparrow} \) and \( L_{\downarrow} \) are modeling operators that produce up-going data, \( \mathbf{d}_{\uparrow} \), and down-going data, \( \mathbf{d}_{\downarrow} \), from the model space (\( \mathbf{m} \)). I use the adjoint of the acoustic reverse time migration (RTM) operator to formulate \( L_{\uparrow} \) and \( L_{\downarrow} \). Two modified computational
grids are used to forward model the lowest order of up- and down-going signals, namely the primary and the receiver ghost. The formulation of the modeling and its adjoint (RTM) operator is summarized in Figures 2.5 and 2.6.

![Figure 2.5: Forward modeling of (a) primary-only and (b) mirror-only data. The algorithm involves cross-correlating the source wavefield \( U_o \) with the reflectivity model \( m \) to generate the receiver wavefield \( U_r \). Reciprocity is used here where the data, in common-receiver domain, are injected at the source location while the source wavelet is injected at the receiver location. Cross-correlation is done only with grid points below the seabed.](chap2/.forward)

In the modified computational grid (Figure 2.5), the primary signal is obtained by the cross-correlation of the source wavefields with the reflectivity estimate. For the down-going receiver ghost, the receiver nodes are placed at twice the water depth, which effectively represents a reflection off the sea-surface.

**COMPUTATIONAL COST OF LSRTM VS RTM**

LSRTM is a computationally expensive algorithm. If an iterative inversion such as the conjugate gradient method is used, the computational cost is \( 2N_{\text{iter}} \) that of regular RTM. \( N_{\text{iter}} \) is the number of iterations in the conjugate gradient. However, there are
Figure 2.6: RTM of (a) primary-only and (b) mirror-only data. The algorithm involves cross-correlating the source wave field ($U_s$) with the receiver wave field ($U_r$) to generate the reflectivity model ($m$). Cross-correlation is done only with grid points below the seabed.

Many ways to make the algorithm more affordable. Morton and Ober (1998); Romero et al. (2000); Leader and Almomin (2012) discuss using phase-encoding to effectively reduce the computational cost of many shots into a single shot. If graphical processing unit (GPU) hardware is available, time-domain finite difference computation time can be significantly reduced (Leader and Clapp, 2013). Sometimes, the acquisition geometry can also make the cost of LSRTM more affordable. There are substantially more shots than receivers in an OBN dataset. Instead of performing each prestack depth migration based on shot gather, it is often performed by the common receiver gather.

**NUMERICAL EXAMPLE**

Next, I will demonstrate the joint inversion of primary and mirror signals using a 2D synthetic example from a modified version of the SEAM model (Pangman, 2007).
Synthetic data are generated using an ocean bottom node-like geometry. Figure 2.8c shows the velocity model used for this example. The model is 4650 m deep and 12 km wide with a spacing of 15 m. The seabed is between 800 and 900 m deep.

For the synthetic data, I used the two-way acoustic wave equation to generate primary and mirror ocean bottom node data. Four-hundred shots extend from 0 to 12000 m along the sea surface at an intervals of 30 m. The receiver geometry consists of a receiver spacing of 105 m with 19 ocean bottom nodes located between 4995 m and 6900 m. Because reciprocity is used later, this geometry is equivalent to having 19 shots at the sea-bottom and 400 receivers at the sea-surface.

Typically, in an ocean bottom node dataset, the pressure and the vertical particle velocity are measured. Up- and down-going data can be extracted from PZ summation. In field data settings, imperfect separation between the up-going and down-going signal can happen. To simulate this situation, I purposely leave 20 percent of the down-going energy in the up-going data and vice versa. Figure 2.7 shows an up-going common receiver gather with and without imperfect up-down separation. We can see the mirror reflection energy arriving at around $t = 4s$. Because I used two-way wave equation modeling, there are also internal multiples in the data.

To compare conventional primary migration, mirror imaging, and joint inversion, I will first present the results of RTM on the conventional primary signal and the mirror signal. The corresponding image will then be compared to the joint inversion result using both signals.

Reverse time migration on conventional primary and mirror signals

In this section, I define the term primary-RTM ($m_\uparrow$) to be applying the adjoint of $L_\uparrow$ to the up-going data. The term mirror-RTM ($m_\downarrow$) is defined similarly. In equation form, this is written as:

$$m_\uparrow = L_\uparrow d_\uparrow^{mod},$$
$$m_\downarrow = L_\downarrow d_\downarrow^{mod}. \quad (2.12)$$
Figure 2.7: A common-receiver gather taken at x=5835 m: (a) only up-going energy is present in this gather and (b) with 20 percents of down-going energy remaining in the gather. The figure is displayed with a tpow setting of 3 to highlight the mirror signal noise. Direct arrival and refraction energy are muted.
Figures 2.8 b and c show the corresponding primary-RTM and mirror-RTM images. The mirror image has wider illumination than the primary image. The benefit of the wider aperture is directly correlated with the depth of the sea-bottom. The deeper the sea-bottom, the wider the illumination. The extent between the source and receiver spread also plays a role in the illumination aperture. If the spread of the source position is laterally much wider than the spread of the ocean-bottom receivers, the mirror image will illuminate a much wider area than the primary image. By comparing the two images, against the velocity model (Figure 2.8a), we can see some of the crosstalk noise as a result of imperfect up-down separation and internal multiples. The crosstalk noise can be represented in the following form,

\[
m_{\text{mig}} = L'_t (d_\uparrow + \alpha d_\downarrow),
\]

\[
= L'_t d_\uparrow + L'_t \alpha d_\downarrow,
\]

\[
= m_{\text{signal}} + m_{\text{crosstalk}}.
\]

where \(m_{\text{signal}}\) represents the part of the image that carries the true signal while \(m_{\text{crosstalk}}\) represents the part of the image that is the result of migrating the down-going data (\(\alpha d_\downarrow\)) with the primary kinematics. The crosstalk noise in the primary-RTM appears in the deeper part of the image. On the other hand, the crosstalk noise in the mirror-RTM appears in the shallower part of the image. In this way, the degradation from the crosstalk noise due to imperfect separation affects different parts of the migration images between the two datasets. This characteristic that is beneficial in joint imaging later on.

The primary and mirror events illuminate the same subsurface location at different reflection angles (Figure 2.2). Figure 2.9 shows the primary and mirror image when illuminated at a subsurface angle of \(-50\) degrees. The annotated circle shows the illuminated region from the mirror-RTM image that is not present in the primary-RTM image. The primary-RTM image is able to illuminate the sediment layers at a larger reflection angle than the mirror-RTM image as annotated by the pointers. Different parts of the subsurface are illuminated at different angles in the two datasets.
Figure 2.8: (a) The velocity model, (b) primary RTM image obtained by calculating $L_i^\prime d_i^{\text{mod}}$, and (c) mirror RTM image obtained by calculating $L_i d_i^{\text{mod}}$. [CR]
Figure 2.9: (a) Primary RTM image and (b) mirror RTM image. The front panel shows the image illuminated at a subsurface angle of -50 degrees.
A joint image by least-squares reverse time migration could be superior to conventional mirror imaging in several ways. It will contain the wider illumination of the mirror-RTM and the illumination contribution at different subsurface angles. In addition, the crosstalk artifacts due to imperfect separation will be suppressed because these noises are not consistent between the two images.

**Least-squares RTM Result**

A joint inversion is performed in a least-squares sense with the objective goal described in equation 2.11. Figure 2.10 shows the inversion results for the primary, the mirror, and the joint image. There is an overall improvement from the migration images in Figure 2.8b and c to the inversion image in Figure 2.10a and b, respectively. In general, the LSRTM algorithm improves the image by reducing migration artifacts, balancing the relative amplitude of the reflectors, and increasing the resolution of the image.

I have identified some areas of improvement with two close-up sections shown in Figures 2.11 and 2.12.

1. By comparing the RTM and LSRTM images in both Figure 2.11 and Figure 2.12, we can see that the overall resolution of the image is higher in the LSRTM case.

2. The annotated region A in Figure 2.11 shows a steeply dipping salt flank. It is better illuminated in the primary images than in the mirror images. The joint LSRTM image also illuminates the dipping salt flank well.

3. The annotated region B in Figure 2.11 shows a region of the sediment against a salt flank. It is better illuminated in the mirror images than in the primary images. The joint LSRTM image benefits from the mirror signal and also illuminates the sediment-salt region well.

4. Figure 2.12 shows the overall improved signal-to-noise ratio of the joint LSRTM as compared to the individual primary-LSRTM and mirror-LSRTM images above salt.
Figure 2.10: (a) Primary LSRTM image and, (b) mirror LSRTM image and (c) the joint LSRTM image.
Figure 2.11: A section at x=3800 to 5800 m and z=1000 to 3000 m of (a) primary RTM, (b) mirror RTM, (c) primary LSRTM, (d) mirror LSRTM, (e) joint LSRTM, and (f) velocity model.
Figure 2.12: A section at x=4400 to 6400 m and z=1600 to 3600 m of (a) primary RTM, (b) mirror RTM, (c) primary LSRTM, (d) mirror LSRTM, (e) joint LSRTM, and (f) velocity model. [CR] [chap2/. zoomSap2v2]
5. The annotated region underneath the salt in Figure 2.12 also shows that the joint-LSRTM image has more continuous reflectors there with less artifacts than individual mode imaging.

This example shows that joint inversion coherently combines information from the primary and the mirror signals to produce a better illuminated and better resolved image.
2D FIELD EXAMPLE

Next I will present the results from applying LSRTM on an OBS survey located at the northern Cascadia continental margin offshore of western Canada. The area contains gas hydrates, which have a characteristic structure known as the bottom-simulating reflector (BSR), that marks the base of the hydrate stability zone. OBS data were collected along five parallel lines normal to the margin (Figure 2.13). Line spacing was 500 m with ten ocean-bottom seismometers deployed with 100 m spacing at a water depth of about 1300 m.

Pre-processing

In the 2D study, I extracted a 2D shot line from directly above the OBS line. After completing the processing step described in Dash et al. (2009), I applied a gapped deconvolution with a 12-ms gap length and 300-ms filter length to suppress the source bubble. Only six of the ten OBS receivers were used for final imaging. These receivers were selected according to their data quality. Three receivers were rejected due to large tilt angles, and one receiver was rejected because it was saturated with noisy traces.

I used the adaptive decomposition method of Schalkwijk et al. (2003) to separate energy into up- and down-going wavefields. These data were bandpassed between 5 and 45 Hz to avoid dispersion in the time-domain finite-difference calculation. These data, before and after pre-processing, are shown for one common receiver gather (Figure 2.14).

A velocity model was supplied by the University of Victoria to use for imaging. Velocity values range from 1480 m/s in the water column to 1820 m/s in the sediment layer. The BSR lies at an approximate depth of 1500 m. There is a velocity inversion where the velocity value drop is going deeper across the BSR.
Figure 2.13: The geometry of the Northern Cascadia dataset. Ten ocean-bottom seismometers were deployed with 100 m spacing at a water depth of 1300 m. The shot line spacing was 500 m. [NR] chap2/. geo

Conditioning of the LSRTM problem for the 2D Cascadia data

Seismic inversion is an ill-posed problem. To prevent divergence to unrealistic solutions, I added an additional term to our objective function, \( S(m) \). In Nemeth et al. (1999); Ronen and Liner (2000), data weighting operators were used to remove the acquisition footprint by applying zero weighting on regions corresponding to the
acquisition gap. The overall fitting goal becomes

\[ S(m) = \| W(Lm - d) \|^2 + \epsilon^2 \| m \|^2, \]  
(2.15)

where \( W \) is a diagonal data-weighting matrix, and \( \epsilon \) tunes the level of damping in our objective function. The criterion for choosing \( \epsilon \) is:

\[ 0.2 = \frac{\| \epsilon m \|^2}{\| W(L^T(d_{mod} - d)) \|^2}. \]  
(2.16)

In the first iteration, \( \epsilon \) is defined by limiting the gradient contribution from the damping term to be 20% of the gradient contribution from the data fitting term. From experiments with both synthetic and field dataset, I found that this limit yields satisfactory results. For this dataset, the damping term alone seems to adequately regularize the inversion and yield a satisfactory result. Other regularization terms
are needed to further constrain the inversion for more complex survey regions.

