Handling salt reflection in Least-squares RTM with Salt-dimming

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ABSTRACT

The challenge for applying least-squares reverse-time migration (LSRTM) to area with sharp velocity contrast is discussed. Least-squares migration (LSM), also known as linearized inversion, is an advanced imaging technique. It provides true relative amplitude information while suppressing acquisition footprints and migration artifacts. In most cases when the velocity is smoothly varying, the observed data can be used directly as input for LSRTM. However, when the velocity field has a sharp contrast like in the transition from sediment velocity of salt velocity, a background data term needs to be calculated and subtracted from the observed data before supplying as input for LSRTM. Therefore, special care is needed when performing LSM in regions like Gulf of Mexico where strong salt-reflection is presence. While straight forward with synthetic, subtracting the background data is a non-trivial problem for field data. I introduce a salt-dimming technique for handling sharp velocity contrast in LSM. I demonstrate the concept and methodology in 2D with a modified version of the Sigsbee2B model.

INTRODUCTION

In seismic exploration, geological interpretations require a subsurface image that accurately displays structural information. The conventional practice of obtaining such image is by prestack depth migration. However, depth migration images are often distorted by uneven subsurface illumination from acquisition geometry, complex overburden and band-limited wavefields. To correct the effects of uneven illumination, the imaging problem can be posed as a linear inverse problem. Instead of using the adjoint operator, I use the pseudo-inverse of the Born-modeling operator to optimally reconstruct the reflectivity. This inversion-based imaging method is also widely known as least-squares migration (LSM) (Lambare et al., 1992; Nemeth et al., 1999) or linearized wavefield inversion (Clapp, 2005; Valenciano, 2008).

LSM can be implemented using different types of wave propagation operators. Starting with the ray-based operator in least-squares Kirchoff migration. Kuehl and Sacchi (2003) and Clapp (2005) have worked on LSM with the one-way operator. Until recently, least-squares reverse-time migration (LSRTM), which uses the two-way propagator, becomes computationally affordable (Dai et al., 2010; Wong et al., 2011;

Yao and Jakubowicz, 2012). Two-way propagator is considered the most accurate way of representing wave propagation because it allows bandlimited wave-propagation in all direction with no dip limitation. When LSM is applied to regions like Gulf of Mexico where there is a sharp velocity contrast, a background data term needs to be calculated and subtracted from the observed data. I will show by synthetic example that this subtle point affects LSM result. When the background data is properly subtracted from the observed data.

THEORY

I define my model ($m(\mathbf{x})$) to be a weighted difference between the migration slowness ($s_o(\mathbf{x})$) and the true slowness ($s(\mathbf{x})$):

$$m(\mathbf{x}) = (s(\mathbf{x}) - s_o(\mathbf{x}))s_o(\mathbf{x}) \tag{1}$$

The model ($m(\mathbf{x})$) is directly proportional to the perturbation (or deviation) between the migration slowness and the true slowness. In the synthetic case, this quantity should be zero if the migration velocity is exactly the true velocity. In LSM, the forward modeling operator is linearized with respect to $m(\mathbf{x})$.

$$d^{mod} = F(s_o^2 + m) \approx F(s_o^2) + \mathbf{L}m \tag{2}$$

where $\mathbf{L}$ is the linearized forward-modeling operator and d^{mod} is the synthetic data. The equation for the linearized forward modeling is:

$$\mathbf{L}m = \Delta d(\mathbf{x}_r, \mathbf{x}_s) = \sum_{\mathbf{x}} \omega^2 f_s(\omega) G(\mathbf{x}_s, \mathbf{x}) m(\mathbf{x}) G(\mathbf{x}, \mathbf{x}_r) \tag{3}$$

where ω is the temporal frequency, $\mathbf{x}$ is the image point, $f_s(\omega)$ is the source waveform, and $G(\mathbf{x}_s, \mathbf{x})$ is the Green function of the two-way acoustic constant-density wave equation over the migration slowness s_o. (Note that G is actually ω-dependent.) It is important to point out that the adjoint of the linearized forward-modeling operator is the migration operator:

$$\mathbf{m}_{mig}(\mathbf{x}) = \sum_{\mathbf{x}_r, \mathbf{x}_s} \omega^2 f_s^*(\omega) G^*(\mathbf{x}_s, \mathbf{x}) G^*(\mathbf{x}, \mathbf{x}_r) d(\mathbf{x}_r, \mathbf{x}_s) \tag{4}$$

