t-squared gain for deep marine seismograms

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ABSTRACT

Kjartansson's all-purpose gain function t^2 is updated for deep water and large offset. We examined several deep-water gain possibilities and finally nominate $(t-t_e+\Delta t)t$. The water path time (first-earth-arrival time) is t_e. The thickness Δt is defined as that of a $Q = 100$ layer required to convert a mathematically infinite-band source signal to a realistic band. A suitable value for Δt is 350ms. Using the nominated gain function reduces the need for plotting at various clip values, muting, or AGC. We wish we had code for routinely identifying t_e. Present code assumes the vertical travel time depth is known and assumes normal moveout predicts water-bottom travel time for non-zero offset. Program and code are ready for your use.

INTRODUCTION

The goal is to scale deep marine data so it is visible at all times and offsets. Ideally the scaling function, like Kjartansson's t^2 is independent of any parameters, but easily known parameters like offset and water-depth are acceptable. In zero-order seismology rays propagate vertically. In first-order seismology rays propagate on straight lines defined by shot and receiver locations. In second-order seismology one includes RMS velocity. Here we are looking for something like a first-order calculation of expected wave amplitude so we may divide data by it bringing a screen full of data to visibility everywhere at once. What we are looking for is a general purpose scaling function of time and space not tailored to a particular data set, but useful for most data sets. Our goal is to reduce need for AGC, for clip, and for mute. Besides general purpose data viewing, this scaling may be a useful weighting function for inverse problems.

Kjartansson (1979) points to the mathematical function t^2 as a general purpose gain function for seismic data. A simple mathematical model suggests his function. His model assumes constant Q absorption along the entire ray path. But there is no absorption on the water path to bottom which can be quite long in deep water, especially at wide offset. That fact motivates development of the improved gain function I find here. The water-path time t_e is a function of both shot and geophone locations. To find t_e, here I make simplifying assumptions that vertical water depth is known and normal moveout predicts t_e with offset. I'd rather have a code that for t_e estimates the time the first event reaching the bottom arrived back.

REVIEW OF KJARTANSSON'S T-SQUARED

One power of t arises from spherical spreading. Energy spreads out on a sphere whose area grows as t^2. But we are interested in amplitude, not energy, which implies the square root of t^2 namely t.

The other power of t comes from the constant Q model. Seismograms have their highest frequencies at early times. These damp out leaving only lower frequencies at later times. So data may be gained more at late times. Beginners often believe the way to compensate for such absorption is with exponential gain, but that is wrong because exponentials describe sinusoidal waves, not the broad spectral band that our data has.

The most basic absorption law is the *constant* Q model. According to it, energy diminishes in proportion to the number of wavelengths in space, or in proportion to the number of periods in time, the factor of proportionality being $1/Q$. For a downgoing wave the absorption is proportional to the frequency ω and proportional to time in the medium which is the distance z divided by the velocity v. Altogether the spectrum of a wave passing through a thickness z will be changed by the factor $e^{-|\omega|(z/v)/Q}$ where Q is called the Quality factor of the medium.

We may define spectral bandwidth by setting $e^{-|\omega|(z/v)/Q}$ to be some arbitrary cutoff constant, say, e^{-3}.

$$
\begin{aligned}
.05 &\approx e^{-3} = e^{-|\omega|_{\text{cutoff}}(z/v)/Q} &(1)\\
\omega_{\text{cutoff}} &= 3Qv/z = 3Q/t &(2)
\end{aligned}
$$

This says the later you look on a seismogram, the narrower the spectral bandwidth. You compensate for this by gaining your data by another power of t, hence Kjartansson's t^2.

Fortuitously, this result is independent of velocity and the cutoff threshold e^{-3}; and it depends on Q only as a scaling factor (which merges itself with the usually-irrelevant plot scaling factor). As absorption $1/Q$ transits from zero to nonzero, the amplitude (plane wave) damping transits from 1 to Q/t. Not easy to connect those two functions!

NON-ZERO OFFSET AND DEEP WATER

For waves in the water path, regardless their direction of propagation we wish to delay the absorption effect until the time the waves first enter earth sediment t_e. My first guess at the gain rule $G(t)$ was this:

$$
G(t) = \begin{cases} t & \text{for } t < t_e \\ t^2/t_e & \text{for } t > t_e \end{cases} \qquad (3)
$$

Notice $G(t)$ is continuous at $t = t_e$, has t behavior before, and t^2 after. I believed equation (3) for some weeks, but when time came to write about it I found I could not derive it or defend it.

We want a gain function that grows most rapidly where the energy dissipation is strongest — just below the water bottom. It feels like the gain function needs the time of the in-earth ray path $t - t_e$. So I considered this:

$$G(t) \;=\; \begin{cases} 0 \times t & \text{for } t < t_e \\ (t-t_e)t & \text{for } t > t_e \end{cases} \tag{4}$$

Equation (4) is also continuous at $t = t_e$ because it is zero there. The idea is that there is infinite bandwidth in the water path and at the water bottom reflection so waves there should be multiplied by zero. Observers would be upset if they could no longer see the water bottom! Head waves either? Well, they'd see both the head waves and the water bottom if they chose their t_e a little earlier—but how much?