Migration and Inversion results

Figure 2.15a shows the primary-RTM image and Figure 2.15b shows the mirror-RTM image. I can identify some migration artifacts associated with RTM, such as several high-amplitude low-frequency artifacts. The resolution is relatively low, and the structural information below the BSR is generally difficult to identify. Similar to Figure 2.3, the primary-RTM illuminates a much smaller area compared to the mirror-RTM image. Due to the small numbers of shots, the primary-RTM also suffers from apparent edge artifacts. Figure 2.15c shows the joint-RTM, which is a simple sum of up-going and down-going signal.

I applied LSRTM for three cases as follow (1) using the up-going primary signal (primary-LSRTM), (2) using the down-going mirror signal (mirror-LSRTM), and (3) using both primary and mirror signals (joint-LSRTM). The results after 20 iterations are shown in Figure 2.16a, b, and c respectively. Overall, the inversion images have higher resolution and fewer artifacts than the RTM images. To study the benefit of joint-LSRTM, I focused on the center region where both the primary and mirror signals contribute. Figure 2.17 shows an enlarged section of the image with four different scenarios. The LSRTM results are better than the RTM results. However, when comparing the two LSRTM cases (Figures 2.17 c and d), the joint-LSRTM is far superior than the mirror-LSRTM below the BSR. An arrow highlights that a dipping reflector below the BSR is much more clearly imaged in the joint-LSRTM than in all the other scenarios.
Discussion

Edge artifacts from the primary image have been passed onto both the joint-RTM and joint-LSRTM images. The Cascadia dataset has a shallow target area, which results in strong edge artifacts. I expect that edge artifacts will be less apparent when the target area is deeper or when more ocean-bottom nodes are included in the calculation.

The Cascadia result shows that information from the primary signal can help us to image dipping reflectors better (Figure 2.17). We can understand why better imaging of dipping reflectors is possible by observing the ray paths which illuminate a dipping reflector beneath an ocean bottom node. In Figure 2.18, the primary reflection (red) requires a shorter offset range than the mirror reflection (blue) to illuminate the target. Given that all surveys have a finite maximum offset, this translates to dipping reflectors being better imaged by the primary signal than by the mirror signal.

CONCLUSION

While direct migration of the primary data has limited illumination aperture, direct migration of the mirror signal is less resolved in complex areas. Joint-LSRTM can image geologically complex areas with better illumination, improved resolution, and more balanced amplitude. Although only 2D modeling and migration are used, we see improvements in the joint-LSRTM image over conventional methods. Such improvements include suppression of migration artifacts, enhancement of amplitudes along true reflectors, and better resolution.

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to publish the OBS data. I thank Ranjan Dash for information and previous work on the field dataset.
Figure 2.15: (a) the primary-RTM, (b) the mirror-RTM, and (c) the joint-RTM, which is the sum of primary- and mirror-RTM. [CR] chap2/. RTMcas
Figure 2.16: (a) LSRTM with the primary data (primary-LSRTM), (b) LSRTM with the mirror data (mirror-LSRTM), and (c) joint-LSRTM with both primary and multiple data.

[CR] [chap2/. LSRTMcas]
Figure 2.17: A section of the image cut from x=7600-9200 m and z=1400-1750 m. (a) shows the mirror-RTM, (b) joint-RTM, (c) mirror-LSRTM, and (d) joint-LSRTM.
Figure 2.18: Ray paths for illuminating a dipping reflector underneath an ocean bottom node by the primary (red) and the mirror (blue) reflection.
CHAPTER 2. JOINT LSRTM ON SYNTHETIC OBN DATA
Chapter 3

3D field data applications - the Deimos Dataset

In this chapter, I apply least-squares reverse-time migration (LSRTM) to a three-dimensional ocean-bottom node dataset. Applying imaging algorithms onto 3D field datasets is often more challenging than applying on synthetic datasets. When applying least-squares migration (LSM) to a field dataset, a background data component must be subtracted from the observed data. I introduce salt-dimming data weighting to address some of the issues associated with a strong background data term for my study area. The Deimos data set has complex salt structure that obscures part of the image deep down. I use a target-oriented data-reweighting to emphasize deeper parts of the image near the salt. Oftentimes, a shortfall in the theory or challenges in pre-processing would generate unwanted noise in the LSRTM image. To make the algorithm more robust, I utilize the prestack extended-angle domain to filter some of the noise in the image space.

I will begin by introducing the 3D field dataset called Deimos. I will then illustrate some processing challenges associated with the dataset. I will discuss some methods I used to condition the LSRTM inversion with the field dataset. Finally, I will show
some of the RTM and LSRTM results, comparing between using primary-only, mirror-only, and joint datasets.

**DEIMOS 3D OCEAN-BOTTOM NODE DATASET**

The Deimos data set was recorded in an area south-east of New Orleans in the Gulf of Mexico. The field was discovered in 2002, with first oil production in 2007 (Burch et al., 2010; Smit et al., 2008; Stopin et al., 2008). In 2007, Shell Exploration and Production Company and their partner BP Americas commissioned Fairfield Industries to conduct a 3D ocean-bottom node survey over the Deimos field in the Mississippi Canyon protraction area. Prior to the 3D survey, 16 nodes were deployed on a single 2D line at their normal 3D grid locations (Hays et al., 2008). A swath of seven dual source sail lines nearest the node line were shot. There were fourteen source lines on the nominal 50x50 m grid with approximately 3300 shots. The ocean-bottom receiver was deployed by a remotely operated vehicle (ROV) at approximately 400 m spacing. Figure 3.1 shows the locations of the sources and receivers.

This was effectively a 2.5D survey where the horizontal (in-line) extent is much greater than the vertical (crossline) extent. As a result, inline dipping reflectors should be much better resolved than crossline dipping reflectors. Prior knowledge of this field suggests that there are fewer structural variations along the crossline direction compared to the inline direction. Figure 3.2 shows the migration velocity model used in this study.

Shell Exploration and Production Company performed some pre-processing of this ocean-bottom node dataset. Hydrophone and geophone recording were decomposed into up-going and down-going signal by PZ summation. Multiple removal were also performed using a modeled-based prediction and subtraction technique. Figure 3.3 shows the up-going and down-going data after pre-processing. Next, I will discuss some of the imaging challenges associated with this field dataset.
Figure 3.1: The acquisition geometry for the Deimos ocean-bottom data set. The 14 source lines span a 50x50 m grid. The 16 ocean-bottom node receivers were deployed on a 2D line with an approximate spacing of 400 m. [NR] chap3/. SouRecDeimos
Figure 3.2: The migration velocity model used for the Deimos ocean-bottom node survey.
Figure 3.3: One common receiver gather of the (a) up-going primary and (b) down-going mirror data after pre-processing.
Imaging challenges

Incomplete removal of noise in the image

The conventional imaging scheme for ocean-bottom node datasets migrates only the down-going mirror signal (Ronen et al., 2005; Dash et al., 2009). More care is often given to the down-going mirror signal than the up-going signal. I observe that the down-going data is better separated than the up-going data. Figure 3.4a shows the crosstalk image, $\mathbf{m}_{xalk}$, when we apply the down-going migration operator, $\mathbf{L}_\downarrow^T$, onto the up-going data, $\mathbf{d}_\uparrow$. It can be represented mathematically as:

$$
\mathbf{m}_{xalk} = \mathbf{L}_\downarrow^T \mathbf{d}_\uparrow. \tag{3.1}
$$

Figure 3.4b shows the image produced from applying the down-going migration operator onto the down-going data, $\mathbf{d}_\downarrow$. We can identify some of the events that are present in both of the two images. For example, the reflectors annotated in Figure 3.4 are in both images. This suggests that some residual down-going energy remains in the up-going data.

When wrongly attributed energy is present in the field dataset, it compromises both the RTM and the LSRTM results. When we migrate the down-going data with the kinematics of a primary reflection, the image will contain false reflectors at a depth that is deeper than the depth of the corresponding true reflectors. Similarly, when we migrate the up-going data with the kinematics of the mirror reflection, the image will contain false reflectors at a depth that is shallower than the depth of the corresponding true reflectors. Sometimes these false reflectors can be suppressed with LSRTM. However, the inversion can also adjust the solution to try to explain some of the wrongly attributed energy in the data. To alleviate this problem, I utilized the prestack extended-angle domain to discriminate some of the unwanted energy in the image space. I will discuss more about this noise filtering scheme in the next section.
Figure 3.4: (a) The image depicting crosstalk energy when applying the down-going migration operator onto the up-going data; $\mathbf{m}_{\text{crosstalk}} = \mathbf{L}_m^T \mathbf{d}_i$. (b) The image when applying the down-going migration operator onto the down-going data; $\mathbf{m}_{\text{RTM}} = \mathbf{L}_d^T \mathbf{d}_i$. The annotation is showing some of the events that are present in both images.
Incomplete theory

The wave theory we used might not adequately explain the entire physics of the subsurface. I assumed the Earth behaves as an acoustic isotropic medium. The assumption of an acoustic isotropic medium vastly simplifies the complexities happening within the Earth. In LSRTM, we hope to explain the entire wave-form of the observed data with the modeled data. In practice, it is naive to expect that the recorded data can be completely explained when the theory is over simplified.

The Earth is at least a visco-elastic medium with density variations. One can use more accurate wave theory in their LSRTM algorithm. It requires additional parameters such as the density or modulus to be estimated. In order for LSRTM to converge to a meaningful solution, the estimated parameters need to be sufficiently close to the true parameters, which can be challenging. To explore the algorithm in a robust way while obtaining useful subsurface information, I work with the acoustic isotropic wave equation.

**IMPROVING LSRTM RESULT IN FIELD DATASET**

With the imaging challenges described in the previous section, it is difficult to obtain desirable result using only the standard LSRTM objective function in field datasets. I introduce three ways to improve the LSRTM results, they are (1) salt-dimming, (2) target-oriented data reweighting, and (3) extended-angle domain noise removal.

**Salt Dimming**

The LSRTM objective function, $S(m)$, requires the background data term, $d_o$, to be subtracted from the observed data, $d_{obs}$.

$$S(m) = \| d_{mod} - d_{obs} \|^2 = \| Lm - (d_{obs} - d_o) \|^2 .$$  \hspace{1cm} (3.2)
where \( \mathbf{d}_{\text{mod}} \) is the modeled data. \( \mathbf{Lm} \) represents the Born-modeled data and \( \mathbf{d}_o \) is the background data. The modeled data is the sum of the Born-modeled data and the background data. In practice, \( \mathbf{d}_o \) is the full-wave forward modeling using the migration velocity model. When the velocity varies smoothly, this term is usually negligible and we can ignore it in the inversion. The background data term is non-trivial when the velocity field has a sharp contrast such as a salt body. In the field data case, there are always errors in the background velocity, positioning, and reflectivity of the salt. As a result, an accurate estimation of the background data term is difficult. This makes following equation 3.2 impractical. Salt-dimming (Wong, 2013) is introduced as a way to bypass this problem.

