Appendix A

Elastodynamic OBC seismic interferometry

The following discussion follows and summarizes Wapenaar and Fokkema (2004, 2010); Wapenaar et al. (2006) and Wapenaar (2007) with the intention of specifying the equations for seismic interferometry in the particular case of buried ocean-bottom cable (OBC) recordings.

OBC GREEN'S FUNCTION REPRESENTATIONS

The field quantities capturing elastodynamic wave propagation in an anisotropic, dissipative medium are the three components of particle velocity, υ_1, υ_2 and υ_3, and the six unique components of the stress-tensor $-\tau_{11}$, $-\tau_{22}$, $-\tau_{33}$, $-\tau_{23}$, $-\tau_{31}$ and $-\tau_{12}$. Each field quantity has a corresponding source quantity, the three components of external volume force density f_1, f_2 and f_3, and the six unique elements of the external deformation rate tensor, h_{11}, h_{22}, h_{33}, h_{23}, h_{31} and h_{12}. The medium properties of an anisotropic, dissipative elastic medium are described by the space dependent density $\rho = \rho(\mathbf{x})$ and the space- and frequency-dependent compliance tensor $s_{ijkl} = s_{jikl} = s_{ijlk} = s_{klij}$. The compliance tensor is related to the stiffness tensor through $c_{ijkl}s_{klmn} = s_{ijkl}c_{klmn} = \frac{1}{2}\left(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}\right)$ (de Hoop, 1995).

The elastodynamic Green's matrix, $\mathbf{G} = \mathbf{G}(\mathbf{x}_r, \mathbf{x}_s, \omega)$, is organized by rows that contain recordings at $\mathbf{x}_r$ in each of the field quantities, ordered as ($\upsilon_1,\ \upsilon_2,\ \upsilon_3,\ -\tau_{11},\ -\tau_{22},\ -\tau_{33},\ -\tau_{23},\ -\tau_{31},\ -\tau_{12}$), and organized by columns that contain the sources at $\mathbf{x}_s$, ordered in the columns as ($f_1,\ f_2,\ f_3,\ h_{11},\ h_{22},\ h_{33},\ 2h_{23},\ 2h_{31},\ 2h_{12}$). Representation theorems for the Green's functions can be derived from energy considerations following the Gauss divergence theorem. One representation that is particularly useful for seismic interferometry is the correlation-type reciprocity theorem. For the elastodynamic Green's matrix in a non-flowing, dissipative and inhomogeneous medium $\mathcal{D}$ with boundary $\partial\mathcal{D}$, we have[1]:

$$\mathbf{G}^\dagger(\mathbf{x}_B, \mathbf{x}_A, \omega) + \mathbf{G}(\mathbf{x}_A, \mathbf{x}_B, \omega) = i\omega \int_{\mathcal{D}} \mathbf{G}^\dagger(\mathbf{x}, \mathbf{x}_A, \omega) \left[\mathbf{A} - \mathbf{A}^\dagger\right] \mathbf{G}(\mathbf{x}, \mathbf{x}_B, \omega)\ d^3\mathbf{x} + \oint_{\partial\mathcal{D}} \mathbf{G}^\dagger(\mathbf{x}, \mathbf{x}_A, \omega)\, \mathbf{N}\, \mathbf{G}(\mathbf{x}, \mathbf{x}_B, \omega)\ d^2\mathbf{x}, \tag{A.1}$$

where $\dagger$ denotes complex conjugation and matrix transposition, i.e. $\{\}^\dagger = \{\{\}^*\}^T$. The Matrix $\mathbf{N} = \mathbf{N}(\mathbf{x})$ contains the elements of the normal vector, $\mathbf{n} = (n_1, n_2, n_3)$, to the domain boundary arranged as

$$\mathbf{N} = \begin{pmatrix} \mathbf{O} & \mathbf{N}_1 & \mathbf{N}_1 \\ \mathbf{N}_1 & \mathbf{O} & \mathbf{O} \\ \mathbf{N}_2 & \mathbf{O} & \mathbf{O} \end{pmatrix}, \tag{A.2}$$

where

$$\mathbf{N}_1 = \begin{pmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{pmatrix} \quad \text{and} \quad \mathbf{N}_2 = \begin{pmatrix} 0 & n_3 & n_2 \\ n_3 & 0 & n_1 \\ n_2 & n_1 & 0 \end{pmatrix},$$

where $\mathbf{O}$ are appropriately sized matrices containing zeros only. The matrix $\mathbf{A} =$

[1]This equation compares to Equation 14 in Wapenaar and Fokkema (2004) but expanded for the full Green's matrix and with the medium parameters in their state A and B set equal.