Decon-friendly gain

Antoine Guitton suggested a "decon-friendly" gain function. By being non-zero before time t_e, it does not offer a hiding place for non-causal optimization decons to hide information before t_e.

$$G(t) \;=\; \begin{cases} 1 & \text{for } t < t_e \\ t^2/t_e^2 & \text{for } t > t_e \end{cases} \tag{5}$$

Unfortunately, on a seismic section where water depth is increasing along the traverse, water bottom arrivals are not getting their basic geometrical t gain.

Continuity gain function

Shuki Ronen came up with a gain function that starts from linear gain in the water, then converts to parabolic (polynomial containing t^2) in the sediment, with the condition that the two functions match in slope as well as value at t_e.

$$G(t) \;=\; \begin{cases} t & \text{for } t < t_e \\ t + (t-t_e)^2/t_e & \text{for } t > t_e \end{cases} \tag{6}$$

This gain function is linear before $t = t_e$ and quadratic after. It is continuous in value and derivative at $t = t_e$. There is no problem in having a continuous derivative, but there seems no reason for the derivative to be continuous.

An all-purpose source spectrum

Equation (4) seems best in theory, but worst in practice. Data is multiplied by zero at the water bottom. We won't see data there! Furthermore, we don't know exactly where the water bottom is. Getting started I simply picked t_e a little early. That expedient is barely suitable for personal software and wholly unsuitable for shared software.

After pondering this conundrum some weeks I came to realize it's connected with the assumption that the wave begins with infinite bandwidth. What bandwidth should I assume for the initial shot waveform? Rephrasing the question, suppose we begin with infinite bandwidth. How much earth of $Q = 100$ must that infinite bandwidth propagate through to reach the kind of bandwidth we normally see? Let us say we would like the spectrum to drop down to 5% at 150Hz.

$$\begin{aligned} .05 \quad &\approx \quad e^{-\pi} = e^{-2\pi f \Delta t/Q} && (7) \\ \Delta t \quad &= \quad Q/2f = 100/300 \ \approx 350\text{ms} && (8) \end{aligned}$$

That's it in a nutshell. Put an infinite bandwidth into 1/3 second of earth and get out our defaulted standard seismic source spectrum. (Not a big reward for the builder of that infinite bandwidth source!) I nominate this default gain for our deep marine data:

$$G(t) \;=\; \begin{cases} 0 & \text{for } t < t_e - \Delta t \\ (t - t_e + \Delta t)t & \text{for } t > t_e - \Delta t \end{cases} \qquad (9)$$

By this logic Kjartansson's all-purpose gain t^2 would be improved by $(t + \Delta t)t$.

RESULTS

Figure 1 shows the result on a shot gather from offshore western Australia. This result is delightful showing more improvement than expected. We might wish to incorporate this gain as an option in our routine plot facilities. Signal at inner offsets has grown to be comparable to that at wider offsets. There are dominating multiples in the middle, but primaries may be found from top to bottom. Critical-angle events formerly dominated and still do, but much less so. Back scatter events at 5s are now more evident. The need for subsequent mute, or AGC, or revised clip is now reduced or eliminated. Subsequent processes such as velocity analysis, migration, and stack should change in subtle, perhaps useful ways.

Figure 2 shows a near-trace section. Results are similar, but here the new deep-water gain is better in a simple way — stronger at early time. I had anticipated this good result, but had not anticipated the even better result in Figure 1.

At wide offset in Figure 1 we see events beyond the water asymptote, both head waves and deep reflections. Here is why: To avoid suppressing such good signal, I boosted the mute velocity parameter to 2000m/s. Formerly we might play with gain via the parameter `tpow`=γ in t^γ where we had no physical model for γ. Now we may play with gain where we have two physical parameters: (1) vertical travel time to water bottom `tau`$=\tau$, and (2) `velocitymute` v, for a rough guess of first earth arrival time t_e as a function of offset.

Better yet should be an along-path first-arrival-time finding code. That would simplify our lives leaving only the minor sensitivity to the spectrally dependent Δt. Not only would we see our data better, but we might be more attuned to the notion that anomalous amplitudes have real meaning.

An expeditious approximation

Figure 3 shows the result of the expeditious approximation that the water path is only its horizontal distance ignoring the actual slanted path. The first earth reflection time t_e is not difficult to obtain, but it's not effortless either in view of the many complications arising in volume production. Since the offset is always known, this expeditious approximation $t_e = |x|/v$ is nearly effortless to install and use by default. Serendipitously, Figure 3 seems even better than the more accurate calculation in Figure 1! This result, however, is not likely to be typical. West Australia off-shore data is strongly dominated by multiples. The theory here is designed for primaries. Any gain function designed with multiples in mind must be customized according to their local strength. Examining the data here closely, we recognize weak primaries all throughout the range of time. Without the approximation the early primaries are gained to about the strength of the late ones. So the expedient result is not so desirable as it might first appear. Still, it is an expedient result, perhaps needed where depth varies within a survey and water-bottom tracking information is not available.