Salt-dimming aims to down-weight the salt reflection energy in the data space so that the inversion can minimize other regions in the model. This corresponds to the objective function,

\[
S(\mathbf{m}) = \| \mathbf{W}_s(\mathbf{Lm} - \mathbf{d}_{\text{obs}}) \|^2,
\]

where \( \mathbf{W}_s \) is the data weighting function that down-weights the salt reflection energy. This can be done by forward-modeling the salt reflection using the migration velocity. The next step is to calculate an envelope around the salt energy. The data weighting function can then be defined by assigning small values to the salt reflection envelope. The objective function with salt-dimming becomes:

\[
S(\mathbf{m}) = \| \mathbf{W}_s^\uparrow(\mathbf{L}_\uparrow\mathbf{m} - \mathbf{d}_{\text{obs}}^\uparrow) \|^2 + \| \mathbf{W}_s^\downarrow(\mathbf{L}_\downarrow\mathbf{m} - \mathbf{d}_{\text{obs}}^\downarrow) \|^2.
\]

(3.3)

where the subscripts \( \uparrow \) and \( \downarrow \) represent the up-an down-going direction in OBN data. Figure 3.5a shows the forward modeling of one common-receiver gather in the synthetic Sigsbees model (Paffenholz et al., 2002). The salt reflection is then used to derive an envelope region to be down-weighted. One example of a weighting function is shown in Figure 3.5b, where the down-weighted region (blue) corresponds mostly to the salt reflector.
Figure 3.5: (a) Background data created by full-wave forward modeling using the migration velocity and (b) the corresponding salt-dimming weight ($W_s$) generated with the background data. [CR] chap3/Vfig5wgt
Data reweighting to emphasize shadow zone

In addition to down-weighting the contribution from salt reflections, we can also use data weighting to emphasize an important part of the data. Due to geometric spreading and attenuation, reflection energy from the deeper part of the image is often significantly weaker than the reflection energy from the shallower part. In a least-squares inversion, the inversion algorithm often focuses on minimizing the strong energy in the data, which corresponds to the shallow reflection. This can be problematic when the deeper reflection energy is neglected. For the synthetic case, we have the exact operator that matches the physics of the recorded data. In this scenario, the strong shallow reflection energy can be fully explained by our modelling and migration operator. The deeper reflection in the data will subsequently be explained later. For the field data case, our operator might not exactly match the physics that generated the field data. As a results, it is impossible to expect the shallow reflection to be fully explained by the inversion. Often, the contribution to the objective function from the deeper part of the data will not be significant enough for some of the deeper reflector to be fully imaged.

I allow the inversion to fit the shallower part of the data for a few iterations and then reweight the entire inversion to focus onto the deeper part. The procedure to generates this weighting is relatively simple. The first step is to pick a few reflectors that are poorly illuminated in the RTM image (Figure 3.6). Next, perform Born modeling of the picked reflector (Figure 3.7a) and then extract an envelope around the forward modeled energy to produce a diagonal data weighting function (Figure 3.7b). The objective function then becomes,

\[
S(m) = \begin{cases} 
||W_s^\dagger(L_s^\dagger m - d_{obs}^\dagger)||^2 + ||W_s^\dagger(L_s^\dagger m - d_{obs}^\dagger)||^2 & : \text{iter} \leq n_{rw} \\
||W_B^\dagger W_s^\dagger(L_s^\dagger m - d_{obs}^\dagger)||^2 + ||W_B^\dagger W_s^\dagger(L_s^\dagger m - d_{obs}^\dagger)||^2 & : \text{iter} > n_{rw},
\end{cases}
\]

where \(W_{B}^{1/1}\) is the data weighting that emphasize picked reflectors. \(I_{iter}\) is the current iteration number in the conjugate gradient scheme. I use the objective function with
Figure 3.6: Reflector picks used to generate data reweighting.

chap3/vreflpick
Figure 3.7: (a) Born modeled data of the picked reflectors in Figure 3.6 and (b) the associated data weighting. The weighting is extracted by an envelope around the prominent energy in the Born modeled data. [CR] chap3/.VbmDeiA8
salt-dimming for \( n_{rw} \) iterations. The objective function is then reweighted at iteration \( n_{rw} \) to include the new weighting to emphasize the deeper picked reflectors. I used \( n_{rw} = 10 \) as the reweighting limit for the Deimos field dataset.

**Noise removal in the prestack extended-angle domain**

The assumption behind extended-angle domain filtering is that noise and signal are formed at different angles. While this assumption is not true for all kinds of noise, it is useful as a way to alienate certain types of noise in the images. Figure 3.8a shows an OBN image gather generated by a single ocean-bottom node (OBN) at \( x_{OBN} = 54150 \) m and \( y_{OBN} = 34800 \) m. Figure 3.8b, c, and d are displaying the depth-angle panel of Figure 3.8a at 3 different inline locations of 53000 m, 54000 m, and 55000 m. These depth-angle panels are located to the left, around, and to the right of the ocean bottom receiver along the inline direction. There are several characteristics based on the image gather. Image points near the receiver are mostly illuminated by reflections with small aperture angles. On the other hand, image points located to the left of the receiver are predominantly illuminated by reflections with negative aperture angles as shown in Figure 3.8b. Similarly, image points located to the right of the receiver are predominantly illuminated by reflections with positive aperture angles as shown in Figure 3.8d.

We can identify areas of signal and noise in Figure 3.8 by slicing through the angle domain. Figure 3.9a shows the image extracted at a subsurface angle of 15 degrees. At this illumination angle, the image is predominantly signal (label 1). Figure 3.9c is showing an area (label 2) that is believed to be migration artifacts and is illuminated at -35 degrees.

It is important to highlight the benefit of filtering at each prestack OBN image gather instead of at a poststack image gather. This extra degree of freedom allows us to isolate noise that would have otherwise be buried with the signal in a poststack image gather. Given that we have identified an angular range to be signal for each OBN image gather, it is straight-forward to remove the noise. In two dimension,
Figure 3.8: An 2D RTM image generated with an ocean-bottom receiver located at $x_{OBN} = 54150m$ and $y = 34800m$. Depth-angle panels are taken at inline locations of (b) 53000 m, (c) 54000 m, and (d) 55000 m. The prominent energy in the depth-angle panel is shifted based on its relative position from the source. [CR]
Figure 3.9: The same image gather from Figure 3.8. (a) is showing the image illuminated at an angle of 15 degrees. Label 1 highlights an area that is predominantly signal. (b) is showing the corresponding depth-angle panel extracted at midpoint $x = 55000$ m with a line indicating the slicing of the image cube at 15 degrees. (c) is showing the image illuminated at an angle of -35 degrees. Label 2 highlights an area that has conflicting dips with the sediment and is believed to be noise. (d) The same depth-angle panel as in (b) but with a line indicating the -35 degrees slicing.

[CR]

chap3/. dmAngbefore-explain
filtering in the angle domain is equivalent to filtering the dips in the depth-offset domain. The relationship between the dips in the depth-offset domain to the aperture angle ($\gamma$) is,

$$\frac{k_{hx}}{k_z} = - \tan \gamma.$$  \hfill (3.4)

where $k_{hx}$ and $k_z$ are wavenumber along the subsurface offset ($hx$) and depth $z$ directions. In practice, I apply the filter in the depth-offset domain by finding equivalent dips ranges based on the angle ranges using equation 3.4. Although equation 3.4 is only true in 2D, there is an equivalent expression in 3D that includes the reflector’s tilt. For this particular dataset, there is not enough crossline aperture to obtain a meaningful extended image gather in the crossline direction. An equivalent filtering procedure involving $k_{hy}$ can be applied in 3D.

Figure 3.10a shows the result of extended domain filtering on a prestack OBN image. Most of the noise is removed above the salt reflection at $z=4000$ m. Figure 3.10b, c, and d, are displaying the depth-angle panel of Figure 3.10a at $x=53000$ m, 54000 m, and 55000 m. The angle range are chosen to preserve the prominent energy in the image. Figure 3.11a shows the corresponding filtered noise. Figure 3.11b, c, and d, are displaying the depth-angle panel of Figure 3.11a at $x=53000$ m, 54000 m, and 55000 m. The original prestack RTM image (Figure 3.8a) is decomposed into the signal part (Figure 3.10a) and the noise part (Figure 3.11a).

**RESULTS**

**Imaging with a single mode**

In conventional imaging, the mirror signal is used for migration because it provides a wider illumination area. Often time, the primary signal is not used in imaging. This is because the illumination area of the primary reflections is often much narrower than that from the mirror reflections. In chapter 2, Figure 2.4 shows the relative illumination area (highlighted in yellow) of the primary and mirror events. The mirror signal can illuminate the sea-bottom region much better than the primary signal.
Figure 3.10: The same prestack RTM image as in Figure 3.8 after extended-angle domain filtering. Depth-angle panels are taken at inline location of (b) 53000 m, (c) 54000 m, and (d) 55000 m.
Figure 3.11: The filtered noise from the same prestack RTM image as in Figure 3.8 after extended-angle domain filtering. Depth-angle panels are taken at inline location of (b) 53000 m, (c) 54000 m, and (d) 55000 m. The sum of Figure 3.10a and Figure 3.11a should be the same as Figure 3.8a [CR]
Figure 3.12 shows the RTM image using the primary-only signal and the mirror-only signal. The illumination area between Figure 3.12a and b is not as dramatically different as in Figure 2.4. This is because the lateral extent of the source grid is only slightly larger than the lateral extent of the receiver grid along the inline direction.

Although the illumination between the primary and mirror signals are similar for this survey, the angular coverage of the primary and mirror signals are different. Figure 3.13 shows the comparison of an angle-domain common image gather between the primary and the mirror signals. The side panel shows that the same reflector is illuminated at different angles by the primary and the mirror signals.

I ran LSRTM using the conjugate gradient method for 25 iterations. Figures 3.14 and 3.15 compares between the results of RTM with spectral balancing and LSRTM. In both cases, the LSRTM image is superior than the RTM image with higher resolution and better amplitude information. The improvements are more dramatic in regions that are 4000 m or deeper. Figure 3.16 shows the vertical wave-number amplitude spectrum for the primary-image between the RTM and the LSRTM case. The amplitude increases in both the higher end and the lower ends of the spectrum. With iterations, the LSRTM algorithm gradually recovered a banded white spectrum.

Joint imaging of primary and mirror signals with LSRTM

Figure 3.17a shows the joint-RTM image and Figure 3.17b shows the joint-LSRTM image. The joint-RTM image is essentially the first gradient of the joint-LSRTM algorithm. It can also be viewed as the sum of the primary-RTM image and the mirror-RTM image. In the joint image, we see similar improvement when using LSRTM as compared to using RTM.

Figure 3.18 shows a section of the Deimos image from Figures 3.14, 3.15, and 3.17; The four images are calculated using the mirror RTM with spectral balancing, primary LSRTM, mirror LSRTM, and joint LSRTM. I also observe better delineation of the dipping reflector in the joint-LSRTM image as highlighted by circle A. This is because the primary signal has a decreasing illumination aperture in region close to the
Figure 3.12: RTM image using (a) the primary signal and (b) the mirror signal. The front face shows the image slice located at a crossline location of $y = 34450$ m. [CR] chap3/. updownimagedeimos
Figure 3.13: (a) Angle domain common image gather for mirror RTM and (b) primary RTM. The angular coverage is different between the two signals.
Figure 3.14: A enlarged section of the (a) the mirror-RTM image with spectral balancing and (b) the mirror-LSRTM image after iteration 25.
Figure 3.15: A enlarged section of the (a) the primary-RTM image with spectral balancing and (b) the primary-LSRTM image after iteration 25.
Figure 3.16: Graph showing the amplitude spectrum of the vertical wavenumber for the primary-RTM (solid-line) and primary-LSRTM (dashed-line) images. With LSRTM, the amplitude information increases at both the higher and lower end of the spectrum. [CR chap3/. spectrum]
Figure 3.17: (a) Joint-RTM image with AGC and (b) joint-LSRTM image after iteration 25. [CR] chap3/. jointdeimosimage
ocean bottom node receiver. In chapter 2, the Cascadia field data result shows that information from the primary signal can help image dipping reflector better. Circle B shows a region where faulting is better recognized in the joint-LSRTM image than the mirror-LSRTM image. Circle C highlights a region where the sediment layers near the salt is better imaged with joint-LSRTM than the other three images.

