Multiple realizations of equiprobable velocity fields using sequential Gaussian simulation

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ABSTRACT

For a given seismic dataset, there may be many equally reasonable seismic velocity models that could be used as input into imaging algorithms such as migration. Small variations in velocity models can have a substantial impact on the seismic image structure that is produced, and eventually interpreted. Here, we focus our study on building a geostatistical workflow that will later enable us to better assess the impact of velocity uncertainty on seismic image structures. Using sequential Gaussian simulation, we produce a range of equiprobable and geologically consistent realizations of RMS velocity fields.

INTRODUCTION

Quantifying uncertainty associated with seismic imaging is a key challenge for subsurface characterization. Within this process, a particular source of uncertainty arises during the estimation of seismic velocities. Errors in velocity model estimation can create and propagate errors in our final migrated images which, in turn, can lead to an erroneous estimation of our reservoir volumetrics. Therefore, finding a way to quantify uncertainty in subsurface structures can help interpreters and petroleum engineers in their decision making process.

Bube et al. (2004a) studied how the errors in stacking velocity lead to errors in the positioning of structural events for an isotropic and homogeneous medium. In an analogous paper, Bube et al. (2004b) performed a similar analysis, but for the uncertainty of anisotropic parameters in a weakly anisotropic and homogeneous medium. However, both analyses are done locally, and around a single point in the subsurface. In a series of publications, Clapp (2000, 2001a,b); Chen and Clapp (2002); Clapp (2003) developed a method to modify a least-squares geophysical inversion technique by adding random noise to both terms of the objective function (model fitting term and data fitting term). This allowed them to generate multiple realizations of equiprobable smooth RMS velocity models.

In this paper, we decide to tackle this problem from a geostatistical point of view by using sequential Gaussian simulation (SGS) to generate a set of equiprobable 2D RMS velocity fields. The reason for choosing this geostatistical approach is that it allows us to maintain spatial continuity and consistency in our model. Each realization

should be a realistic representation of the Earth. This approach lays the foundation for future studies, where we intend to use those realizations to obtain different migrated seismic images. Eventually, we want to assess the sensitivity of these migrated image structures with respect to variations in the input RMS velocity field.

We begin by remarking that estimating RMS velocity using semblance plots on common midpoint (CMP) gathers is a source of uncertainty, and that we can interpret it from a statistical point of view. Then we present the workflow we developed to generate realistic and equiprobable 2D RMS velocity profiles. Finally, we apply this workflow to a 2D marine seismic dataset acquired in the Gulf of Mexico (Claerbout, 2011), and show various realizations of velocity maps. Finally, we conclude on our results and discuss the steps going forward.

GEOSTATISTICAL INTERPRETATION OF SEMBLANCE ANALYSIS PLOTS

In this study, we focus our analysis on quantifying the risk inherent to RMS velocity (V_{rms}) estimation. A major source of uncertainty is introduced when picking V_{rms} profiles by using a semblance analysis plot. On such a plot, the zones of high semblance values have a certain width along the velocity axis (usually increasing with depth), which makes the interpreter uncertain of which velocity value to pick for a given two-way traveltime τ (Figures 1(b) and 1(c)). The consequence is that we can extract multiple and equally reasonable V_{rms} profiles from each CMP gather (Figure 1(d)). While it is common practice to choose only one V_{rms} profile among all the possible ones, and then move on with a standard processing workflow, we decide to model this uncertainty by designing an algorithm that generates geostatistically consistent 2D V_{rms} profiles. In other words, we create models that ensure realistic spatial variations from a geological standpoint. To do so, we first assume that the V_{rms} field in the Earth's subsurface was generated by a realization of a family of random variables (stochastic process). Hence, the spatial variability of the velocity value between two points in the subsurface will be fully determined by the covariance of the random variables at these two locations. We also assume that the 1D semblance function (for a given CMP gather, and a two-way traveltime), displayed in Figure 1(c), is a good approximation of a non-normalized probability density function (PDF) of the random variable that models V_{rms} at that location.