The objective function ($S(m)$) of LSM is defined by minimizing the least-squares difference between the synthetic (d^{mod}) and the observed data (d^{obs}):

$$S(\mathbf{m}) = \|d^{mod} - d^{obs}\|^2 = \|\mathbf{L}m - (d^{obs} - F(s_o^2))\|^2 \tag{5}$$

Notice in the above objection function, there is a background data term $F(s_o^2)$ that needs to be subtracted from the observed data. Essentially, $F(s_o^2)$ is the forward modeling using the migration slowness s_o. In most cases when the velocity is smoothly varying, this term is trivial and we can just ignore this term in the inversion.

However, when the velocity field has a sharp contrast, this term is non-trivial. Therefore, special care is needed when performing LSM in regions like Gulf of Mexico where strong salt-reflection is presence. I will illustrate this point with a simple synthetic model in the next section.

LSRTM with and without background data subtraction

I use a simple 2D synthetic model with a salt structure and two subsalt reflectors (Figure 1(a)). The contrast from the sediment to the salt is sharp and the velocity transition across the two subsalt reflectors is weak. In a typical model-building flow, the salt structure will be included in the migration slowness (Figure 1(b)). Therefore, the background data term ($F(s_o^2)$) is non-trivial. This simple example serves as a good exercise to test the importance of the background data term in LSM.

I will compare two LSRTM inversions, with the following objective functions:

$$S_1(\mathbf{m}) = \|\mathbf{L}m - d^{obs}\|^2 \tag{6}$$

$$S_2(\mathbf{m}) = \|\mathbf{L}m - (d^{obs} - F(s_o^2))\|^2 \tag{7}$$

In the first inversion as shown by the objective function $S_1(m)$, I try to fit the observed data with the linearized synthetic data ($\mathbf{L}m$). That is, I ignore the background data term. Figure 2(a) shows the RTM result or equivalently, the first iteration output of LSRTM. In this model, there are two subsalt reflectors located at depth 4000m and 5000m. From the true model (figure 1(a)), I expect the impedance across the two subsalt reflectors to be constant. Figure 3(a) shows the amplitude along these two reflectors from the RTM image. Blue corresponds to the shallower reflector at $z = 4000m$ while the red corresponds to the reflector at $z = 6000m$. Due to uneven illumination and the complex salt structure, the amplitude along the reflectors is not constant. In the ideal case, we want to see two horizontal lines in Figure 3(a). By using LSRTM, we hope to correct the affect of uneven illumination. Figure 2(b) shows the result of LSRTM at iteration 40 using the objective function $S_1(m)$. Figure 3(b) shows the corresponding subsalt reflector amplitude, notice that the amplitude at the shadow zone of the two subsalt reflectors is not well compensated by the inversion. On the other hand, using the objective function $S_2(m)$, the amplitude along the two subsalt reflectors is better corrected by LSRTM as shown in Figure 2(c) and 3(c).

This example shows that subtracting the background data is crucial in correcting for uneven illumination for the subsalt reflectors. The next question is how to apply it in field data? It is actually a difficult task. The theory that generates the background data $F(s_o^2)$ is just an approximation to the complex earth mechanism that generates the observed data $F(s^2)$. That makes obeying equation $S_2(\mathbf{m})$ impossible.

Let try to understand the inversion in a different way. Figure 4 (a) and (b) shows the square of one common receiver gather data-residual at iteration 1 and at iteration 40 using objective function $S_2(m)$. By looking at the square of the data residual, we get a sense of the highest value in the data-residual space This also tells us where (in the data space) the inversion tries to minimize. Between iteration 1 and 40, the highest value still remains to be the salt reflections. This means that the inversion is still predominantly trying to čorrectön the salt-reflector. I will introduce a simple 'method' to address this issue in the next section.