Figure 3.19 shows a section of the Deimos image close to the left salt. The four images are calculated using primary RTM with spectral balancing, primary LSRTM, mirror LSRTM, and joint-LSRTM. Circle A highlights a region where the sedimentary reflector is better imaged with joint LSRTM (Figure 3.19d) than using primary signal ((Figure 3.19a and b). Circle B shows a region where the sedimentary layer truncate against a salt flank on the left. We can better delineate the reflectors in the joint-LSRTM image than the mirror-LSRTM image. Overall, joint LSRTM is consistently better as a whole than single-mode imaging. Besides studying the zero-subsurface offset image, we can also study the differences in angular illumination. Figure 3.20 and 3.21 shows the angular illumination of reflectors at two different lateral locations. The joint LSRTM image has the widest range of coverage.

CONCLUSION

Least-squares reverse time migration is an advanced imaging technique that can improve imaging with better relative amplitude information, fewer artifacts, and reduced noise. When applying to field data sets, the recorded data departs from the theory and assumptions of the LSRTM operator. A simple data-fitting objective function in LSRTM is insufficient. I used salt-dimming data weighting, extended domain noise filtering, and target-oriented data reweighting to improve the convergence of the LSRTM algorithm when applied to the 3D Deimos ocean-bottom field data set. The inverted LSRTM image has better relative amplitude balance for the reflectors and improved illumination near the salt than the conventional RTM image.

Instead of treating the primary image and the mirror image separately, I combine
Figure 3.18: A zoomed section of the Deimos image calculated using (a) mirror RTM with spectral balancing, (b) primary-LSRTM, (c) mirror-LSRTM, and (d) joint-LSRTM. The annotations highlight several area where joint-LSRTM shows improvement over either mirror LSRTM or primary LSRTM. [CR]
Figure 3.19: The zoomed section of the (a) primary RTM with spectral balancing, (c) primary LSRTM, (c) mirror-LSRTM, and (d) joint-LSRTM. [CR] chap3/. comparedeimos2
CHAPTER 3. 3D FIELD DATA APPLICATIONS - THE DEIMOS DATASET

Figure 3.20: Angular illumination of reflectors at x=54000 m and y=34450 m from the (a) up-going LSRTM, (b) down-going LSRTM, and (c) joint LSRTM. The joint LSRTM has the widest range of coverage. [CR]chap3/. CompareAng1

Figure 3.21: Angular illumination of reflectors at x=57000 m and y=34450 m from the (a) up-going LSRTM, (b) down-going LSRTM, and (c) joint LSRTM. The joint LSRTM has the widest range of coverage. [CR]chap3/. CompareAng2
the information from the two sets of data coherently using joint least-squares reverse-time migration (LSRTM). The image calculated using joint-LSRTM of primary and mirror signals is superior than conventional single-mode imaging.

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Chapter 4

Imaging with multiples using LSRTM

In this chapter, I present a technique for imaging both primaries and higher-order multiples using joint least-squares reverse-time migration (joint-LSRTM). As discussed in chapter 1, one critical challenge with multiple imaging is the presence of crosstalk artifacts in the migration image. By enforcing the consistency between synthetics and observed data, joint-LSRTM can better delineate between true reflectors and noise in the image. When used with a suitable migration velocity model, this technique uses the multiple energy as signal. I can image a class of multiply scattered events by using the two-way propagator in both modeling and migration. Such events can scatter off sharp-interfaces in the migration velocity as well as the sea surface.

I will begin by discussing the theory for the ocean bottom node (OBN) acquisition geometry. An illumination analysis of the primary and multiples will be presented. I will show the effect of joint-LSRTM as compared to migration on a simple 2D layered model. Finally, I will demonstrate the concept and methodology in 2D with a synthetic Sigsbee2B model. By including the multiples into imaging, I found that the image quality improved in areas close to the complex salt structures.
CHAPTER 4. IMAGING WITH MULTIPLES USING LSRTM

THEORY

Least-squares migration poses the imaging problem as an inversion problem by linearizing the wave-equation with respect to the image model ($m$) In Appendix A, I show how to linearize the wave-equation to obtain the forward modeling and its adjoint operator. For simplicity, I will refer to the Born modeling operator as $L$. The standard objective function measures the least-squares difference between the modeled data ($d_{mod}$) and the observed data ($d_{obs}$),

$$S(m) = \|d_{mod} - d_{obs}\|^2 = \|Lm + d_o - d_{obs}\|^2. \quad (4.1)$$

where the modeled data is the sum of the background data ($d_o$) and the Born modeled data ($Lm$). The background data is the full-wave modeling of the data based on the migration velocity. This background data term was discussed in detail in relation to salt-dimming in Chapter 3.

Multiple imaging with ocean-bottom node

In an ocean-bottom survey, the signal can be classified into up-going (Figure 4.1a) and down-going signal (Figure 4.1b-d) with respect to the receivers. The lowest order of the up-going signal is the primary reflection (Figure 4.1a). The lowest order of the down-going signal is the direct-arrival. Since the direct-arrival does not carry any information about the subsurface, the next order of down-going event, the mirror signal (Figure 4.1b), is used for conventional migration of down-going OBN data.

The focus of this chapter is on imaging the higher-order surface-related multiples (Figure 4.1c and d) for OBN data. In particular, I compare the image output between migration with the mirror data and joint-LSRTM with both mirror and multiple data. For the remaining of this chapter, I will refer to the first order down-going reflection (Figure 4.1b) as the mirror reflection and the surface-related down-going multiple reflections (Figure 4.1c and d) as the double-mirror reflections. Although
Figure 4.1: Ray-paths for (a) a primary reflection, (b) a mirror reflection, and (c, d) higher-order surface-related multiple reflections.
what I call the double-mirror reflections actually include all higher-order surface-related reflections, the strongest of such multiples involve two bounces off the sea-surface. Hence the word 'double' will be used to mark the most significant part of the higher-order multiple energy.

To jointly image with both mirror and double-mirror signals, I use the following objective function:

\[ S(m) = \|L_{\text{mirror}}m - d_{\text{mirror}}\|^2 + \|L_{\text{double}}m - d_{\text{double}}\|^2. \]

\[ (4.2) \]

\(L_{\text{mirror}}\) and \(L_{\text{double}}\) are the linearized Born modeling operators for the mirror reflection and the double-mirror reflection events respectively. The recorded mirror and double-mirror data are represented by \(d_{\text{mirror}}\) and \(d_{\text{double}}\), respectively. Figure 4.2 describes the procedure for modeling mirror data. The procedure uses the symmetry across the sea surface to simulate one part of the traveling path. This concept was introduced in mirror imaging (Godfrey et al., 1998; Ronen et al., 2005). Figure 4.3 describes the procedure for modeling the double-mirror data. For the incident wavefield, the down-going data is used as an areal shot to simulate the first three legs of the travel path.

At first glance, \(L_{\text{mirror}}\) seems to only generate singly scattered events (e.g. Figure 4.4a). To clarify, the term scattering includes both diffraction and reflection. However, if I propagate the wavefields using the two-way wave equation, the Born modeling operator can actually generate multiply scattered events. In Figures 4.4b and d, the ray path reflects off a salt flank and then the horizontal reflector. If the sharp salt-flank boundary already exists in the migration velocity, then the scattering off the salt flank is automatically generated by the propagator. By using the data as a source, \(L_{\text{double}}\) can generate all higher-order surface-related multiples (Figure 4.4c).
Figure 4.2: Born modeling of the mirror OBN data. The mirror reflection ray path can be divided into the incident part and the scattered part. To simulate the down-going leg of the ray path, OBN are placed at the mirror position with respect to the sea surface. The algorithm involves cross-correlating the incident wavefield with the image model to generate the input for the scattered wavefield. Synthetic mirror data is then collected at the sea surface (green box).

**ILLUMINATION ANALYSIS OF MULTIPLES**

I performed an illumination analysis of the mirror and the multiples signal from the Sigsbee2B model. Figure 4.5 shows two sets of illumination angle gathers (Gherasim et al., 2014). The illumination angle gathers are calculated by placing scatterers near and under a complex salt body in the Sigsbee2B model. Each scatterer has a perfect amplitude-versus-angle (AVA) signature between 0 to 80 degrees. I apply Born modeling followed by reverse-time migration based on two types of reflection events. One type of event contains only the mirror reflection, and the other type contains only the double-mirror reflections. Figures 4.5a and b show the resulting illumination angle-gathers based on the mirror reflections and the double-mirror reflections, respectively. The calculated AVA signature is displayed to the right of each scattering point. Bright areas are angle ranges that are well illuminated. Label 1 highlights an area that is poorly illuminated by the double-mirror signal but is well illuminated by the mirror signal. On the other hand, label 2 highlights an area that is poorly illuminated by the mirror signal but is well illuminated by the double-mirror signal.
For the incident wavefield, the down-going data is used as an areal shot to simulate the first three legs of the travel path. To generate the incident wavefield, the areal shot is injected at the sea-surface with a -1 factor. The procedure to generate the scattered wavefield is similar to that for the mirror reflection (Figure 4.2).
Figure 4.4: The ray-path for (a) a singly scattered event, (b) a doubly scattered event and (c,d) triply scattered events. Single circles (in purple) indicate scattering off the migration velocity and the sea surface. Double circle (in green) indicate scattering off the image model.

[NR] chap4/. chap4-figure4
illuminated by the mirror signal but is well illuminated by the double-mirror signal. Label 3 points to areas that are illuminated by both the mirror and the double-mirror but at a different angular range.

We can better understand the illumination difference between the mirror and the double-mirror signal by examining the incident wavefield. Figure 4.6 shows a snapshot of the incident wavefield from the mirror and the double-mirror signals. Due to the rugosity of the salt, energy that enters the salt is often bent at different angles. In the mirror wavefield (Figure 4.6a), we can see that the energy that enters the annotated shadow zone is weaker than energy in other parts of the model. On the other hand, the bending from the complex salt structure causes the energy from the double-mirror wavefield (Figure 4.6b) to enter the shadow zone at a different angle. This results in different angular illumination between the two types of signal as observed in Figure 4.5.

In general, the double-mirror illumination response (Figure 4.5) has a lower resolution than the mirror-illumination response. This is because of the cross-correlation imaging condition in RTM. Essentially, the double-mirror illumination response is composed of correlating multiple source wavelets and subsurface reflectivities. This problem will be addressed when using least-squares migration.

SYNTHETIC EXAMPLE

I apply joint-LSRTM on two models. The first one is a simple one-layered model that allows us to keep track of different kinds of migration artifacts. The second model is the more complicated Sigsbee2B model.

One-layered model

I construct a one-layered model (Figure 4.7 a) with ocean-bottom geometry. Figure 4.7 b shows the synthetic data. Label $d_1$ points to a mirror reflection, while labels $d_2$ and $d_3$ indicate higher order multiple events. Because I use Born modeling to generate
Figure 4.5: Illumination angle gathers based on (a) the mirror and (b) the multiple reflections. A set of scatterers are placed throughout the Sigsbee2B model. Each scatterer has a AVA signature from 0 to 80 degrees displayed to the right of the scattering point. Label 1 and 2 show areas that are poorly illuminated by the double-mirror and the mirror signals, respectively. Label 3 points to areas that are illuminated by both type of signals but at different angular range.  