WORKFLOW

The objective of our workflow is to start from an empty V_{rms} model, and assign a geostatistically meaningful value to each location. In this section, we present the workflow used to generate one realization of a V_{rms} field. This workflow can be decomposed into three main steps:
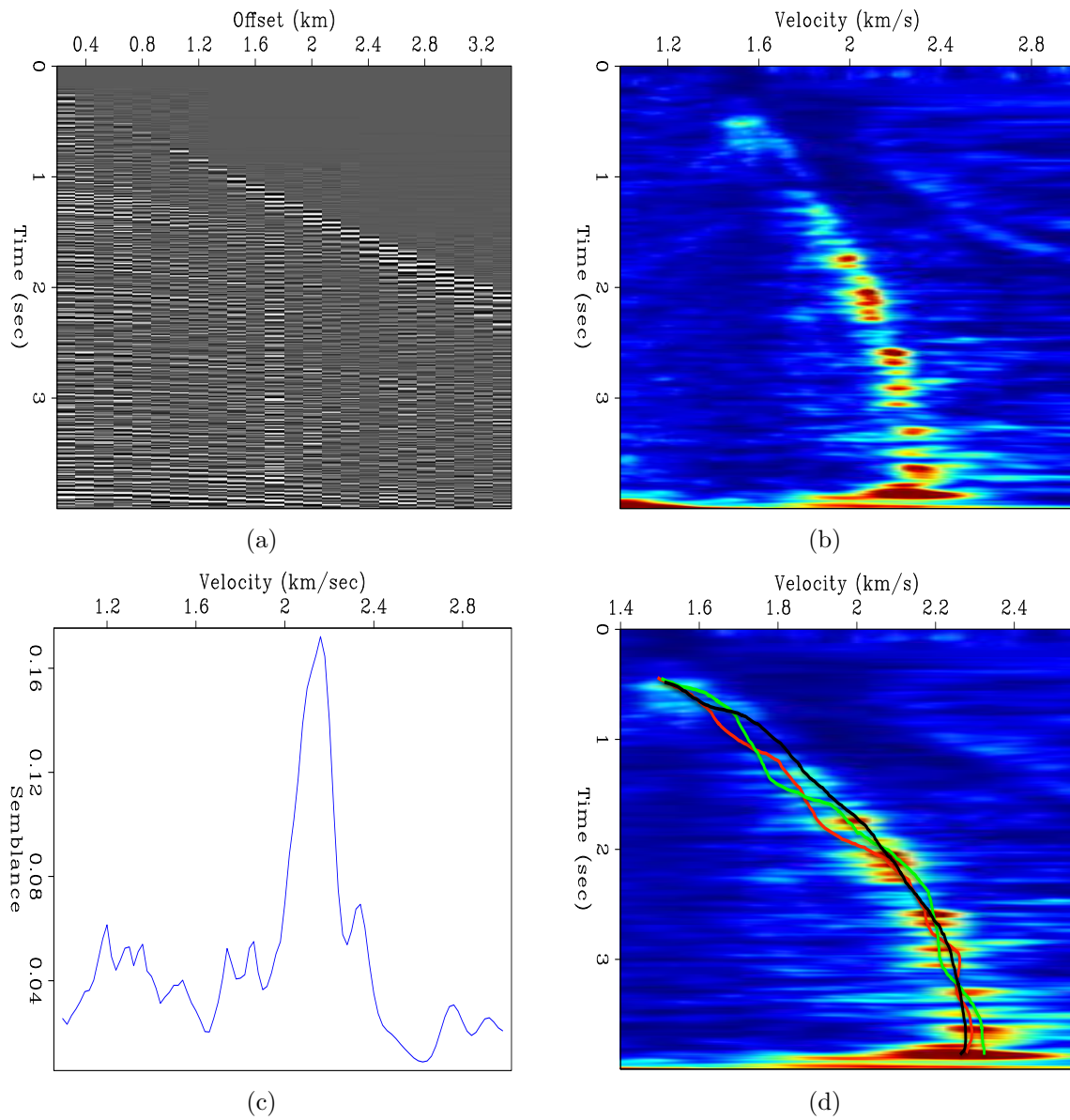
Figure 1: Uncertainty introduced by using semblance analysis plots for RMS velocity picking. (a) One CMP gather for a Gulf of Mexico marine seismic dataset. (b) Semblance analysis plot of (a). (c) 1D semblance profile for $\tau = 2.4\,sec$. (d) Equally reasonable V_{rms} profiles for the CMP gather in (a). [**NR**]