$\mathbf{A}(\mathbf{x},\omega)$ contains the medium parameters organized as:

$$\mathbf{A} = \begin{pmatrix} \rho\mathbf{I} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{s}_{11} & 2\mathbf{s}_{12} \\ \mathbf{O} & 2\mathbf{s}_{21} & 4\mathbf{s}_{22} \end{pmatrix}, \tag{A.3}$$

where $\mathbf{s}_{11}$, $\mathbf{s}_{12}$, $\mathbf{s}_{21}$ and $\mathbf{s}_{22}$ contain the elements of the space- and frequency-dependent compliance tensor:

$$\mathbf{s}_{11} = \begin{pmatrix} s_{1111} & s_{1122} & s_{1133} \\ s_{1122} & s_{2222} & s_{2233} \\ s_{1133} & s_{2233} & s_{3333} \end{pmatrix}, \quad \mathbf{s}_{12} = \begin{pmatrix} s_{1123} & s_{1131} & s_{1112} \\ s_{2223} & s_{2231} & s_{2212} \\ s_{3323} & s_{3331} & s_{3312} \end{pmatrix},$$

$$\mathbf{s}_{21} = \mathbf{s}_{12}^T \qquad \text{and} \qquad \mathbf{s}_{22} = \begin{pmatrix} s_{2323} & s_{2331} & s_{2312} \\ s_{2331} & s_{3131} & s_{3112} \\ s_{2312} & s_{3112} & s_{1212} \end{pmatrix}.$$

I consider the special case of marine seismic where the solid subsurface is overlain by a homogeneous (isotropic) water layer that is bounded by a free surface on the upper-domain boundary. (Figure A.1). Water has zero shear modulus ($\mu = 0$) and as a consequence cannot sustain shear-stresses ($\tau_{23} = 0$, $\tau_{31} = 0$ and $\tau_{12} = 0$). The space- and frequency-dependent compliance tensor reduces to $s_{ijkl} = \delta_{ij}\delta_{kl}\ \kappa$, where κ is the compressibility. In water, the domain integral over an elastic system reduces to an integral over an acoustic system. The shear tractions are zero on the solid-fluid interface however the vertical component of particle velocity is continuous. Additionally, the normal traction is continuous with negative the pressure in water. The normal traction (pressure) in water is zero on the free surface. The field quantities in water are pressure, p, and the three components of particle velocity ($\upsilon_1,\ \upsilon_2,\ \upsilon_3$). The corresponding source quantities in water are volume injection-rate density source, q, and the three components of external volume force density ($f_1,\ f_2,\ f_3$). Thus the full representation reduces to

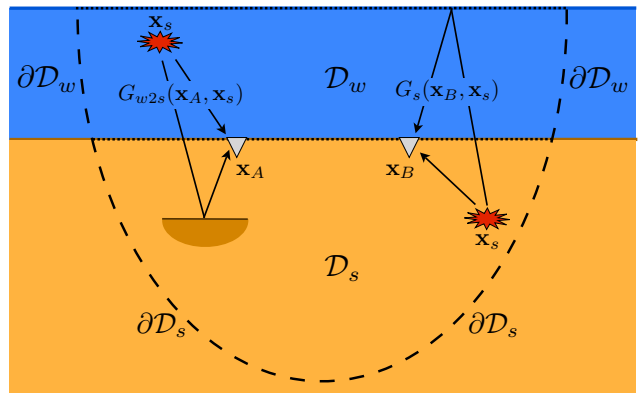

Figure A.1: Setting and variable definitions of buried OBC Green's function representation in a marine setting. The domain enclosing two stations, at $\mathbf{x}_A$ and $\mathbf{x}_B$, consist a portion in water $\mathcal{D}_w$ and a portion in solid $\mathcal{D}_s$. The domain boundary consists of a boundary in water, $\partial\mathcal{D}_w$, and a boundary in the solid subsurface, $\partial\mathcal{D}_s$. Green's functions G_s and G_{w2s} are Green's functions for the full elastodynamic response measured in the solid of a source in respectively solid or water. [**NR**] marinesetting

