Appendix A

Implicit finite differencing

Assuming the S-wave velocity is much slower than the P-wave velocity, we can approximate the dispersion relationship for VTI media as follows (Shan, 2009):

$$S_z = \sqrt{\frac{1 - (1 + 2\epsilon)S_r^2}{1 - 2(\epsilon - \delta)S_r^2}}, \tag{A.1}$$

where $S_z = \frac{k_z}{\omega / v_v}$, $S_r = \frac{k_r}{\omega / v_v}$ with $k_r = \sqrt{k_x^2 + k_y^2}$, ω is the angular frequency, v_v is the vertical velocity, and ϵ and δ are the Thomsen parameters.

Shan (2009) suggests that the exact dispersion relationship A.1 can be approximated by a rational function $R_{n,m}(S_r)$:

$$R_{n,m}(S_r) = \frac{P_n(S_r)}{Q_m(S_r)}, \tag{A.2}$$

where

$$P_n(S_r) = \sum_{i=0}^{n} a_i S_r^i \tag{A.3}$$

and

$$Q_m(S_r) = \sum_{i=0}^{m} b_i S_r^i. \tag{A.4}$$

Moreover, when the polynomials in equations A.3 and A.4 are of the same degree, namely $m = n$, dispersion relationship A.2 can be further split as follows:

$$S_z = 1 - \sum_{i=1}^{n} \frac{\alpha_i S_r^2}{1 - \beta_i S_r^2}. \tag{A.5}$$

The coefficients α_i and β_i can be obtained by solving the least-square problem below:

$$\min \sum_{S_r} \left(\sqrt{\frac{1 - (1 + 2\epsilon) S_r^2}{1 - 2(\epsilon - \delta) S_r^2}} - \left(1 - \sum_{i=1}^{n} \frac{\alpha_i S_r^2}{1 - \beta_i S_r^2} \right) \right)^2 . \tag{A.6}$$

The tables for coefficients α and β for ϵ ranging from 0 to 0.2 and δ ranging from -0.004 to 0.2 are shown in Figure A.1. In general, coefficent α is more sensitive to the change in δ than to the change in ϵ. Coefficient β has similar sensitivities to both ϵ and δ.

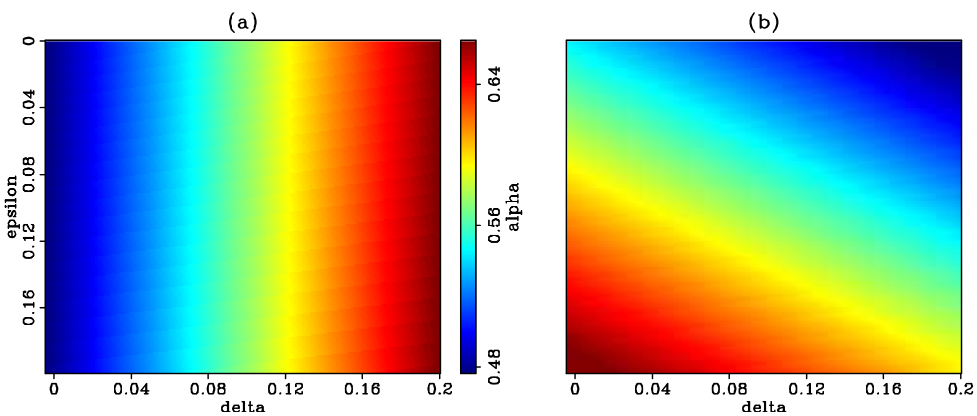

Figure A.1: (a) Table for α and (b) table for β at discrete ϵ and δ values. [**ER**] append1/. coef

In the downward extrapolation, the wavefield at the next depth (P_{z+1}) can be computed from the wavefield at the current depth (P_z) according to the following equation:

$$P_{z+1} = P_z e^{i k_z dz}, \tag{A.7}$$

where $i = \sqrt{-1}$, dz is the extrapolation distance in depth, and k_z can be obtained from the first-order approximation of the dispersion relation A.5:

$$k_z = \frac{w}{v_v}\left(1 - \frac{\alpha\frac{k_r^2}{(w/v_v)^2}}{1-\beta\frac{k_r^2}{(w/v_v)^2}}\right). \tag{A.8}$$

Dispersion relation A.8 can be further simplified to polynomials using Taylor expansion:

$$\begin{aligned} k_z &= \frac{w}{v_v}\left(1 - \alpha\frac{k_r^2}{(w/v_v)^2}\left(1+\beta\frac{k_r^2}{(w/v_v)^2}\right)\right) \\ &= \frac{w}{v_v}\left(1 - \alpha\frac{k_r^2}{(w/v_v)^2} - \alpha\beta\frac{k_r^4}{(w/v_v)^4}\right). \end{aligned} \tag{A.9}$$