- Evaluation and extraction of the 2D V_{rms} trend as a function of traveltime and midpoints, which we refer to as "detrending"

- Generation of a 2D residuals field using SGS, which we refer to as "geostatistical simulation of residuals"

- Adding the trend back to the simulated 2D residuals field, which we refer to as "retrending"

Detrending

SGS theory requires that the variable we would like to simulate can be modeled (or explained) by a stationary random process. This implies that its mean and variance do not vary spatially. V_{rms} usually increases with depth, and therefore its mean has a trend. Moreover, semblance energy zones usually become wider as depth increases, which implies that the variance is also not stationary. In this paper, we neglect the non-stationarity of the variance, and decide to leave it for future studies. The first part of our algorithm takes care of the non-stationarity of the mean, and we use semblance analysis plots to extract the trend in the velocity. Once we have extracted this trend (a smooth 2D surface), we can move on to the second step of the workflow.

On the semblance plots, we specify a range of possible velocity values around the zone of maximum energy as a function of traveltime, referred to as the "fairway" (Figure 2(a)). For each traveltime, we pick the velocity with the highest semblance value inside the fairway, and we obtain a spiky V_{rms} profile. Figure 2(b) shows the rough 2D profile after performing this operation for all available CMPs. The 2D trend is then obtained by smoothing both along the CMP and traveltime axes (Figures 2(c) and 2(d)).

Sequential Gaussian simulation (SGS) of residuals

Assumptions

Throughout our study, we will assume that we can express the random variable $V_{rms}(x, \tau)$ as a sum of two terms: a deterministic trend $m(x, \tau)$, and a random variable with zero mean, referred to as the residuals velocity $R(x, \tau)$ (equation 1). Here, x corresponds to the position on the CMP axis, whereas τ is the two-way traveltime.