$$\begin{aligned}\mathbf{G}_s^\dagger(\mathbf{x}_B, \mathbf{x}_A, \omega) + \mathbf{G}_s(\mathbf{x}_A, \mathbf{x}_B, \omega) = i\omega \int_{\mathcal{D}_s} \mathbf{G}_s^\dagger(\mathbf{x}, \mathbf{x}_A, \omega) \left[\mathbf{A}_s - \mathbf{A}_s^\dagger\right] \mathbf{G}_s(\mathbf{x}, \mathbf{x}_B, \omega)\ d^3\mathbf{x} \\ + i\omega \int_{\mathcal{D}_w} \mathbf{G}_{s2w}^\dagger(\mathbf{x}, \mathbf{x}_A, \omega) \left[\mathbf{A}_w - \mathbf{A}_w^\dagger\right] \mathbf{G}_{s2w}(\mathbf{x}, \mathbf{x}_B, \omega)\ d^3\mathbf{x} \\ + \oint_{\partial\mathcal{D}_s} \mathbf{G}_s^\dagger(\mathbf{x}, \mathbf{x}_A, \omega)\, \mathbf{N}_s\, \mathbf{G}_s(\mathbf{x}, \mathbf{x}_B, \omega)\ d^2\mathbf{x} \\ + \oint_{\partial\mathcal{D}_w} \mathbf{G}_{s2w}^\dagger(\mathbf{x}, \mathbf{x}_A, \omega)\, \mathbf{N}_w\, \mathbf{G}_{s2w}(\mathbf{x}, \mathbf{x}_B, \omega)\ d^2\mathbf{x}. \qquad \text{(A.4)}\end{aligned}$$

In this expression $\mathbf{G}_s$, $\mathbf{A}_s$ and $\mathbf{N}_s$ are defined as as $\mathbf{G}$, $\mathbf{A}$ and $\mathbf{N}$ before. The Green's matrix, $\mathbf{G}_{s2w} = \mathbf{G}_{s2w}(\mathbf{x}_r, \mathbf{x}_s, \omega)$, is organized by rows that contain recordings at $\mathbf{x}_r$ in each of the field quantities in water, ordered as $(\upsilon_1,\ \upsilon_2,\ \upsilon_3,\ p)$, and organized by columns that contain the sources in a solid at $\mathbf{x}_s$, ordered in the columns as $(f_1,\ f_2,\ f_3,\ h_{11},\ h_{22},\ h_{33},\ 2h_{23},\ 2h_{31},\ 2h_{12})$. The matrix $\mathbf{A}_w = \mathbf{A}_w(\mathbf{x}, \omega)$ contains

the medium parameters in water organized as:

$$\mathbf{A}_w = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & \kappa \end{pmatrix} \quad \text{and} \quad \mathbf{N}_w = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}. \tag{A.5}$$

Dissipation can be accounted for by complex-valued compliance and compressibility. Equation A.4 is a representation for the Green's matrix $\mathbf{G}(\mathbf{x}_A, \mathbf{x}_B, \omega)$ and the time reverse of its reciprocal $\mathbf{G}^\dagger(\mathbf{x}_B, \mathbf{x}_A, \omega)$ in terms of crosscorrelations of source responses recorded throughout a domain and on its boundary of sources at $\mathbf{x}_A$ and $\mathbf{x}_B$.