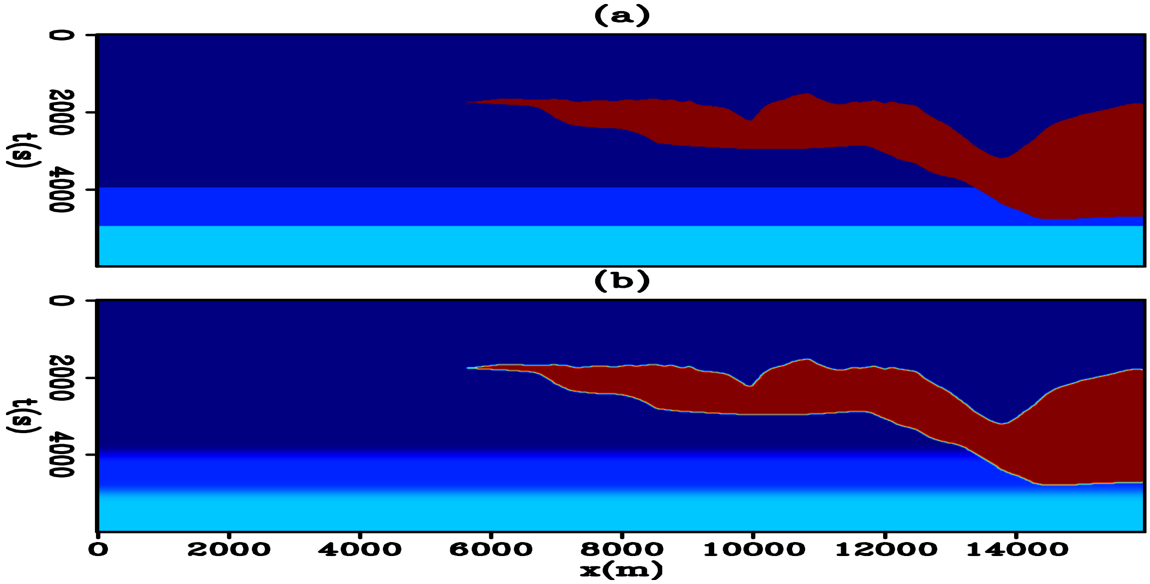

Figure 1: (a) True velocity model and (b) migration velocity model. Notice how the salt boundary is slightly smoothed in the migration velcoity model. This is to simulate the situation where salt reflection is not correctly estimated.

How to subtract background data in field datasets

One way to work around this problem is by salt-dimming. Salt-dimming aims to down-weight the salt reflection energy in the data space so that the inversion can try to minimize other region in the model like the subsalt reflectors. This corresponds to the following objective function.

$$S_3(\mathbf{m}) = \|W_s(\mathbf{L}m - d^{obs})\|^2.$$

W_s is the data weighting function that down-weight the salt reflection energy. This can be done by forward modeling the salt reflection using the migration velocity $F(s_o^2)$. Next is to calculate an envelope around the salt energy. The data weighting function can then by derived by assigning small value to the salt reflection envelope. Figure 5(a) shows the forward modeling of one CRG. The salt reflection is then used to derive an envelope region to be down-weighted. The resulting weighting function is shown in Figure 5(b). As shown in Figure 5(b) the down-weighted region (blue) corresponds mostly to salt reflector. It is important to point out that those blue regions are not muted. Instead, it is given a very small value of 0.01. By applying salt dimming, I guide the inversion to focus on other parts of the model (like subsalt reflectors) instead of the salt reflection itself. This data weighting function can be used for LSRTM using objective function $S_3(m)$.

Figure 3(d) shows the result of using salt-dimming with observed data. Notice in figure 4(d), the amplitude of the shadow zone along the subsalt reflectors is better recovered than in figure 4(c). Figure 4(c) and 4(d) shows the data residual squared plot at iteration 1 and iteration 40 of LSRTM with salt-dimming. Notice how the salt-dimming part completely changes the highest value in the least-square residual.