Figure 4.6: A snapshot of the incident wavefield from (a) the mirror and (b) the double-mirror signal. The two sets of wavefield enter the shadow zone with different angles and strength.
the synthetic data, internal multiples are absent. Figure 4.7 c shows the migration image $m_{mig}$. The migration image is made up of signal $m_{sig}$ and crosstalk artifacts $m_{xtalk}$. In the figure, label $A$ marks spurious reflectors generated by migrating the mirror signal ($d_1$) as if it were a multiple. Label $B$ highlights the correct reflector in the image. Label $C$ points to an artifact generated by migrating the multiple signal ($d_2$ or $d_3$) as if it were a mirror reflection. In equation form, they are denoted as follows:

\[
\begin{align*}
    m_{mig} &= m_{signal} + [m_{xtalk}] = m_B + [m_A + m_C], \\
    m_A &= L'_2d_1 + L'_3d_1 + L'_4d_1 + ..., \\
    m_B &= L'_1d_1 + L'_2d_2 + L'_3d_3 + ..., \\
    m_C &= L'_1d_2 + L'_1d_3 + L'_2d_3 + ..., 
\end{align*}
\]

where $m_A$, $m_B$, and $m_C$ correspond to the parts of the image labeled with $A$, $B$, and $C$ in Figure 4.7 c. $L'_1$, $L'_2$, and $L'_3$ are migration operators that correspond to different orders of reflection events. Figure 4.7 d shows the inversion result. Notice that the artifacts are removed from the image. In conventional RTM, if there is residual multiple energy in the data, artifacts of type $C$ would appear in the image. Treating it as real signal would negatively affect the interpretation of the sub-surface. Figure 4.7d shows the joint-LSRTM result. Joint-LSRTM poses the imaging problem as an inversion problem. Therefore, it enforces the consistency between observed and modeled data. Since the Born modeling operator has incorporated the kinematics of the multiples, the multiple energy in the data ($d_2$ and $d_3$) can now be properly explained. As a result the crosstalk artifacts in the RTM image is removed. Beside removing artifacts in the image, joint-LSRTM can improve the imaging in some poorly illuminated areas. In the next section, I will test the method on the 2D Sigsbee2B synthetic model where there are shadow zones underneath a complex salt overburden.
Figure 4.7: (a) Original one layered model, (b) synthetic data, (c) migration image and (d) inversion image.

Sigsbee2B model

I apply joint-LSRTM to the Sigsbee2B model with the ocean-bottom geometry. Figure 4.8 shows the migration velocity and the reflectivity model used for this study. There are two interfaces in the migration velocity. One comes from the salt and the other comes from the basement reflector. These sharp interfaces along with the free surface boundary condition will generate strong multiples in the data. I first generate the mirror data ($d_{\text{mirror}} = L_{\text{mirror}} m$) using the procedure in Figure 4.2. It requires
positioning the nodes at the mirror point across the sea surface. In standard processing flow, surface-related multiples are often predicted and subtracted. Next, I use the synthetic double-mirror data \( d_{\text{double}} = L_{\text{double}} m \) to represent the predicted multiples from the standard processing flow. I then add 20% of the double-mirror data to the mirror data. This is to simulate the case when multiple-elimination techniques cannot completely remove the multiples. I will refer to this synthetic simply as the noisy-mirror data (Figure 4.9a). Figure 4.9 (b) shows the synthetic that are the sum of mirror and double-mirror data.
Next, I perform three different types of inversion. The first inversion, termed as the Mirror-LSRTM, tries to match the noisy-mirror data using mirror-modeling operator. The double-LSRTM tries to match the double-mirror data with the double-mirror modeling operator. Finally, the joint-LSRTM uses both the mirror and the double-mirror signal as described in equation 4.2 to produce a single image.

Figure 4.10 shows the result of applying RTM and LSRTM. Figure 4.10a is the mirror-RTM image. Label 1 indicates the artifacts in the migration image that correspond to crosstalk noise. The artifacts overshadow a bright spot in the image. Figure 4.10b is the mirror-LSRTM result. Note the substantial improvement in the crosstalk noise between migration and inversion. Although LSRTM can enhance the resolution of the image ((Wong et al., 2011)), poorly illuminated areas still struggle. The subsalt area is not well illuminated (Box 2). Next, Figure 4.10c shows the result obtained
by applying LSRTM to the double-mirror signal. Even with inversion, the sediment area remains poorly illuminated (Box 3). Finally, Figure 4.10d is calculated using joint-LSRTM (equation 4.2). The image is superior to other imaging result in all 3 highlighted regions.

Figure 4.11 shows a deeper area of the Sigsbee2B model. Label 1 indicates an area underneath the complex salt structure, while label 2 highlights an area against a salt flank. Both areas are better imaged with joint-LSRTM of mirror and double-mirror signals than by other techniques (Figure 4.11a-d).

In terms of convergence, our objective function decreases to two percent of its initial value after 40 iterations. The inversion converges pretty quickly because the modeling operator used in the inversion is exactly the same as the modeling operator used to generate the observed data. For field dataset application, additional weighting and regularization to the inversion problem are needed for faster convergence.

**DISCUSSION**

Joint LSRTM does not migrate all orders of multiples. It only migrates multiples associated with surface-related reflection or scattering off sharp velocity interfaces such as the sediment-salt transition. Considering that multiples with high-amplitude in the data are often generated by subsurface interfaces of high impedance contrast, this technique can account for most of the significant multiples in the data.

This method is model-based. One consideration is the accuracy of the migration velocity. Because the operators in LSRTM are based on the linearization of the wave-equation, strong deviation between the migration velocity and the true velocity would hamper the effectiveness of this technique. However, using the double-mirror in LSRTM as described is no more sensitive to the velocity than primary-LSRTM or mirror-LSRTM. This is because field datasets are used as an areal source in the multiple modeling operator. Both the waveform and the travel time is correct up to the last down-going and up-going legs of a multiple reflection. If the migration velocity is good enough for mirror-LSRTM to improve over RTM, then double-LSRTM
or joint-LSRTM should also produce improved images.

**CONCLUSION**

I demonstrated a method for imaging with surface-related multiples using joint-LSRTM for ocean bottom node datasets. This technique increases the subsurface illumination by using the multiple energy as signal and it also addresses the issue of crosstalk in the image. I demonstrate the concept and methodology with a 2D layered model and the Sigsbee2B model.

**ACKNOWLEDGMENTS**

I thank Dave Nichols and Robert Clapp for suggestions and comments.
Figure 4.10: A section of the Sigsbee2B model from $z=1950-7185$ m and $x=1950-7185$ m calculated using (a) Mirror-RTM, (b) Mirror-LSRTM, (c) Double-mirror LSRTM, and (d) joint-LSRTM. Label 1 indicates a strong crosstalk noise in the Mirror-RTM. Label 2 highlights a subsalt area that is poorly illuminated by the mirror signal but is well illuminated by the double-mirror signal. Label 3 marks a sediment area that is poorly illuminated by the double-mirror signal but is well illuminated by the mirror signal. \[CR\] \[chap4/. chap4-sigsbee1\]
Figure 4.11: A section of the Sigsbee2B model from x=6000-10485 m and z=4050-10785 m calculated using (a) mirror-RTM, (b) mirror-LSRTM, (c) double-mirror-RTM, (d) double-mirror-LSRTM, and (e) joint-LSRTM. (f) is the original image. Labels 1 and 2 highlight two areas against the salt that are better imaged with joint-LSRTM. [CR] chap4/ chap4-sigsbee2
Chapter 5

3D Gulf of Mexico data example

In this chapter, I apply joint least-squares reverse time migration (joint LSRTM) to a synthetic three-dimensional ocean-bottom node (OBN) dataset. The model used to generate the synthetic data is based on the Deimos dataset in Chapter 3. The dataset is designed to simulate a range of imaging challenges in the Gulf of Mexico. Instead of using traditional ocean-bottom node acquisition geometry, I used a pseudo-random distribution of ocean-bottom receivers. Despite the fact that relatively few OBN receivers are used in the survey, joint LSRTM can still generate a relatively well-illuminated subsurface image when higher-order surface related multiples are included in imaging.

To speed up the rate of convergence, I have applied some of the techniques introduced in Chapter 3. One technique addresses the issue of shadow zones in the subsurface due to a complex salt structure. To emphasize deeper parts of the image near the salt, I use a target-oriented data-reweighting in least-squares reverse-time migration. To extract the most information from the LSRTM algorithm with the fewest number of iterations, I include salt dimming in the inversion.

I will begin by introducing the 3D synthetic dataset. I will then describe some of the processing and workflow applied before running migration. I will show some of the reverse time migration (RTM) and LSRTM results using different modes of
reflection data as operators. The joint LSRTM algorithm can properly combine the information from first and higher order surface-related multiples energy. Compared to using only the first order reflection, I will show that joint LSRTM can provide crosstalk noise reduction and illumination improvement in the image.

3D SYNTHETIC GULF OF MEXICO MODEL

This synthetic model is created by Robert Clapp which is based on a user specified sequence of geological events (Clapp, 2014). The salt structure in this dataset is modeled after the salt structure from the Deimos field dataset from Chapter 3. The model contains various geological features including deformation, folds, faults, and planes. Some unconventional structures are also included to test the capability of my imaging algorithms. The goal is to create many areas that are difficult to image using tradition migration. In particular, I am interested in seeing how different imaging processes will perform in shadow zones. Figure 5.1a shows the velocity profile used to create the synthetic data. As part of the blind test, I do not have access to the true velocity mode. Instead, some errors were introduced to the migration velocity model for the subsequent migration and LSRTM calculation. Figure 5.1b shows the velocity error (in percents) between the migration velocity and the true velocity. Negative errors mean that the migration velocity is underestimated while positive errors mean that the migration velocity is overestimated. The errors in the migration velocity model are clustered in several regions and deviate as much as 20\% from the true velocity model.

The depth of the model is 7km, with a lateral size of 13.5km x 13.5km. The datasets are then simulated using the Born forward modeling method and the constant-density acoustic wave equation.
Figure 5.1: (a) Velocity model used to generate the synthetic data. (b) Velocity error (in percent) in the migration velocity model.
Figure 5.2: The study area is 13.5km x 13.5km wide in both the inline (x) and crossline (y) directions. (a) shows the distribution of ocean-bottom nodes for this survey. A total of 60 nodes are used. (b) shows the location of the physical shots in red. The shot spacing is 25m in both inline and crossline directions.
Acquisition geometry

We use the ocean-bottom node acquisition geometry for this dataset to generate different order of reflection arrivals. In practice, the data are simulated per receiver gather using reciprocity, in which the role of the shots and receivers are exchanged. An unconventional distribution of ocean-bottom nodes is used instead of a traditional spreading of OBN nodes. The industry standard typically has a node spacing of 400m and the OBNs are distributed in either a rectangular or hexagonal grid. The rectangular or hexagonal grid is centered at the target region. In this dataset, we employ node spacing that are at least 400m; the nodes are distributed in a pseudo-random fashion with higher density in the target region. The nodes are then gradually spaced further apart. Figure 5.2a shows the distribution of the 60 OBNs in this synthetic dataset. This type of distribution allows us to simulate with far fewer receivers. In an actual field acquisition, this will translate into shorter acquisition time.

Synthetic dataset

For this dataset, we simulate the pressure wavefield directly using Born forward modeling. To verify the theory discussed in Chapter 4, I will be studying the contribution from the first two orders of down-going reflection (Figure 5.3). For simplicity, I will refer to the first order of down-going reflection (Figure 5.3a) as the mirror reflection. The second order of down-going reflection (Figure 5.3b) will be called the double-mirror reflection.

To generate the mirror reflection, we used the procedure described in Figure 4.2 where the OBN node is placed at the mirror position with respect to the sea surface. We used the procedure in Figure 4.3 to generate the double-mirror reflection. Figure 5.4a and 5.4b show a receiver gather with mirror-only reflection and double-mirror only reflection. The sum of the two modes (Figure 5.4c) should adequately represent the total down-going energy as higher order surface-related multiples will become
The pressure data are simulated per receiver gather using reciprocity, in which the role of the shots and receivers are exchanged. Figure 5.2b shows the distribution of the shot carpet (in red) for this survey. The shots are spaced 25m apart in both the inline and the crossline directions.

Since the data are prepared synthetically, a minimal amount of pre-processing is needed. One pre-processing step I made was to apply static time shifts in the OBN gather to move the dataset from a grid of 12.5m sampling to 25m sampling. I found that this provide sufficient correction.

![Figure 5.3](chap5/MirrorDoubleRay)

**Figure 5.3:** (a) The first-order of down-going reflection. It will be referred to as the mirror reflection. (b) The second-order of surface-related down-going reflection. It will be referred to as the double reflection.