The Green's matrix from solid to water, $\mathbf{G}_{s2w}$, has a reciprocal Green's matrix from water to solid, $\mathbf{G}_{w2s}$, which is organized by rows that contain recordings at $\mathbf{x}_r$ in each of the field quantities in solid, ordered as $(\upsilon_1,\ \upsilon_2,\ \upsilon_3,\ -\tau_{11},\ -\tau_{22},\ -\tau_{33},\ -\tau_{23},\ -\tau_{31},\ -\tau_{12})$, and organized by columns that contain the sources in water at $\mathbf{x}_s$, ordered in the columns as $(f_1,\ f_2,\ f_3,\ q)$. To arrive at a Green's matrix representation useful for seismic interferometry, we apply source-receiver reciprocity of the Green's matrices and transpose both sides of the equation[2,3]:

$$\begin{aligned} \mathbf{G}_s(\mathbf{x}_B, \mathbf{x}_A, \omega) + \mathbf{G}_s^\dagger(\mathbf{x}_A, \mathbf{x}_B, \omega) = i\omega \int_{\mathcal{D}_s} & \mathbf{G}_s(\mathbf{x}_B, \mathbf{x}, \omega) \left[\mathbf{A}_s - \mathbf{A}_s^\dagger\right] \mathbf{G}_s^\dagger(\mathbf{x}_A, \mathbf{x}, \omega)\ d^3\mathbf{x} \\ + i\omega \int_{\mathcal{D}_w} & \mathbf{G}_{w2s}(\mathbf{x}_B, \mathbf{x}, \omega) \left[\mathbf{A}_w - \mathbf{A}_w^\dagger\right] \mathbf{G}_{w2s}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega)\ d^3\mathbf{x} \\ - \oint_{\partial\mathcal{D}_s} & \mathbf{G}_s(\mathbf{x}_B, \mathbf{x}, \omega)\, \mathbf{N}_s\, \mathbf{G}_s^\dagger(\mathbf{x}_A, \mathbf{x}, \omega)\ d^2\mathbf{x} \\ - \oint_{\partial\mathcal{D}_w} & \mathbf{G}_{w2s}(\mathbf{x}_B, \mathbf{x}, \omega)\, \mathbf{N}_w\, \mathbf{G}_{w2s}^\dagger(\mathbf{x}_A, \mathbf{x}, \omega)\ d^2\mathbf{x}. \end{aligned} \tag{A.6}$$

[2]From a convolution-type reciprocity theorem it can be derived that the Green's matrices, $\mathbf{G}_s$ and $\mathbf{G}_{s2w}$, obey the reciprocity relations $\mathbf{G}_s(\mathbf{x}_A, \mathbf{x}_B, \omega) = \mathbf{J}_s \mathbf{G}_s^T(\mathbf{x}_B, \mathbf{x}_A, \omega)\mathbf{J}_s$ and $\mathbf{G}_{s2w}(\mathbf{x}_A, \mathbf{x}_B, \omega) = \mathbf{J}_w \mathbf{G}_{w2s}^T(\mathbf{x}_B, \mathbf{x}_A, \omega)\mathbf{J}_s$, where $\mathbf{J}_s = \mathrm{diag}\,(1,\ 1,\ 1,\ -1,\ -1,\ -1,\ -1,\ -1,\ -1,\ -1,\ -1,\ -1)$ and $\mathbf{J}_w = \mathrm{diag}\,(1,\ 1,\ 1,\ -1)$, thus $\mathbf{J}_{s,w} = \mathbf{J}_{s,w}^T = \mathbf{J}_{s,w}^\dagger$. The medium parameter matrices, $\mathbf{A}_s$ and $\mathbf{A}_w$, obey $\mathbf{A}_{s,w} = \mathbf{J}_{s,w}\mathbf{A}_{s,w}^T\mathbf{J}_{s,w} = \mathbf{A}_{s,w}^T$, and the matrices, $\mathbf{N}_s$ and $\mathbf{N}_w$ obey $\mathbf{N}_{s,w} = -\mathbf{J}_{s,w}\mathbf{N}_{s,w}^T\mathbf{J}_{s,w} = \mathbf{N}_{s,w}^T$. This result compares to Equation 5 in Wapenaar et al. (2006).

[3]This equation compares to Equation 7 in Wapenaar et al. (2006) but with the medium parameters in state A and B set equal.

Equation A.6 is a representation for the Green's matrix $\mathbf{G}_s(\mathbf{x}_B, \mathbf{x}_A, \omega)$ and the time-reverse of its reciprocal $\mathbf{G}_s^\dagger(\mathbf{x}_A, \mathbf{x}_B, \omega)$ in terms of crosscorrelations of observed responses at $\mathbf{x}_A$ and $\mathbf{x}_B$ to sources distributed throughout the domain and on its boundary.