DISCUSSION

Convergence plot

Figure 6 shows the convergence curve for all three inversions. The residual is normalized by the residual value from the first iteration. In the $S_1(\mathbf{m})$ inversion, the background data is ignored. The inversion fails to properly correct for the uneven illumination (figure 3(b)) and this is supported by the fact that the residual barely goes down. (the diamond-blue curve in figure 6). When LSM is handled properly by using $S_2(\mathbf{m})$, the residual drops down to 50 percents over 200 iterations as shown by the square-red curve. Finally, salt-dimming is applied using the observed data and the objective function $S_3(\mathbf{m})$, the residual drops steadily over iterations as indicated by the triangle-green curve.

For the $S_2(\mathbf{m})$ inversion, it is surprising to see that the residual doesn't drop down rapidly to zero. One reason for this is because linearization requires that the perturbation $(s(\mathbf{x}) - s_o(\mathbf{x}))$ perfect salt

Why salt-dimming works?

In standard GOM model building, the salt-structure is usually picked and included in the velocity output. The background data contain salt reflection energy. Ideally, we want to assume that we picked the salt correctly. That means the salt reflection energy in the perturbed data should disappear completely. Salt-dimming works because it simulates the same affect by making the amplitude of salt-reflection energy small. Alternatively, one might consider muting out all the salt reflection in the data space completely. However, I found that it is less effective than salt dimming.

Why salt-dimming is better than salt-muting?

Although I want all the salt-reflection energy to disappear in the observed data, it is impossible to achieve this by muting out all the salt reflection in the data-space. This is because the salt-reflection energy in the data space becomes progressively complex as time increases. Ultimately, some of the salt-reflection energy remains in the data. When LSM is performed on such a dataset, the inversion becomes problematic because only partial information about the salt structure is presence in the dataset. In addition, many truncations in the data space (as a result of the muting) create artifacts in the image space. The cause of such artifacts is similar in logic to that of having acquisition footprint in migration images. Salt-dimming does not have the same problem. Essentially all the information about the salt-structure still exists. Even when the inversion tries to recover some of those salt structures, there are still information to reconstruct it in the dataset.

CONCLUSION

In regions like Gulf of Mexico where there is a sharp velocity contrast, a background data term needs to be calculated and subtracted from the observed data. While this is a straight forward task in the synthetic case, it is challenging in the field data case. Synthetic examples show that salt-dimming in LSRTM can correct amplitude information for subsalt reflectors much better than conventional LSRTM. Salt-dimming is a viable solution for addressing the issue of subtracting background data in field datasets.