$$V_{rms}(x, \tau) = m(x, \tau) + R(x, \tau) \tag{1}$$

Step one of our workflow was intended to take care of the trend $m(x, \tau)$. In this section, we generate multiple realizations of the residuals $R(x, \tau)$ by SGS, using a
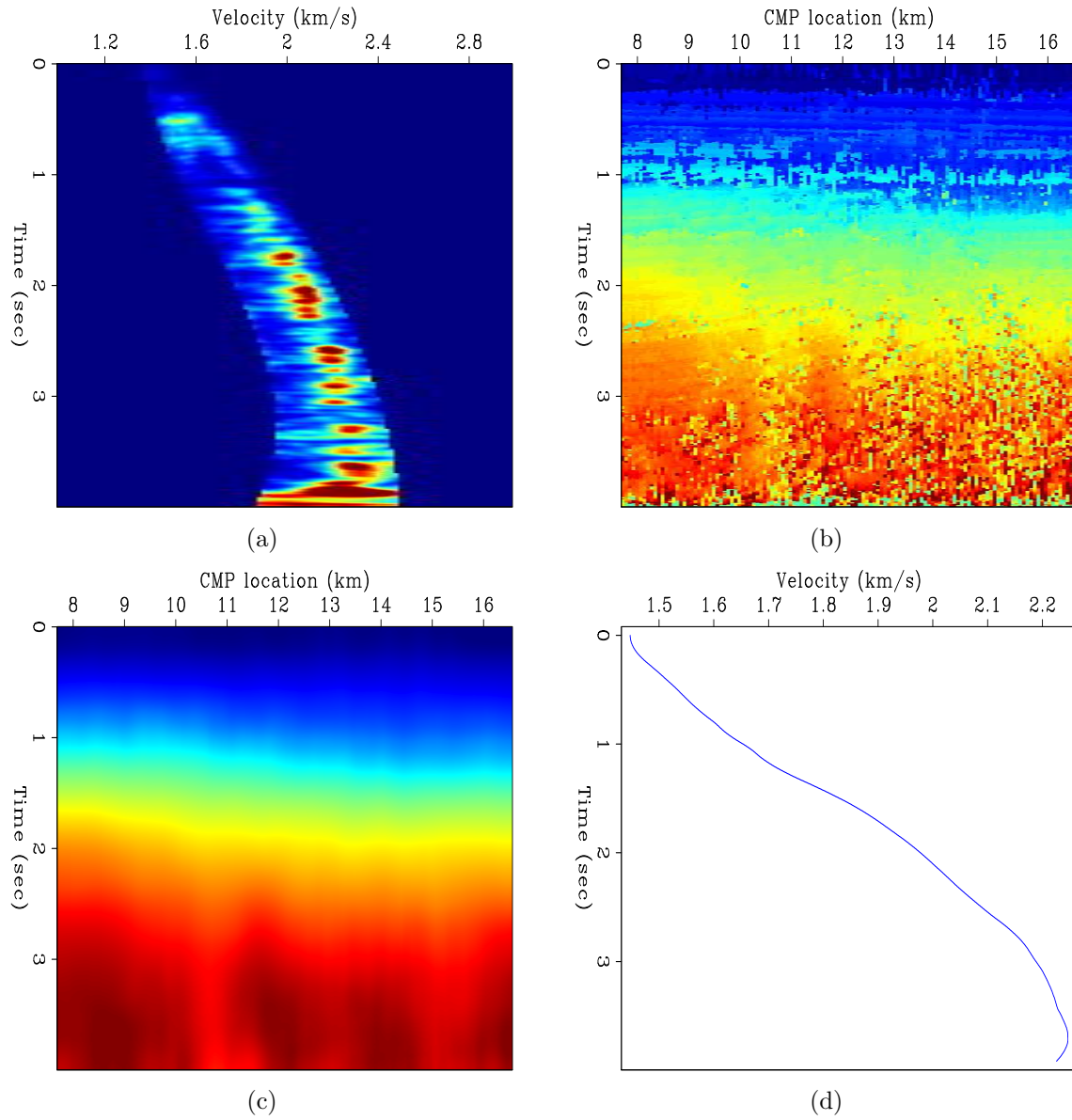
Figure 2: Various steps for velocity trend estimation. (a) Fairway for one CMP. (b) Rough 2D velocity profile. (c) Extracted 2D trend (smoothed version of (b)). (d) Trend profile for one CMP. [**ER**]

stationary random process with zero mean. In order to obtain our final velocity model, we will eventually need to add the trend back to the residuals (Isaaks and Srivastava, 1989).

Theoretical review of SGS

SGS is a stochastic simulation that enables us to generate multiple equiprobable realizations of the property we are trying to study. We use the assumption that the true Earth model is one realization of a stationary stochastic process, and we try to generate more realizations from it. This process enables us to obtain a better representation of the natural variability of the property being studied (Bohling, 2005).

Unlike geostatistical interpolation methods such as ordinary kriging (OK), we are not trying to get an estimation of the property value at the simulated point. Instead, we are aiming to obtain its conditional PDF given values of the property already simulated (or known) at locations nearby. Moreover, the assumption that each variable follows a Gaussian distribution makes the estimation of the conditional PDF straightforward. At a given point, its conditional PDF is also normally distributed, with mean the estimated kriged value (OK) at that point, and variance the error variance from the kriging process.

The SGS process can be decomposed into four steps (Bohling, 2005):

- Generating a random path through the model points

- Performing OK at each model point to estimate its conditional distribution (mean and variance)

- Drawing a random sample from this conditional PDF, and assigning the value to the model point

- Visiting each successive point in the random path and repeating this process, including previously simulated points as data values in the OK process

Pseudo-code

We start with an empty 2D residual model $R(x, \tau) = 0\ km/s$. Our goal is to generate a value for each location of the model using sequential Gaussian simulation (Mariethoz and Caers, 2014). Our pseudo-code for this workflow can be described as follows:

Realizations of residuals maps

By applying the first two steps of our workflow to a Gulf of Mexico marine seismic dataset (Claerbout, 2011), we obtain and display three realizations of residuals shown

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Algorithm 1 Sequential Gaussian simulation of residuals