In Chapters 3, 4, 5 and 7 I study recordings of OBC geophones that are installed over the Valhall field to measure particle velocity just below the sea floor. Extracting the terms that correspond to the i-th and j-th components of particle velocity recordings at $\mathbf{x}_A$ and $\mathbf{x}_B$ from Equation A.6 we find

$$\begin{aligned} G_{(i,j)}^{(v,f)}(\mathbf{x}_B, \mathbf{x}_A, \omega) + G_{(j,i)}^{(v,f)*}(\mathbf{x}_A, \mathbf{x}_B, \omega) = \\ - \oint_{\partial \mathcal{D}_s} \mathbf{G}_{s,(i)}^{(v)}(\mathbf{x}_B, \mathbf{x}, \omega)\, \mathbf{N}_s\, \mathbf{G}_{s,(j)}^{(v)\dagger}(\mathbf{x}_A, \mathbf{x}, \omega)\; d^2\mathbf{x} \\ - \oint_{\partial \mathcal{D}_{w2s}} \mathbf{G}_{w,(i)}^{(v)}(\mathbf{x}_B, \mathbf{x}, \omega)\, \mathbf{N}_w\, \mathbf{G}_{w,(j)}^{(v)\dagger}(\mathbf{x}_A, \mathbf{x}, \omega)\; d^2\mathbf{x} \\ + i\omega \int_{\mathcal{D}_s} \mathbf{G}_{s,(i)}^{(v)}(\mathbf{x}_B, \mathbf{x}, \omega) \left[\mathbf{A}_s - \mathbf{A}_s^\dagger\right] \mathbf{G}_{s,(j)}^{(v)\dagger}(\mathbf{x}_A, \mathbf{x}, \omega)\; d^3\mathbf{x} \\ + i\omega \int_{\mathcal{D}_w} \mathbf{G}_{w2s,(i)}^{(v)}(\mathbf{x}_B, \mathbf{x}, \omega) \left[\mathbf{A}_w - \mathbf{A}_w^\dagger\right] \mathbf{G}_{w2s,(j)}^{(v)\dagger}(\mathbf{x}_A, \mathbf{x}, \omega)\; d^3\mathbf{x}, \end{aligned} \tag{A.7}$$

where $\mathbf{G}_{s,(i)}^{(v)} = \begin{pmatrix} G_{(i,1)}^{(v,f)} & G_{(i,2)}^{(v,f)} & G_{(i,3)}^{(v,f)} & G_{(i,11)}^{(v,h)} & G_{(i,22)}^{(v,h)} & G_{(i,33)}^{(v,h)} & G_{(i,23)}^{(v,2h)} & G_{(i,31)}^{(v,2h)} & G_{(i,12)}^{(v,2h)} \end{pmatrix}$ and $\mathbf{G}_{w2s,(i)}^{(v)} = \begin{pmatrix} G_{(i,1)}^{(v,f)} & G_{(i,2)}^{(v,f)} & G_{(i,3)}^{(v,f)} & G_{(i)}^{(v,q)} \end{pmatrix}$.

In Chapters 6 and 7 I study recordings of OBC sensors installed over the Ekofisk field that measure the pressure[4] just below the sea floor, i.e. $-\frac{1}{3}(\tau_{11} + \tau_{22} + \tau_{33})$. To extract the relevant terms from the Green's function representation, I left multiply Equation A.6 with $\mathbf{K} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix}$ and I right multiply Equation A.1 with $\mathbf{K}^T$. This constructs a composite-receiver recording pressure, $\bar{p} = -\frac{1}{3}(\tau_{11} + \tau_{22} + \tau_{33})$, and a corresponding composite-source of volume injection type, $\bar{q} = \frac{1}{3}(h_{11} + h_{22} + h_{33})$. This leads to

[4]Pressure in an elastodynamic system means the sum of the trace of the stress tensor; a composite field denoted with a bar.