ACKNOWLEDGMENTS

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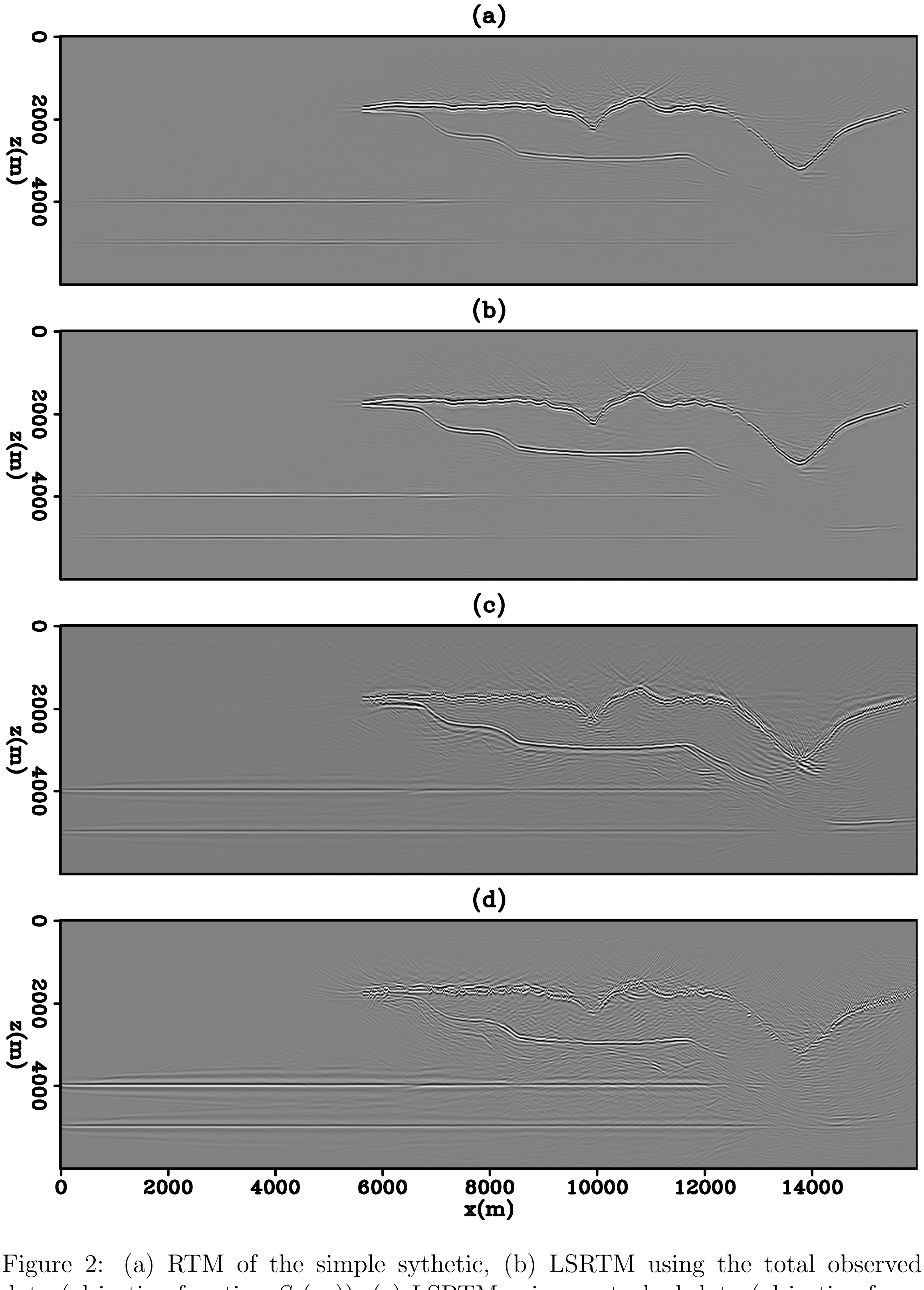

Figure 2: (a) RTM of the simple sythetic, (b) LSRTM using the total observed data (objective function $S_1(\mathbf{m})$), (c) LSRTM using perturbed data (objective function $S_2(\mathbf{m})$), (d) LSRTM using observed data with salt dimming (objective function $S_3(\mathbf{m})$.