Therefore, the perturbed wavefield is

$$\Delta P_{z+1} = e^{ik_z dz} i dz P_z \Delta k_z, \tag{A.10}$$

with

$$\Delta k_z = \frac{\partial k_z}{\partial v_v}\Delta v_v + \frac{\partial k_z}{\partial \epsilon}\Delta\epsilon + \frac{\partial k_z}{\partial \delta}\Delta\delta, \tag{A.11}$$

$$\frac{\partial k_z}{\partial v_v} = -\frac{w}{v_v^2}\left(1 + \alpha\frac{k_r^2}{(w/v_v)^2} + 3\alpha\beta\frac{k_r^4}{(w/v_v)^4}\right), \tag{A.12}$$

$$\frac{\partial k_z}{\partial \epsilon} = -\frac{w}{v_v}\left(\frac{\partial\alpha}{\partial\epsilon}\frac{k_r^2}{(w/v_v)^2} + \left(\frac{\partial\alpha}{\partial\epsilon}\beta + \alpha\frac{\partial\beta}{\partial\epsilon}\right)\frac{k_r^4}{(w/v_v)^4}\right), \tag{A.13}$$

and

$$\frac{\partial k_z}{\partial \delta} = -\frac{w}{v_v}\left(\frac{\partial\alpha}{\partial\delta}\frac{k_r^2}{(w/v_v)^2} + \left(\frac{\partial\alpha}{\partial\delta}\beta + \alpha\frac{\partial\beta}{\partial\delta}\right)\frac{k_r^4}{(w/v_v)^4}\right). \tag{A.14}$$

Since the finite difference parameters α and β are obtained by optimization, the derivatives in Equation A.13 and Equation A.14 are obtained numerically by taking derivatives along the ϵ and δ axis in Figure A.1. The tables of the derivatives of the coefficients with respect to the anisotropic parameters are shown in Figure A.2.

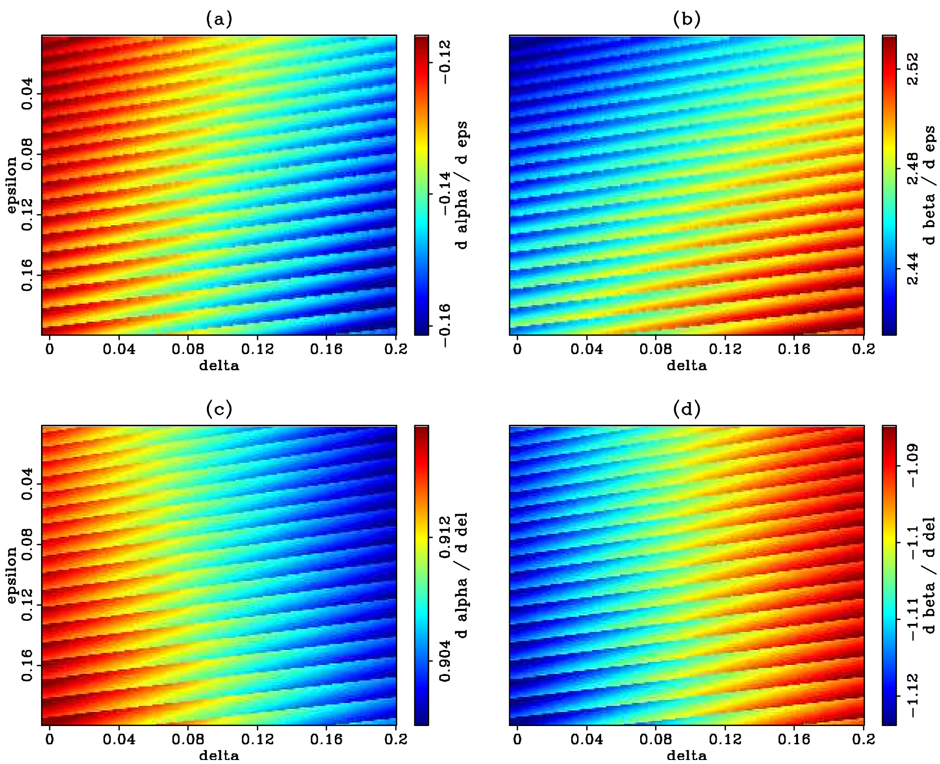

Figure A.2: Derivatives of the coefficients with respect to ϵ and δ; (a) table of $\frac{\partial\alpha}{\partial\epsilon}$; (b) table of $\frac{\partial\beta}{\partial\epsilon}$; (c) table of $\frac{\partial\alpha}{\partial\delta}$, and (d) table of $\frac{\partial\beta}{\partial\delta}$. [**ER**] append1/. dr-coef