$$
\begin{aligned}
G^{(\bar{p},\bar{q})}(\mathbf{x}_B,\mathbf{x}_A,\omega) + G^{(\bar{p},\bar{q})*}(\mathbf{x}_A,\mathbf{x}_B,\omega) = \\
- \oint_{\partial\mathcal{D}_s} \mathbf{G}_s^{(\bar{p})}(\mathbf{x}_B,\mathbf{x},\omega)\,\mathbf{N}_s\,\mathbf{G}_s^{(\bar{p})\dagger}(\mathbf{x}_A,\mathbf{x},\omega)\;d^2\mathbf{x} \\
- \oint_{\partial\mathcal{D}_w} \mathbf{G}_{w2s}^{\bar{p}}(\mathbf{x}_B,\mathbf{x},\omega)\,\mathbf{N}_w\,\mathbf{G}_{w2s}^{(\bar{p})\dagger}(\mathbf{x}_A,\mathbf{x},\omega)\;d^2\mathbf{x} \\
+ i\omega\int_{\mathcal{D}_s} \mathbf{G}_s^{(\bar{p})}(\mathbf{x}_B,\mathbf{x},\omega)\left[\mathbf{A}_s - \mathbf{A}_s^{\dagger}\right]\mathbf{G}_s^{(\bar{p})\dagger}(\mathbf{x}_A,\mathbf{x},\omega)\;d^3\mathbf{x} \\
+ i\omega\int_{\mathcal{D}_w} \mathbf{G}_{w2s}^{(\bar{p})}(\mathbf{x}_B,\mathbf{x},\omega)\left[\mathbf{A}_w - \mathbf{A}_w^{\dagger}\right]\mathbf{G}_{w2s}^{(\bar{p})\dagger}(\mathbf{x}_A,\mathbf{x},\omega)\;d^3\mathbf{x},
\end{aligned}
\tag{A.8}
$$

where

$$
\mathbf{G}_s^{(\bar{p})T} = \begin{pmatrix} G_{(1)}^{(\bar{p},f)} \\ G_{(2)}^{(\bar{p},f)} \\ G_{(3)}^{(\bar{p},f)} \\ G_{(11)}^{(\bar{p},h)} \\ G_{(22)}^{(\bar{p},h)} \\ G_{(33)}^{(\bar{p},h)} \\ G_{(23)}^{(\bar{p},2h)} \\ G_{(31)}^{(\bar{p},2h)} \\ G_{(12)}^{(\bar{p},2h)} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} G_{(11,1)}^{(-\tau,f)} + G_{(22,1)}^{(-\tau,f)} + G_{(33,1)}^{(-\tau,f)} \\ G_{(11,2)}^{(-\tau,f)} + G_{(22,2)}^{(-\tau,f)} + G_{(33,2)}^{(-\tau,f)} \\ G_{(11,3)}^{(-\tau,f)} + G_{(22,3)}^{(-\tau,f)} + G_{(33,3)}^{(-\tau,f)} \\ G_{(11,11)}^{(-\tau,h)} + G_{(22,11)}^{(-\tau,h)} + G_{(33,11)}^{(-\tau,h)} \\ G_{(11,22)}^{(-\tau,h)} + G_{(22,22)}^{(-\tau,h)} + G_{(33,22)}^{(-\tau,h)} \\ G_{(11,33)}^{(-\tau,h)} + G_{(22,33)}^{(-\tau,h)} + G_{(33,33)}^{(-\tau,h)} \\ G_{(11,23)}^{(-\tau,2h)} + G_{(22,23)}^{(-\tau,2h)} + G_{(33,23)}^{(-\tau,2h)} \\ G_{(11,31)}^{(-\tau,2h)} + G_{(22,31)}^{(-\tau,2h)} + G_{(33,31)}^{(-\tau,2h)} \\ G_{(11,12)}^{(-\tau,2h)} + G_{(22,12)}^{(-\tau,2h)} + G_{(33,12)}^{(-\tau,2h)} \end{pmatrix}
\tag{A.9}
$$