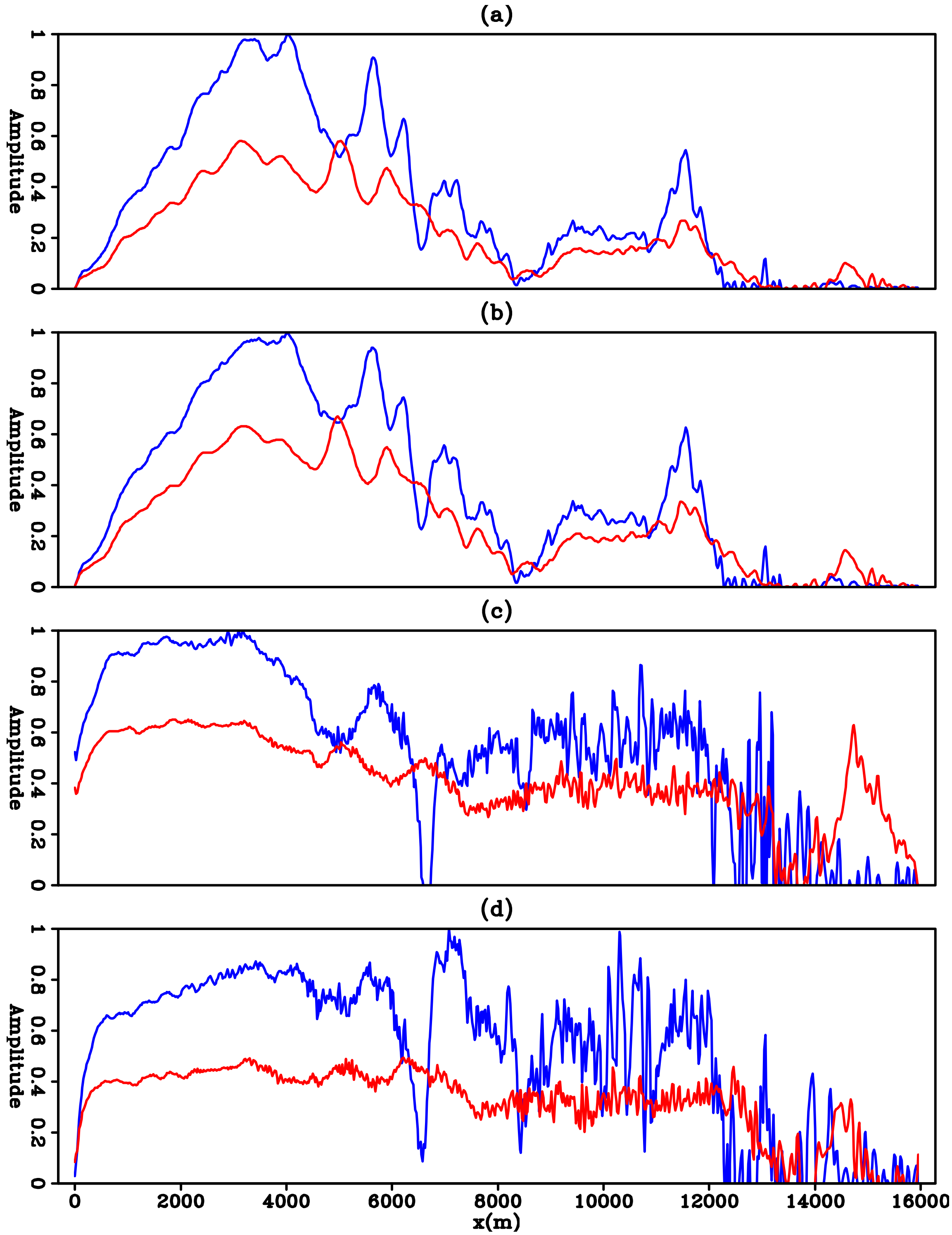

Figure 3: Amplitude along two subsalt reflectors at 4000m and 5000m. (a) RTM, (b) LSRTM using the total observed data (objective function $S_1(\mathbf{m})$), (c) LSRTM using perturbed data (objective function $S_2(\mathbf{m})$), (d) LSRTM using observed data with salt dimming (objective function $S_3(\mathbf{m})$. Blue corresponds to the shallower reflector at $z = 4000m$ while the red corresponds to the reflector at $z = 6000m$. The last case (d) gives the best recovery of uneven illumination.