### MIRROR LSRTM AND JOINT LSRTM

I ran two LSRTMs with the down-going data, containing both the mirror and the double-mirror arrival. The first inversion has an operator that only accounts for the kinematics of the mirror reflection. Its objective function, $S_{\text{mirror}}(\mathbf{m})$, is:

$$S_{\text{mirror}}(\mathbf{m}) = \| \mathbf{L}_{\text{mirror}} \mathbf{m} - \mathbf{d}_d \|^2, \quad (5.1)$$

where $\mathbf{L}_{\text{mirror}}$ represents the Born forward modeling operator for the mirror reflection, and $\mathbf{m}$ is the image model. The down-going data, $\mathbf{d}_d$ (Figure 5.4c) contain both the mirror and the double-mirror reflection.
Figure 5.4: A receiver gather along the inline direction. (a) shows the mirror reflection energy. (b) shows the double-mirror reflection energy. (c) shows the total down-going energy, which is generated by the sum of (a) and (b).
The second inversion has an operator that accounts for the kinematics of both the mirror and the double-mirror reflection. Its objective function, $S_{\text{joint}}(m)$, is:

$$S_{\text{joint}}(m) = \| (L_{\text{mirror}} + L_{\text{double}}) m - d_l \|^2,$$

(5.2)

$$= \| L_{\downarrow} m - d_{\downarrow} \|^2,$$

(5.3)

where $L_{\text{double}}$ represents the Born forward modeling operator for the double-mirror reflection. The sum of $L_{\text{mirror}}$ and $L_{\text{double}}$ becomes $L_{\downarrow}$, which represents the Born forward modeling operator that accounts for both the mirror and the double-mirror reflection. In general, the computational cost of applying $L_{\downarrow}$ is the same as the computational cost of applying $L_{\text{mirror}}$. However, joint LSRTM is applied to a longer recording time to capture some of the later arrivals from the double-mirror reflection. Therefore, the computational cost scales linearly with the number of time steps in the finite difference time-domain calculation. I use 10 iterations of conjugate direction to perform the two inversions. To speed up convergence, I have applied some techniques introduced in Chapter 3. In the next section, I will discuss some of the preparation I made before initiating the inversion.

Figure 5.5: (a) Born modeled data using the migration velocity model for an ocean-bottom node located at $x=3300\text{m}$ and $y=25\text{m}$. (b) The corresponding data-weighting function used for salt dimming. [CR chap5/chap5-dim]
PREPARATION FOR INVERSION

In this dataset, the salt reflection energy is relatively strong when compared to other reflection energy. I included salt dimming, as introduced in Chapter 3, to the LSRTM algorithm. Salt dimming aims to down-weight the contribution of strong reflection energy that corresponds to the sharp velocity contrast in the background (migration) velocity model.

The motivation is to allow the inversion to fit other parts of the image quickly with the fewest number of iterations possible. For LSRTM to perform properly, a sufficiently good migration velocity model that is close to the true velocity model is needed. Later, I will examine how LSRTM performs in areas with velocity errors. The objective functions with salt dimming become:

\[ S_{\text{mirror}}(m) = \| W_{\text{mirror}}(L_{\text{mirror}}m - d) \|_2^2, \]  \hspace{1cm} (5.4)

\[ S_{\text{joint}}(m) = \| W_{\downarrow}(L_{\downarrow}m - d) \|_2^2, \]  \hspace{1cm} (5.5)

where \( W_{\text{mirror}} \) and \( W_{\downarrow} \) represent the salt-dimming data weighting function for the mirror and the down-going signal, respectively. Figure 5.5a shows the Born modeled data for the mirror reflection of an OBN located at \( x=3300 \text{m} \) and \( y=25 \text{m} \). In Figure 5.5a, the energy from the top-of-salt reflection arrives at around \( t=5 \text{s} \) as shown by label 1. There is also strong energy arriving at an earlier time (label 2), this energy corresponds to the reflection off the sea-bottom. Figure 5.5b shows the corresponding data-weighting function. The salt and seabed reflection energy are down-weighted to a value of 0.1. The energy coming from the bottom-of-salt reflection is not affected, because it is relatively weak in the data.

Besides salt dimming, I also applied data reweighting, introduced in Chapter 3, to emphasize the shadow zone in the subsurface. I allow the inversion to fit the shallower part of the data for a few iterations and then reweight the entire inversion to focus onto the deeper part. Due to the complexity of this model, I applied two sets of data reweighting that correspond to two different poorly illuminated areas.
Figure 5.6: Reflectors picks used to generate data reweighting. (a) shows the picks from the left subsalt area. (b) shows the picks from the right subsalt areas. I performed the picking over many inline sections and theses picks are interpolated as a plane along the crossline direction. [CR] chap5/chap5-pick
Figure 5.6a shows the picked reflectors (in red) underneath the left salt structure. Similarly, Figure 5.6b shows the picked reflectors (in red) underneath the right salt structure. I used the OpenCPS seismic processing software to perform the picking over many inline sections. The software then interpolates along the crossline direction and outputs my picks as a plane.

Next, I perform Born modeling of the picked reflectors (Figure 5.7a) and then extract an envelope around the forward modeled energy to produce a diagonal data weighting function (Figure 5.7b). The objective functions for the two inversions become,

\[
S_{\text{mirror}}(m) = \begin{cases} 
\| W^s_{\text{mirror}} (L_{\text{mirror}} m - d_\uparrow) \|^2 & : I_{\text{iter}} < n_{rw1} \\
\| W^{R1}_{\text{mirror}} W^s_{\text{mirror}} (L_{\text{mirror}} m - d_\uparrow) \|^2 & : n_{rw1} \le I_{\text{iter}} < n_{rw2} \\
\| W^{R2}_{\text{mirror}} W^s_{\text{mirror}} (L_{\text{mirror}} m - d_\uparrow) \|^2 & : I_{\text{iter}} \ge n_{rw2},
\end{cases}
\]
\[ S_{\text{joint}}(m) = \begin{cases} \| W^*_1(L_i m - d_i) \|^2 & : I_{\text{iter}} < n_{rw1} \\ \| W^{B1}_i W^*_1(L_i m - d_i) \|^2 & : n_{rw1} \leq I_{\text{iter}} < n_{rw2} \\ \| W^{B2}_i W^*_1(L_i m - d_i) \|^2 & : I_{\text{iter}} \geq n_{rw2} \end{cases} \]

where \( W^{B1}_i \) and \( W^{B2}_i \) are the down-going data weightings that emphasize poorly illuminated areas underneath the right and the left salt structures, respectively. \( W^{B1}_{\text{mirror}} \) and \( W^{B2}_{\text{mirror}} \) are similar to \( W^{B1}_i \) and \( W^{B2}_i \), except that they account for the kinematics of the mirror reflection instead of the down-going reflection. \( I_{\text{iter}} \) is the current iteration number in the conjugate direction scheme. \( n_{rw1} \) and \( n_{rw2} \) are the iteration limits that mark the transition between different data weightings. I used \( n_{rw1} = 4 \) and \( n_{rw1} = 8 \) as the reweighting limits for this dataset.

**RESULTS**

By supplying an operator with the proper kinematics to model the down-going data, the main goal of joint inversion is to achieve the following:

1. Crosstalk suppression
2. Illumination improvement

In the next two sections, I will show a few areas in the image where I observe crosstalk suppression and illumination improvement.

**Crosstalk suppression**

Figures 5.8a and 5.8b show the image cube from the mirror RTM at a depth of \( z=1075 \text{m} \) and \( z=2000 \text{m} \). In the figure, the pattern in the shallower depth section (Figure 5.8a) is repeated as an imprint in the deeper depth section (Figure 5.8b). This is the result of crosstalk artifacts from migrating the double-mirror reflection energy with the kinematics of the mirror RTM operator. The same kind of crosstalk artifacts can be found in the joint LSRTM. Figure 5.9 shows the same depth section
as in Figure 5.8b from the joint LSRTM at iteration 1, 4, and 10. Notice that the imprint has been gradually removed by the inversion algorithm.
Figure 5.8: The image cube calculated using mirror RTM. The top panels show the depth section extracted at (a) $z=1075\text{m}$ and (b) $z=2000\text{m}$. The depth pattern in the shallower section (a) can be found as an imprint in the deeper section (b). This is the result of migrating the double-mirror data with the mirror operator. [CR]
Figure 5.10 shows another example of crosstalk artifacts appearing in the migration image. The annotations point to areas inside the salt where there are spurious reflectors. These spurious reflectors are the result of applying the mirror migration operator onto the double-mirror data. In Figure 5.10b, the coherent energy that forms the crosstalk artifacts has been suppressed by the joint-LSRTM algorithm.

**Illumination improvement**

Beside crosstalk suppression, joint-LSRTM can also enhance the subsurface illumination of the image by including the double-mirror reflection energy as signal. There are a few areas in the mirror RTM image that are poorly illuminated. Usually, those regions are close to or underneath a complex salt structure. The double-mirror reflection has an equivalent geometry of a dense source and receiver arrays. As a result, some of the double-mirror reflection energy can better reach these poorly illuminated areas.

Figure 5.11a, b, and d show a crossline section calculated using three different techniques. They are mirror RTM with spectral balancing, mirror LSRTM, and joint LSRTM. The reflectivity model (Figure 5.11c) is also included for comparison. Label A points to a salt structure. The areas near and underneath the salt structure are poorly illuminated (Circle B). When I perform mirror LSRTM (Figure 5.11b), we can see an improvement in this region. However, joint LSRTM (Figure 5.11d) can further improve the image because the double-mirror reflection energy are also used as signal. The reflectors that terminate against the salt are better delineated. In addition, the anticline below the salt becomes more interpretable. Circle C highlights a region where the sediment laying on top of the salt body is better imaged with joint LSRTM than with the other techniques.

Figure 5.12a, b, and d show an inline section calculated using three different techniques. They are mirror RTM with spectral balancing, mirror LSRTM, and joint LSRTM. Circle A shows a region against the right salt flank that is poorly illuminated. Label B points to an area where the reflectors near the salt flank are
Figure 5.9: The depth section at $z=2000\text{m}$ from the joint LSRTM image at (a) iteration 1, (b) iteration 4, and (c) iteration 10. (d) shows the reflectivity model.
Figure 5.10: (a) The image cube calculated using joint RTM. The arrows point to areas where crosstalk artifacts appear. (b) The image cube calculated after 10 iterations of joint-LSRTM. Notice the crosstalk artifacts have been suppressed. [CR] chap5/.chap5-xtalk-salt
better illuminated with joint LSRTM. In addition, there are some conflicting dips in the image with energy oriented almost perpendicularly to the direction of the true reflectors. These noises are better suppressed by joint LSRTM.

Figure 5.13 shows an example where the base of salt is better imaged with joint LSRTM. When only the mirror signal is used (Figure 5.13a and 5.13b), the base-of-salt boundary in circle A and circle B is not well defined. In the joint LSRTM image (Figure 5.13d), the base-of-salt boundary is more coherent and becomes more interpretable.
Figure 5.11: A crossline section calculated using (a) mirror RTM with spectral balancing, (b) mirror LSRTM, (c) true model, and (d) joint LSRTM. The regions around the salt body (label A) are poorly illuminated. Circle B and C show area of improvement when using joint LSRTM as compared to the other techniques.
Figure 5.12: An inline section calculated using (a) mirror RTM with spectral balancing, (b) mirror LSRTM, (c) true model, and (d) joint LSRTM. The regions around the salt body (label A) are poorly illuminated. Label B points to an area where the reflector near the salt flank are better illuminated. [CR] chap5/.chap5-enhance2
CONCLUSION

In this chapter, I extend the methodology for imaging with surface-related multiples using joint LSRTM to a 3D data example. The physics in the LSRTM operator properly account for both the first order (mirror) and higher order (double-mirror) reflections. Therefore, we observe crosstalk reduction and illumination improvement in the image. In particular, areas near and underneath a complex salt structure are better illuminated with joint LSRTM. In an OBN survey, receivers are sparsely placed and surface-related multiple elimination can be a challenging task. Joint LSRTM eliminates the need to separate out surface-related multiples in the down-going data and can be a viable alternative to traditional processing.