and

$$
\mathbf{G}_{w2s}^{(\bar{p})T} = \begin{pmatrix} G_{(1)}^{(\bar{p},f)} \\ G_{(2)}^{(\bar{p},f)} \\ G_{(3)}^{(\bar{p},f)} \\ G^{(\bar{p},q)} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} G_{(11,1)}^{(-\tau,f)} + G_{(22,1)}^{(-\tau,f)} + G_{(33,1)}^{(-\tau,f)} \\ G_{(11,2)}^{(-\tau,f)} + G_{(22,2)}^{(-\tau,f)} + G_{(33,2)}^{(-\tau,f)} \\ G_{(11,3)}^{(-\tau,f)} + G_{(22,3)}^{(-\tau,f)} + G_{(33,3)}^{(-\tau,f)} \\ G_{(11)}^{(-\tau,q)} + G_{(22)}^{(-\tau,q} + G_{(33)}^{(-\tau,q)} \end{pmatrix}.
\tag{A.10}
$$

UNCORRELATED NOISE SOURCES

There are two end-member approaches to deriving a direct expression for Green's function retrieval by crosscorrelation of noise recordings. The first is to assume the medium is dissipative and the domain spans sufficiently far such that the wavefields emitted by sources placed on or near the boundary dissipate before reaching the receivers. Thus the domain-boundary integrals on the right hand of equations A.7 and A.8 vanish. The second approach is to assume the medium is lossless and thus the domain integrals on the right-hand sides of Equations A.7 and A.8 vanish (for a non-dissipative medium $\mathbf{A} - \mathbf{A}^\dagger = 0$).

Sources distributed over a domain containing the receivers

Assuming the medium is dissipative, the Green's function can be represented by crosscorrelated source responses of sources positioned throughout a domain containing the receivers. The combinations and weights of source-responses that need to be correlated are determined by $\mathbf{A} - \mathbf{A}^\dagger$. I assume a distribution of uncorrelated noise sources of all source types, $\mathbf{s}(\mathbf{x}, \omega)$, throughout $\mathcal{D}$ that satisfy $\left\langle \mathbf{s}(\mathbf{x}', \omega)\mathbf{s}^\dagger(\mathbf{x}, \omega) \right\rangle = \boldsymbol{\lambda}\, \delta(\mathbf{x}' - \mathbf{x}) S(\omega)$, where $\langle\ \rangle$ denotes a spatial ensemble average over source positions, $S(\omega)$ is the power spectrum of the noise, and $\boldsymbol{\lambda} = \boldsymbol{\lambda}(\mathbf{x}, \omega)$ is a diagonal matrix containing the excitation functions. The recordings in the i-th component of particle velocity or a composite pressure at $\mathbf{x}$ can be expressed as $\upsilon_i(\mathbf{x}) = \int_{\mathcal{D}} \mathbf{G}_i^{\upsilon}(\mathbf{x}, \mathbf{x}')\mathbf{s}(\mathbf{x}')d^3\mathbf{x}'$ or $\bar{p}(\mathbf{x}) = \int_{\mathcal{D}} \mathbf{G}^{\bar{p}}(\mathbf{x}, \mathbf{x}')\mathbf{s}(\mathbf{x}')d^3\mathbf{x}'$, respectively. For a particular noise regime where $\boldsymbol{\lambda} = \text{diag}\left[\mathbf{A}_s - \mathbf{A}_s^\dagger\right]$ in the solid and $\boldsymbol{\lambda} = \text{diag}\left[\mathbf{A}_w - \mathbf{A}_w^\dagger\right]$ in the water, and all off diagonal terms of $\left[\mathbf{A}_s - \mathbf{A}_s^\dagger\right]$ and $\left[\mathbf{A}_w - \mathbf{A}_w^\dagger\right]$ are zero, the crosscorrelations of observed signals, $\upsilon_i(\mathbf{x}_A)$ with $\upsilon_j(\mathbf{x}_B)$, or $\bar{p}(\mathbf{x}_A)$ with $\bar{p}(\mathbf{x}_B)$, yield the integral representations of the second terms in Equations A.7 and A.8. Thus

$$\left\langle \upsilon_i(\mathbf{x}_A)\, \upsilon_j^*(\mathbf{x}_B) \right\rangle \propto \left\{ G_{i,j}^{\upsilon,f}(\mathbf{x}_A, \mathbf{x}_B, \omega) + G_{j,i}^{\upsilon,f\,*}(\mathbf{x}_B, \mathbf{x}_A, \omega) \right\} S(\omega) \tag{A.11}$$

and

$$\left\langle \bar{p}(\mathbf{x}_A)\, \bar{p}^*(\mathbf{x}_B) \right\rangle \propto \left\{ G^{\bar{p},\bar{q}}(\mathbf{x}_A, \mathbf{x}_B, \omega) + G^{\bar{p},\bar{q}\,*}(\mathbf{x}_B, \mathbf{x}_A, \omega) \right\} S(\omega). \tag{A.12}$$