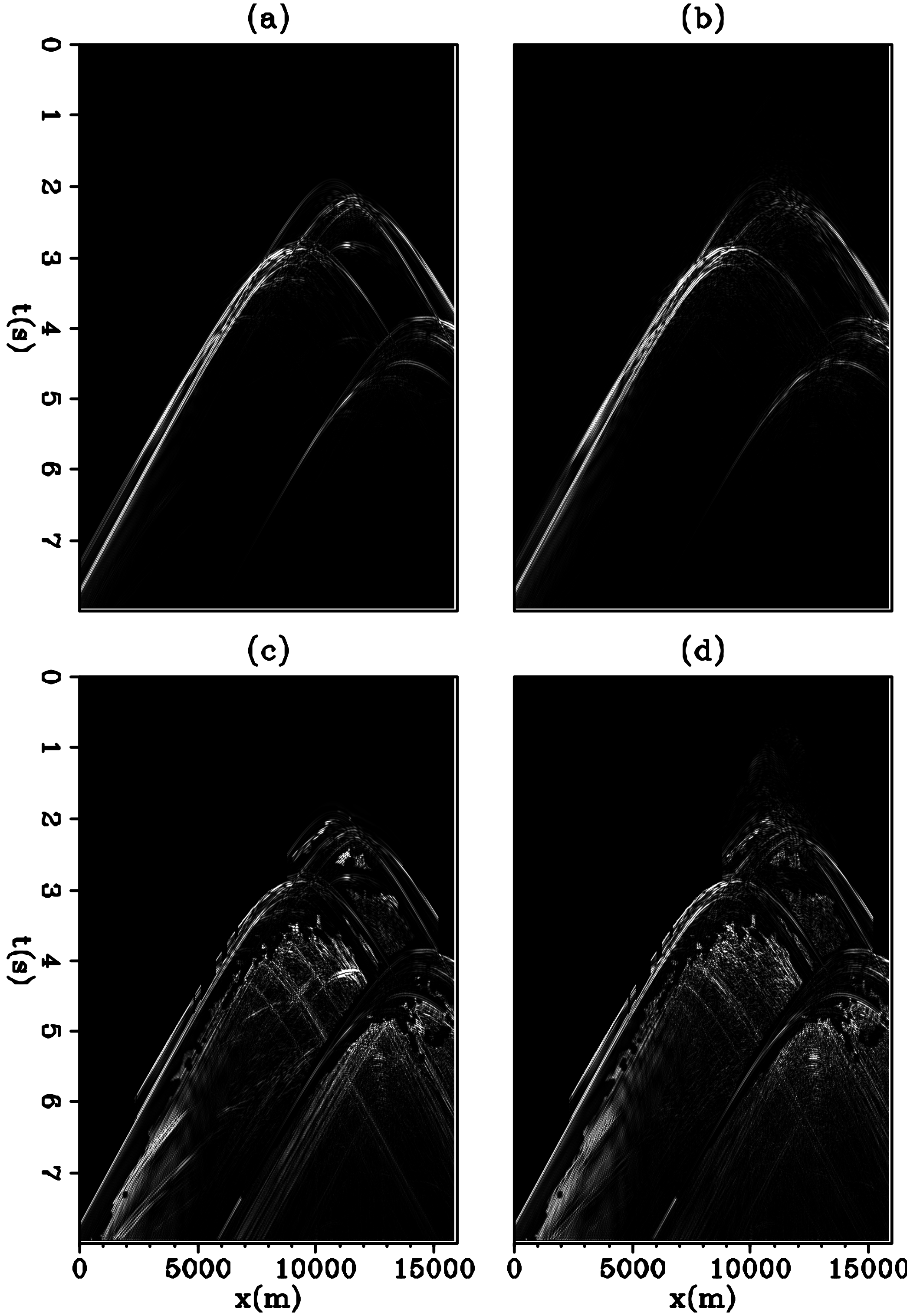

Figure 4: The square of a common receiver gather. It gives a sense of where the inversion is biased towards. (a) Original data (b) LSRTM iteration 20 using perturbed data. (c) LSRTM iteration 1 (d) LSRTM iteration 20 with salt-dimming