    Initialize the model to zero: $\forall\, (x, \tau),\ R(x, \tau) = 0$
    while at least one point in the model has not been assigned a value **do**
        pick a random model location $M_i(x_i, \tau_i)$
        if the value at M_i has already been simulated **then**
            pick another location
        end if
        scan in the vicinity of M_i to see if there are nearby points for which we already know the value
        if there are not enough nearby points that have already been simulated **then**
            get the PDF for location M_i (estimated from the semblance analysis)
            draw a random sample value $r_i{}^{rand}$ from this PDF
            assign this sample value at location M_i. $R(M_i) = r_i{}^{rand}$
        else
            pick the nearby points, and interpolate between them by OK
            store the kriged value $R_{kriged}(M_i)$
            calculate the minimum error variance σ^2_{min} associated with the OK process
            draw a random sample $r_i{}^{norm}$ from a normal distribution $N(R_{kriged}, \sigma^2_{min})$
            assign this sample value at location M_i. $R(M_i) = r_i{}^{norm}$
        end if
    end while

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in Figures 3(a), 3(b), and 3(c). Each of these three realizations was performed using model correlograms with different ranges for the OK process (Figure 4(a)). As we increase the range in the correlograms, we observe more spatial continuity in residuals realizations. Figure 4(b) confirms this observation, and shows that our simulation has preserved the range of the original model correlogram. This is the main advantage of tackling this problem with a geostatistical approach.

Generating V_{rms} models by retrending

In the final step of our workflow, we add the trend back to the residuals to obtain the velocity models. Figures 5(a), 5(b), and 5(c) show the retrended velocity models obtained for the realizations of residuals in Figures 3(a), 3(b), and 3(c). By extracting V_{rms} profiles at $x = 15.75\,km$, we can now display the three realizations overlaid on top of the semblance analysis plot for this CMP (Figure 6). We have generated a set of equiprobable 2D RMS velocity fields, where each realization is spatially consistent with the input covariance model.
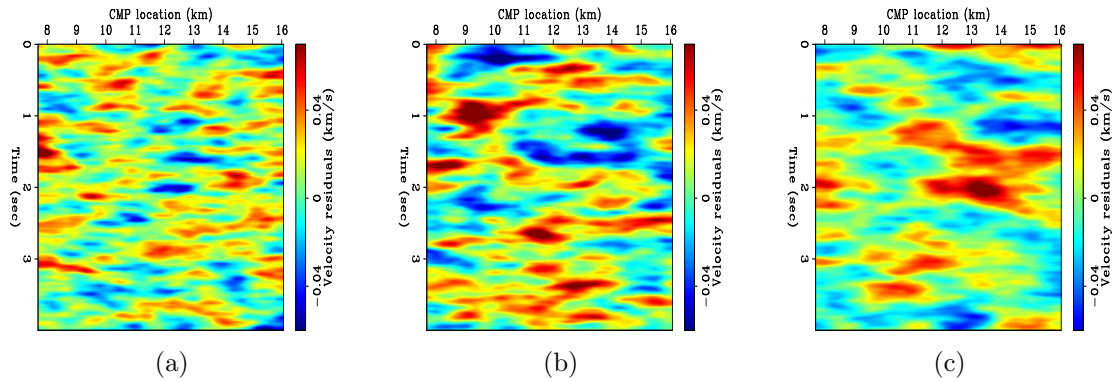
Figure 3: Multiple residuals realizations using correlograms with different ranges. (a) Realization with a range of 20 pixels. (b) Realization with a range of 40 pixels. (c) Realization with a range of 60 pixels. [**ER**]
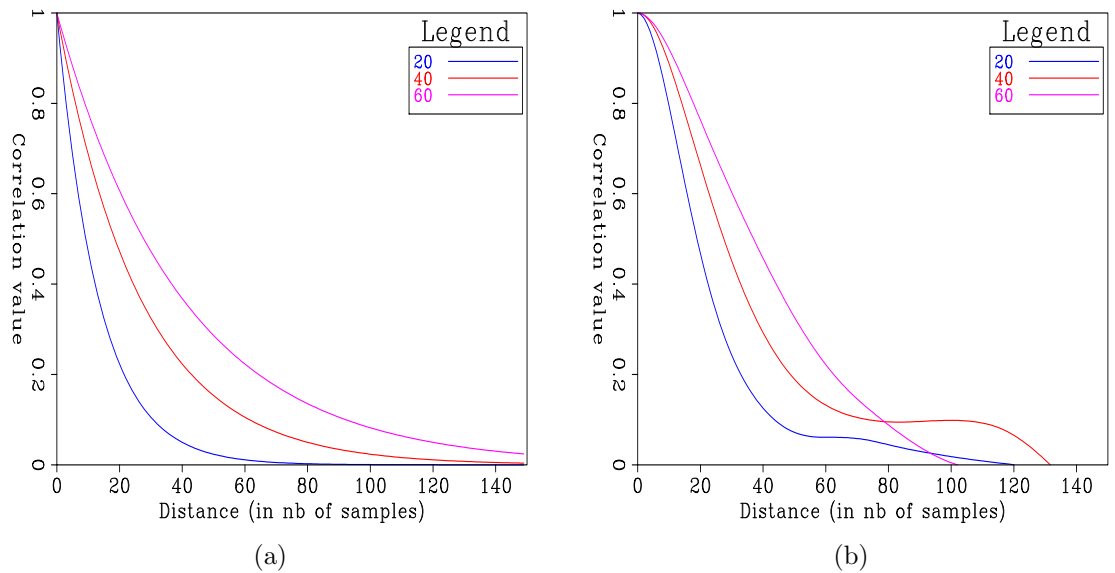