Sources on a boundary surrounding the receivers

Assuming the medium is non-dissipative, the Green's function can be represented by crosscorrelated source responses of sources positioned on a boundary surrounding the receivers. The combinations of source responses that need to be correlated are determined by $\mathbf{N}$ (containing the components of the normal vector to the boundary). However, by definition the matrix $\mathbf{N}$ is not diagonal, meaning the responses of two different source types need to be recorded independently yet simultaneously. Such recordings cannot be made in only one source realization in the earth, thus rendering the need for additional assumptions. Assuming the domain boundary $\partial\mathcal{D}$ is sufficiently far away from the receivers, I can define auxiliary source types (of pressure and shear type) that approximate the source types required by the matrix $\mathbf{N}$ in the boundary integral. This approximation splits the matrix $\mathbf{N}$ into a diagonalized matrix term plus a spurious matrix term with non-desired (spurious) energy. The spurious term contains cross terms between inward and outgoing propagating waves causing energy in the retrieved signal that does not match the desired Green's function. This term vanishes when the medium is homogeneous and isotropic outside the domain $\mathcal{D}$. In addition, the spurious events are non-stationary with position, thus if the sources are randomly positioned in the neighborhood of the boundary the spurious events stack incoherently (Draganov et al., 2006). As before, a source distribution can be envisaged where the direct crosscorrelation evaluates the diagonalized boundary-integral term, similarly leading to Equations A.11 and A.12.

Practical considerations

The earth is dissipative at geophysical seismic frequencies but water is not. Requirements of source positions are thus a mix between boundary and domain distributions.

In practice however, the required source distribution to retrieve the entire Green's function is (almost) never satisfied. The requirements are quite strict, especially for dissipative media. When the source distribution does not compensate for dissipation, one would recover the Green's function with an amplitude error, but kinematically correct (Snieder, 2007). When sources are missing on part of the domain boundary, stationary phases of certain events may not be sampled and those events not correctly distributed (Snieder, 2004).

There are several factors that make the retrieval of body-waves by seismic interferometry problematic. A major difficulty is that for recordings at the earth surface, body wave reconstruction requires source energy to be reflected by the free surface near one receiver, back scattered by the subsurface and recorded by the other station (Forghani and Snieder, 2010). While reconstruction of surface-waves requires energy to simply be transmitted between the stations. Furthermore, body waves require sources distributed in three dimensions, while surface waves can be retrieved by sources distributed in two dimensions (Kimman and Trampert, 2010). But a source distribution with sources on the surface only makes retrieval of higher-modes more difficult (Halliday and Curtis, 2008; Kimman, 2011).

The requirements on seismic noise suitable for Green's function retrieval is generally described as to fulfill energy equipartitioning. I.e. the seismic noise field is in a non-zero-energy state with zero net-energy flow between wave-modes and with zero net-energy flow over space. More simply, all modes are excited equally and energy is evenly distributed over space. Energy equipartitioning alone is insufficient and sources exciting the seismic noise need to be uncorrelated so their response is evaluated independently by crosscorrelating long-time records (Snieder et al., 2010). However, Snieder et al. (2010) also note that energy equiparitioning is too strict of a condition when the desire is to retrieve only a portion of the Green's function. I argue that a requirement for the retrieval of the Green's function between two stations of energy equiparitioning over the entire system is always too strict. Instead, this requirement only needs to be satisfied for the energy that is emitted by a source placed at the position of one the receivers and that is recorded by the other receiver

(i.e. the energy represented by Equation A.4 and A.6). Furthermore, neither station may record any additional non-equipartitioned energy that could be present in an ambient seismic recording.

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