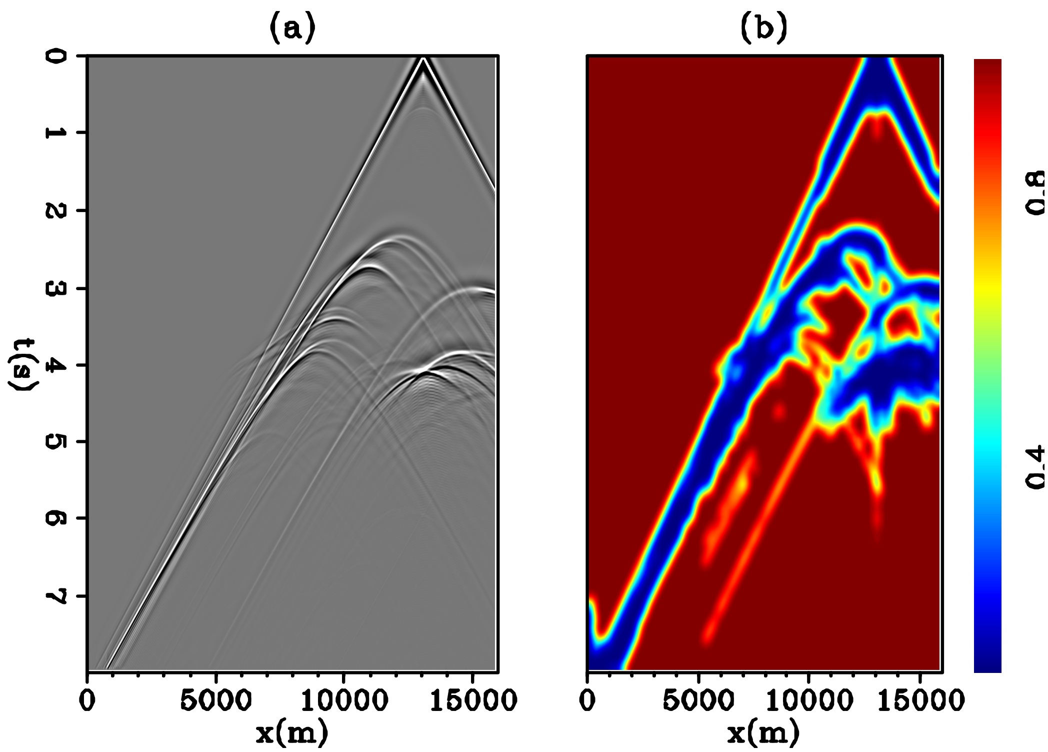

Figure 5: (a) Background data ($F(s_o^2)$) created by forward modeling with the migration slowness and (b) salt-dimming weight (W_s) generated with the background data.

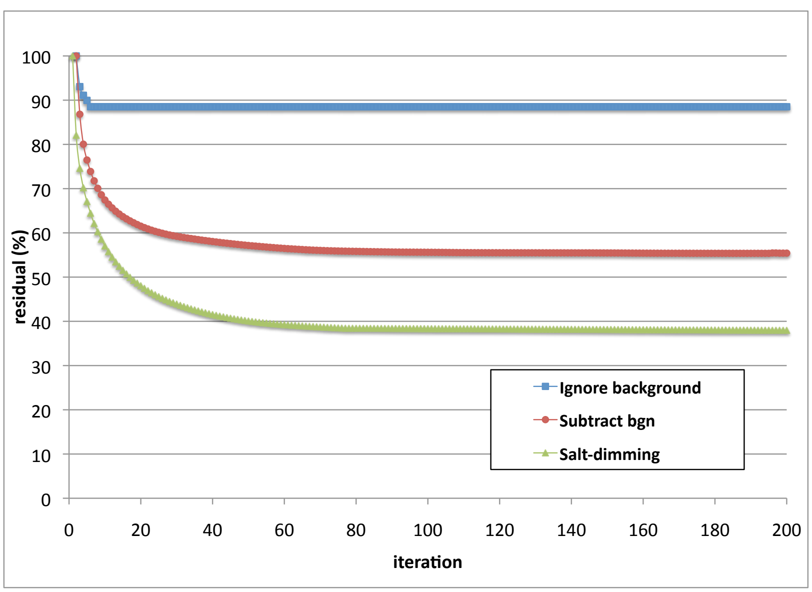

Figure 6: Convergence curve of different cases. Notice that when background data is ignored, the inversion hardly converges. When the problem is handled properly like the case of using the perturbed data, the inversion residual drops down to 50 percents within 200 iterations. Finally, using salt-dimming with the observed data, the residual also drops steadily over iterations.