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Figure 5.13: Image cubes extracted at x=8475m, y=4400m and z=5850m. (a) Mirror RTM with spectral balancing, (b) mirror LSRTM, (c) true model, and (d) joint LSRTM. The base-of-salt boundary is more coherent and better defined with joint LSRTM. [CR chap5/. chap5-enhance3]
Chapter 6

Conclusions

In this dissertation, I introduced a novel technique to incorporate primaries and surface-related multiples into imaging. The technique can properly address the challenges of multiple imaging such as the appearance of crosstalk artifacts in the image. In regions with complex subsurface geology, this technique has been shown to improve imaging in poorly illuminated areas.

In Chapter 2, I discussed the theory of joint imaging of up- and down-going signal for an ocean-bottom dataset. The benefits and characteristics of imaging with either type of signal were highlighted. The formulation of data-domain linearized inversion, also known as least-squares reverse-time migration, was presented. I demonstrated how high quality images can be obtained with results from a 2D synthetic and the 2D Cascadia field datasets.

In Chapter 3, I applied the method developed in Chapter 2 onto the 3D Deimos ocean-bottom field dataset from the Gulf of Mexico. I introduced three techniques to improve the robustness of applying least-squares reverse-time migration onto the field dataset. Salt-dimming data weighting was introduced to address some of the issues associated with a strong background data in the study area. The complex salt structure in the study area created shadow zones in the image. I used a target-oriented data-rewriting to emphasize deeper parts of the image near the salt.
incorporated the prestack extended-angle domain filtering in the least-square reverse-
time migration algorithm to suppress some of the unwanted noise in the image space.
Instead of treating the primary image and the mirror image separately, I combined the
information from the two sets of data coherently using joint least-squares reverse-time
migration (joint LSRTM). The result of the application showed that joint least-squares
reverse-time migration of primary and mirror signals is superior to conventional single-
mode imaging.

In Chapter 4, I discussed the theory for imaging higher-order surface-related mul-
tiples in an ocean-bottom dataset — mirror and double-mirror reflections. The pro-
cedure for constructing the modeling and migration operator for the higher-order
surface-related multiples was presented. The technique utilizes the data as an areal
source, which provides equal sensitivity to the background velocity when compared
to imaging using the primary signal. I demonstrated improvements in illumination
and noise suppression with results from a 2D synthetic dataset.

In Chapter 5, I applied the method developed in Chapter 4 onto a 3D Gulf of
Mexico ocean-bottom node dataset to image with the mirror and double-mirror re-
fection energy. Salt-dimming data weighting and target-oriented data reweighting
introduced in Chapter 3 was incorporated into the joint least-squares reverse-time
migration process. The result of the application shows that crosstalk is reduced and
illumination is improved in the image. In particular, areas near and underneath a
complex salt structure are better illuminated. Joint LSRTM eliminate the need to
separate out surface-related multiples in the down-going ocean-bottom data and can
be a viable alternative to traditional processing.

In this thesis, I introduced a method that highlights the benefits of using surface-
related multiples in imaging while addressing the problem of crosstalk artifacts in the
image. Interesting future work would include surface-related multiples in the velocity
model building process. In particular, for a region with complex salt structures, joint
LSRTM might provide better illumination of salt boundaries and subsequently salt
picking.
Appendix A

Linearizing the acoustic wave equation

In this appendix, I derive the linear forward modeling and adjoint operators for the least-squares reverse-time migration method. Just like in migration, least-squares migration requires a velocity or slowness model as input. In this derivation, I use the acoustic isotropic constant-density wave equation,

\[ \left( s^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2 \right) P(x, t; x_s) = f_s(t) \delta(x - x_s). \]  

(A.1)

where \( f_s(t) \) is the source signature, \( x_s \) is the source position, and \( P(x, t; x_s) \) is the pressure wavefield as a function of subsurface position \( x \) and time \( t \). Let \( s(x) \) represents the true slowness, and \( s_o(x) \) the migration slowness. If the migration slowness deviates from the true slowness, we can introduce a perturbation term, \( \Delta s(x) \), that is defined as,

\[
\begin{align*}
  s(x) &= s_o(x) + \Delta s(x), \\
  s^2(x) &\approx s_o^2(x) + 2\Delta s(x)s_o(x) = s_o^2(x) + m(x),
\end{align*}
\]

(A.2)
where I define our model, $m(x)$, as a product between the migration slowness and the perturbation slowness. Similarly, the wavefield can be broken down into a background component, $P_o(x, t; x_s)$, and a perturbed component, $\delta P(x, t; x_s)$,

$$P(x, t; x_s) = P_o(x, t; x_s) + \delta P(x, t; x_s).$$  \hfill (A.3)

$P_o(x, t)$ is the wavefield that satisfies the acoustic wave-equation with slowness, $s_o(x)$, as described in equation A.4. $\delta P(x, t; x_s)$ represents the deviation between the two wavefields.

$$\left(s_o^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2\right) P_o(x, t; x_s) = f_s(t)\delta(x - x_s).$$  \hfill (A.4)

Substituting equations A.2 and A.3 into equation A.1 yields,

$$\left(s_o^2(x) + m(x)\right) \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \left(P_o(x, t; x_s) + \delta P(x, t; x_s)\right) = f_s(t)\delta(x - x_s) \hfill (A.5)$$

Expanding equation A.1 and cancelling the terms associated with equation A.4 gives,

$$\left(s_o^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta P(x, t; x_s) = -m(x) \frac{\partial^2}{\partial t^2} (P_o(x, t; x_s) + \delta P(x, t; x_s)) \hfill (A.6)$$

The forcing term also contains the perturbed wavefields. In the Born approximation, the perturbed terms are ignored to produce equation A.7.

$$\left(s_o^2(x) \frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta P(x, x_s, t) = -m(x) \frac{\partial^2}{\partial t^2} P_o(x, x_s, t). \hfill (A.7)$$

Notice that the force term in equation A.7, $(-m(x) \frac{\partial^2}{\partial t^2} P_o(x, x_s, t))$, is a product of the source-side wavefield and the image model defined earlier. The Born approximation of the linearized acoustic wave equation, $d_{\text{born}}$, is

$$d_{\text{born}}(x_r, x_s, t) = \delta P(x, t; x_s)\delta(x - x_r), \hfill (A.8)$$

From equation A.8 and A.7, we can see that $d_{\text{born}}$ scales linearly with $m(x)$. 

The adjoint of the Born modeling operation (equation A.8) can be derived by calculating the gradient of the least-squares reverse-time migration objective function.

\[
S(m) = \frac{1}{2} \| \vec{d}_o + \vec{d}_{\text{born}} - \vec{d}_{\text{obs}} \|^2 \quad (A.9)
\]

\[
= \frac{1}{2} \| \Delta \vec{d} \|^2 \quad (A.10)
\]

where \( \vec{d}_{\text{obs}} \) is the observed data, \( \vec{d}_{\text{born}} \) is the linearized Born modeled data, and \( \Delta \vec{d} \) represents the data residual. \( \vec{d}_o \) is the background data, which is calculated by using full wave modeling based on the background slowness, \( s_o \). Next, I rewrite equation A.1 in matrix and vector notations.

\[
F \vec{u} = \vec{f} \quad (A.11)
\]

where \( F \) and \( \vec{f} \) are

\[
F = (s_o^2 \frac{d^2}{dt^2} + \nabla^2) \quad (A.12)
\]

\[
\vec{u} = F^{-1} \vec{f} \quad (A.13)
\]

The equivalent of equation A.8 in matrix-vector notations is,

\[
\vec{d}_{\text{born}} = S \vec{u}, \quad (A.14)
\]

where \( S \) is the selection operator that extracts the wavefield \( \vec{u} \) at the receiver location.
Taking the gradient of the objective function, equation A.9, gives:

\[
\nabla_m S = \langle \frac{\partial \tilde{d}_{\text{born}}}{\partial m}, \Delta \tilde{d} \rangle, \quad (A.15)
\]

\[
= \left( \frac{\partial \tilde{d}_{\text{born}}}{\partial m} \right)^* \Delta \tilde{d}, \quad (A.16)
\]

\[
= - \left( SF^{-1} \frac{\partial F}{\partial m} F^{-1} \tilde{f} \right)^* \Delta \tilde{d}, \quad (A.17)
\]

\[
= - \left( F^{-1} \tilde{f} \right)^* \left( \frac{\partial F}{\partial m} \right)^* (SF^{-1})^* \Delta \tilde{d}, \quad (A.18)
\]

\[
= - \left( F^{-1} \tilde{f} \right)^* \left( \frac{d^2}{dt^2} \right)^* (SF^{-1})^* \Delta \tilde{d}. \quad (A.19)
\]

Notice that the gradient of the objective function is exactly the reverse-time migration operation.

\[
m(x) = - \sum_{x_s, t} P_{o}^*(x, x_s, t) \left( \frac{d^2}{dt^2} \right)^* P_{r}(x, x_r, t; x_s) \quad (A.20)
\]

where \( t \) is time, \( m(x) \) represents the structural image at subsurface location \( x \), \( x_s \) represents the source location and \( x_r \) represents the receiver location. \( P_{o}(x, x_s, t) \) and \( P_{r}(x, x_r, t; x_s) \) are solutions to the two-way acoustic constant density equation.
Appendix B

Relation between LSM model and reflectivity

The goal of this appendix is to define the relationship between the output of least-squares migration (LSM) and the reflectivity for normal incidence wave. There will be three sections. The first section focuses on deriving the acoustic wave equation and the reflectivity expression between an interface with velocity and density contrast. In the second section, I derive the LSM model parameter in terms of velocity and density perturbation. By LSM model, I am referring to the parametrization that is consistent with Chapter 2. Specifically, the relationship between the Born modeled data, $d_b(x_r, x_s, t)$, and the model parameter, $m(x)$, as described by equation 2.6. Finally, the last section focuses on explaining the relationship between the LSM model parameter and the reflectivity output.
I will derive the acoustic wave equation based on two fundamental principles of physics. Suppose there is an external force acting on a volume of particles, Newton’s second law states that

\[
\frac{\text{force}}{\text{volume}} = \frac{\text{mass} \times \text{acceleration}}{\text{volume}}. \quad (B.1)
\]

Consider a simple case where a volume of material resides in a tube under pressure. The tube has a cross-sectional area \(A\) and the length of the tube in consideration is \(\Delta x\). The force applied to this volume is:

\[
\frac{\text{force}}{\text{volume}} = \frac{\text{force}}{A\Delta x} = \frac{P(x + \frac{1}{2}\Delta x) - P(x - \frac{1}{2}\Delta x)}{\Delta x},
\]

\[
= -\frac{\partial P(x)}{\partial x}, \quad (B.2)
\]

where \(P(x)\) is the pressure applied along the tube. Combining equation B.1 and B.2 yields,

\[
\rho \frac{\partial^2 u(x)}{\partial t^2} = -\frac{\partial P(x)}{\partial x}. \quad (B.3)
\]

where \(u(x)\) is the displacement of the particle at location \(x\). The second equation needed to derive the acoustic wave equation is from the Hooke’s Law. Hooke’s Law states that for small displacements, the strain is proportional to the stress.

\[
P = -\kappa \frac{\partial u}{\partial x}, \quad (B.4)
\]

where \(\kappa\) is the bulk modulus. The pressure \(P\) is the pressure variations around a background hydrostatic pressure. If I take the first derivative with respect to \(x\) on
equation B.3 and a second time derivative on equation B.4, I get the following,

\[
\frac{\partial^3 u(x)}{\partial x \partial t^2} = -\frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial P(x)}{\partial x} \right),
\]

\[
\frac{\partial^2}{\partial t^2} \left( \frac{1}{\kappa} P(x) \right) = -\frac{\partial^3 u(x)}{\partial x \partial t^2}.
\]

Equating the above two equations yields,

\[
\frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial P(x)}{\partial x} \right) = \frac{1}{\kappa} \frac{\partial^2}{\partial t^2} P(x).
\]

(B.5)

Next, I express the bulk modulus as a function of density and velocity.

\[
\kappa = v^2 \rho,
\]

\[
\frac{1}{v^2} \frac{\partial^2}{\partial t^2} P(x) = \rho \frac{\partial}{\partial x} \left( \frac{1}{\rho} \frac{\partial P(x)}{\partial x} \right).
\]

(B.6)

The derivation can be extended to 3D, which will produce the familiar acoustic variable-density wave equation,

\[
\frac{1}{v^2} \frac{\partial^2}{\partial t^2} P(x,t) = \rho \nabla \cdot \left( \frac{1}{\rho} \nabla P(x,t) \right).
\]

(B.7)

Next, I will derive the reflectivity expression in one dimension. Suppose a wave that travels in the z direction hits an interface with velocity and density contrast that goes from \(v_1\) and \(\rho_1\) to \(v_2\) and \(\rho_2\). The expressions of the pressure wave above \((P^+)\) and below \((P^-)\) the interface are,

\[
P^+(z,t) = e^{i(\omega t + k z)} + Re^{i(\omega t - k z)},
\]

\[
P^-(z,t) = Te^{i(\omega t + k z)},
\]

(B.8)

where \(R\) and \(T\) are the amplitude of the reflected and the transmitted pressure wave-field, respectively. \(k\) is the wave number. To obtain an expression for the reflectivity, we can enforce two conditions
1. Continuity of pressure across the interface.