Figure 4: Correlograms used in the modeling process. (a) Model correlograms with various ranges used for the sequential Gaussian simulation. (b) Residuals correlograms obtained after simulations using various ranges. [**ER**]
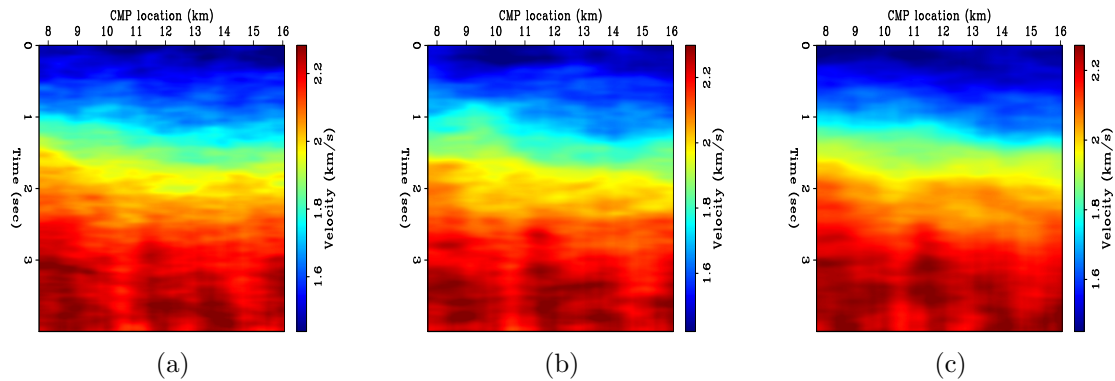
Figure 5: Multiple realizations of RMS velocity models. (a) Realization with a range of 20 pixels. (b) Realization with a range of 40 pixels. (c) Realization with a range of 60 pixels. [**ER**]

CONCLUSIONS AND FUTURE WORK

In this paper, we developed a geostatistical workflow that enables us to generate multiple realizations of spatially consistent RMS velocity fields. First, we extracted a velocity trend from our data. Then, we performed a sequential Gaussian simulation on the residuals. Finally, by adding back the trend to the residuals realizations, we obtained a set of reasonable and equiprobable RMS velocity fields. So far, we have limited our work to the construction of this geostatistical workflow. Moving forward, we would like to use each velocity model realization to obtain a wide range of equiprobable migrated seismic images. By comparing the differences in their structural features, we hope to be able to assess their sensitivity with respect to variations in the RMS velocity model.