2. Continuity of normal acceleration across the interface (equation B.3).

The first condition gives:

\[ P^+(z = 0, t) = P^-(z = 0, t), \]
\[ e^{iwt} + Re^{iwt} = Te^{iwt}, \]
\[ 1 + R = T. \]  
(B.9)

The second condition yields:

\[ a^+(z = 0, t) = a^-(z = 0, t), \]
\[ \frac{1}{\rho_1} \left( \frac{\partial P^+(z, t)}{\partial z} \right)_{z=0} = \frac{1}{\rho_2} \left( \frac{\partial P^-(z, t)}{\partial z} \right)_{z=0}, \]
\[ \frac{1}{\rho_1} (ke^{iwt} - Re^{iwt}) = \frac{1}{\rho_2} Tke^{iwt}, \]
\[ \frac{\omega}{\rho_1 v_1} (1 - R) = \frac{\omega}{\rho_2 v_2} T, \]
\[ \frac{1}{\rho_1 v_1} (1 - R) = \frac{1}{\rho_2 v_2} T. \]  
(B.10)

To obtain an expression for the reflectivity (R), substitute equation B.9 into equation B.10.

\[ \frac{1}{\rho_1 v_1} (1 - R) = \frac{1}{\rho_2 v_2} (1 + R), \]
\[ \rho_2 v_2 (1 - R) = \rho_1 v_1 (1 + R), \]
\[ \rho_2 v_2 - \rho_1 v_1 = (\rho_1 v_1 + \rho_2 v_2) R, \]
\[ R = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_1 v_1 + \rho_2 v_2}. \]  
(B.11)

**LEAST-SQUARES MIGRATION MODEL PARAMETER**

When density variation is introduced into the wave equation, the least-squares migration model parameter will be a function of velocity and density. I will first derive the
forward modeling equation by linearizing the acoustic isotropic variable-density wave
equation using perturbation theory. I will then show the relationship between the
least-squares migration output and the reflectivity model. All the derivations follow
from Zhang et al. (2014). The acoustic isotropic variable density wave-equation with
velocity $v(x)$ and density $\rho(x)$ is:

$$
\left( \frac{1}{v^2(x)} \frac{\partial^2}{\partial t^2} - \rho(x) \nabla \frac{1}{\rho(x)} \cdot \nabla \right) P(x, t; x_s) = f_s(t) \delta(x - x_s).
$$

(B.12)

where $f_s(t)$ is the source signature, $x_s$ is the source position, and $P(x, t; x_s)$ is the
pressure wavefield as a function of subsurface position $x$ and time $t$. To linearize the
wave equation, I introduce perturbation to the velocity and density as follows,

$$
v(x) = v_o(x) + \delta v(x),
$$

$$
\rho(x) = \rho_o(x) + \delta \rho(x),
$$

(B.13)

where I define $v_o(x)$ to be the background velocity and $\delta v(x)$ to be the velocity
perturbation. Similarly, $\rho_o(x)$ is the background density and $\delta \rho(x)$ is the density
perturbation. The wavefield can be broken down into a background component,
$P_o(x, t; x_s)$, and a perturbed component, $\delta P(x, t; x_s),$

$$
P(x, t; x_s) = P_o(x, t; x_s) + \delta P(x, t; x_s).
$$

(B.14)

$P_o(x, t)$ is the background wavefield that satisfies the wave-equation (equation B.12)
with the background velocity and density, $v_o(x)$ and $\rho_o(x)$. $\delta P(x, t; x_s)$ represents
the deviation between the two wavefields. I assume all perturbed quantities are
relatively small as compared to their respective background quantities. In a surface
seismic experiment, if the background velocity is smoothly varying, the background
wavefield at the receiver location, $P_o(x_r, t; x_s)$, would not record any reflection energy.

Substituting equations B.13 and B.14 into equation B.12 yields,

$$
\left( \frac{1}{(v_o + \delta v)^2} \frac{\partial^2}{\partial t^2} - (\rho_o + \delta \rho) \nabla \frac{1}{\rho_o + \delta \rho} \cdot \nabla \right) (P_o + \delta P) = f_s(t) \delta(x - x_s).
$$

(B.15)
Expanding equation B.15, removing second-order perturbation terms, and cancelling the terms associated with $P_o(x, t; x_s)$ give,

$$
\left( \frac{1}{v_o^2} \frac{\partial^2}{\partial t^2} - \rho_o \nabla \frac{1}{\rho_o} \cdot \nabla \right) \delta P = \left( \frac{2\delta v}{v_o^3} \frac{\partial^2}{\partial t^2} - \nabla \frac{\delta \rho}{\rho_o} \cdot \nabla \right) \left( P_o + \delta P \right).
$$

(B.16)

In the Born approximation, we replace the total wavefield on the right hand side with the perturbed wavefield. Equation B.16 in the frequency domain becomes,

$$
\left( \frac{-1}{v_o^2} \omega^2 - \rho_o \nabla \frac{1}{\rho_o} \cdot \nabla \right) \delta P = \left( \frac{-2\delta v}{v_o^3} \omega^2 - \nabla \frac{\delta \rho}{\rho_o} \cdot \nabla \right) P_o,
$$

(B.17)

where $\omega$ represents the temporal frequency. The perturbed wavefield $\delta P(x, \omega; x_s)$ satisfying equation B.17 can be expressed using Green’s function technique,

$$
\delta P(x, x_s, \omega) = \int \left( \frac{-2\delta v(x')}{v_o^3(x')} \omega^2 - \nabla \frac{\delta \rho(x')}{\rho_o(x')} \cdot \nabla \right) P_o(x, \omega; x_s) G(x, x'; \omega) dx',
$$

(B.18)

where $G(x, x', \omega)$ is the Green’s function of the wave equation. It is the solution to equation B.12 with an impulse source. Next, I apply integration by part onto the second term of equation B.18 to get the following,

$$
\delta P(x, x_s, \omega) = \int \left( \frac{-2\delta v(x')}{v_o^3(x')} \omega^2 P_o G + \frac{\delta \rho(x')}{\rho_o(x')} \nabla \cdot (\nabla P_o G) \right) dx'.
$$

(B.19)

To derive an expression of the model parameter for normal incidence, we can represent $P_o(x, \omega)$ and $G(x, x', \omega)$ as plane waves,

$$
P_o(x, \omega) = A_1 e^{i(k_z z - \omega t)},
$$

$$
G(x, x', \omega) = A_2 e^{i(k_z (z - z') - \omega t)}.
$$

(B.20)

where $A_1$ and $A_2$ are constants that represent the amplitude of the plane waves. Notice that the plane waves are traveling vertically, which makes equation B.20 independent from $x$ and $y$. $k_z = \frac{\omega}{v_o}$ is the vertical wavenumber. Applying a gradient to the plane
waves results in the following closed form,

\[
\nabla P_0(x, \omega) = ik_z A_1 e^{i(k_z z - \omega t)} = ik_z P_0(x, \omega),
\]

\[
\nabla G(x, x', \omega) = ik_z A_2 e^{i(k_z (z - z') - \omega t)} = ik_z G(x, x', \omega).
\]  \hspace{1cm} (B.21)

\[
\nabla \cdot (\nabla P_0 G) = (\nabla^2 P_0) G + (\nabla P_0) \cdot (\nabla G),
\]

\[
= -k_z^2 P_0 G - k_z^2 P_0 G,
\]

\[
= -2 \frac{\omega^2}{v_o^2} P_0 G.
\]  \hspace{1cm} (B.22)

Substituting equation B.22 into equation B.19, we reach the final form of the forward modeling expression.

\[
\delta P(x, x_s, \omega) = \int \frac{2}{v_o^2(x')} \left( \frac{\delta v_o(x')}{v_o(x')} + \frac{\delta \rho(x')}{\rho_o(x')} \right) \left(-\omega^2\right) P_0(x', \omega; x_s) G(x, x', \omega) dx'.
\]  \hspace{1cm} (B.23)

The Born modeled data of the acoustic isotropic variable-density wave equation, \(d_b\), is

\[
d_b(x_r, x_s, \omega) = \int \frac{2}{v_o^2(x')} \left( \frac{\delta v(x')}{v_o(x')} + \frac{\delta \rho(x')}{\rho_o(x')} \right) \left(-\omega^2\right) P_0(x', \omega; x_s) G(x, x', \omega) dx'.
\]  \hspace{1cm} (B.24)

Comparing this to the Born forward modeling equation in chapter 2, the model parameter for the case of acoustic isotropic variable-density wave equation would equate to,

\[
m(x') = \frac{2}{v_o^2(x')} \left( \frac{\delta v(x')}{v_o(x')} + \frac{\delta \rho(x')}{\rho_o(x')} \right).
\]  \hspace{1cm} (B.25)

The above equation only holds for normal incidence plane wave. It is possible to derive a more generic expression for different incidence direction. I will direct the reader to Zhang et al. (2014) for further derivation.
APPENDIX B. RELATION BETWEEN LSM MODEL AND REFLECTIVITY

CONNECTION

The reflectivity from equation B.11 can also be expressed in terms of impedance at the two interfaces.

\[
\begin{align*}
I_1 &= \rho_1 v_1, \\
I_2 &= \rho_2 v_2, \\
R &= \frac{I_2 - I_1}{I_1 + I_2}.
\end{align*}
\] (B.26)

Next, I rewrite equation B.26 as a continuous function of variable \(x'\),

\[
R(x') = \frac{\delta I(x')}{2I(x')},
\]

\[
= \frac{\delta(\rho(x')v(x'))}{2\rho(x')v(x')},
\]

\[
= \frac{(\delta\rho(x')v(x') + \rho(x')\delta v(x'))}{2\rho(x')v(x')}.
\]

\[
= \frac{1}{2} \left( \frac{\delta\rho(x')}{\rho(x')} + \frac{\delta v(x')}{v(x')} \right).
\] (B.27)

Comparing between equations B.27 and B.25, I get the following relationship between the least-squares migration model parameter and reflectivity.

\[
R(x') = \frac{1}{4} v_o^2(x') m(x'),
\]

\[
= \left( \frac{v_o(x')}{2} \right)^2 m(x').
\] (B.28)

Equation B.28 suggests that we can scale the LSM output by the square of half of the background velocity to obtain the reflectivity at normal incidence.
Bibliography


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