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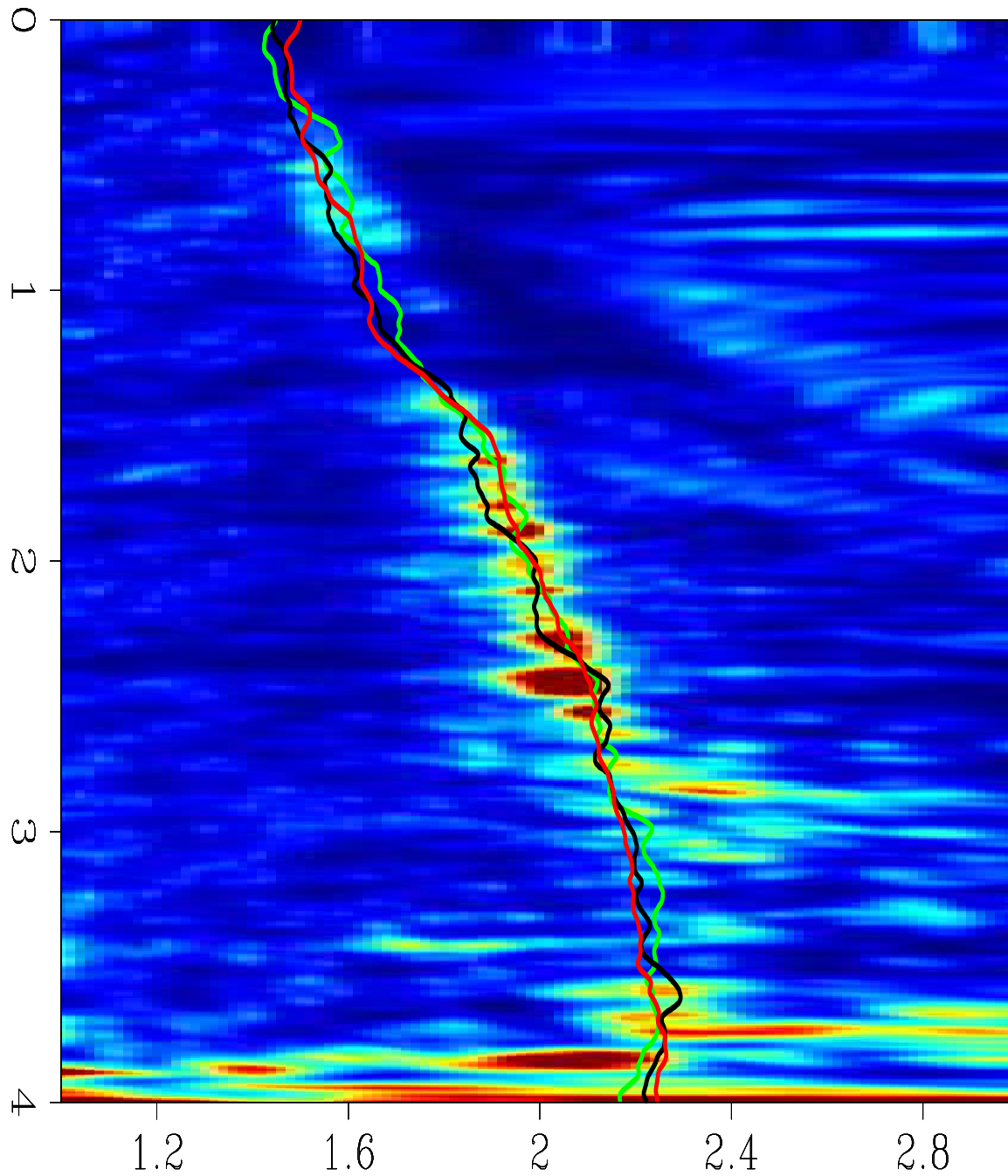———, 2001b, Multiple realizations: Model variance and data uncertainty: 147–158.

Figure 6: V_{rms} profiles for three realizations (using three different ranges), extracted at $x = 15.75\,km$. Black curve was generated with a range of 20 pixels, gray curve with a range of 40 pixels, and red curve with a range of 60 pixels. Background is the semblance analysis plot for this location. [**ER**]

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