Test of the first-guess method

Figure 4 shows a test of the first guess method, equation (3). As expected, water bottom arrivals are very strong, dominating even the later multiples. The mathematical function t^2 has a powerful effect near $t = 0$ but a much weaker one near $t = t_e$. Data sets with weaker multiples should demonstrate more clearly the first guess weighting does not bring primaries into balance.

CODE DOCUMENTATION

We would like to acquire code that deduces the first arrival time, but so far we have not done it. Consequently we require knowledge of `offsetAxis`, a way to know in a hypercube which axis is the offset. Additionally, for the same reason, we require water depth `tau` and a mute velocity `mutevelocity`.

```
#  Deep Marine Gain
#  Dmgain <in.H > out.H tau=0 velocitymute=2000 offsetaxis=2 tspec=.350
#  Gain 3-D data by t*(t-te+tspec) where  te = sqrt(tau**2+x*x/velocitymute**2)
```

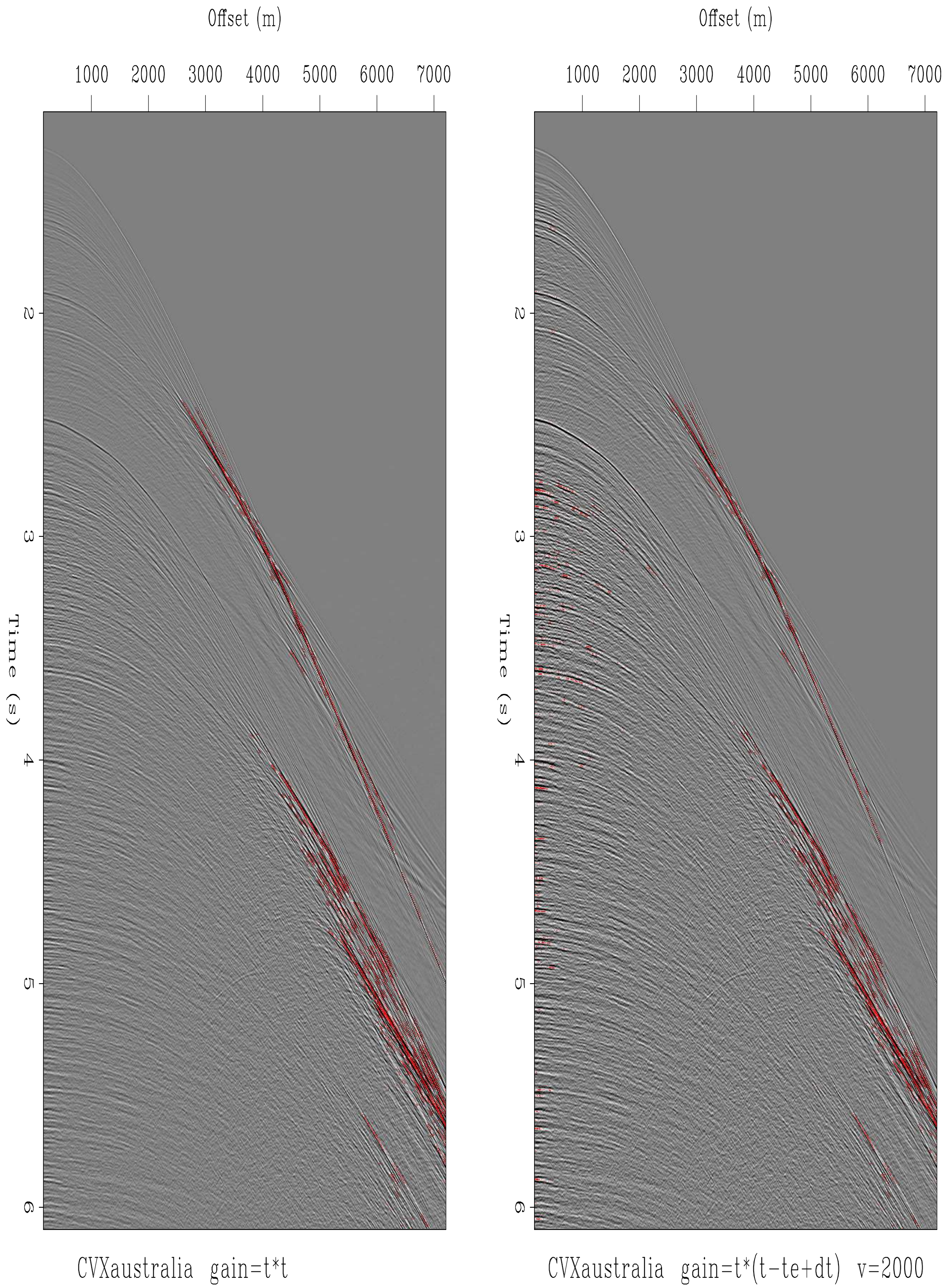

Figure 1: Chevron Australia shot gather gained by t^2 (left) and by $(t - \sqrt{1.2^2 + x^2/v^2} + .35)t$ (right). These plots use our default clip percentile, 99%. If you see the color red, you see where it is clipped. [**ER**]

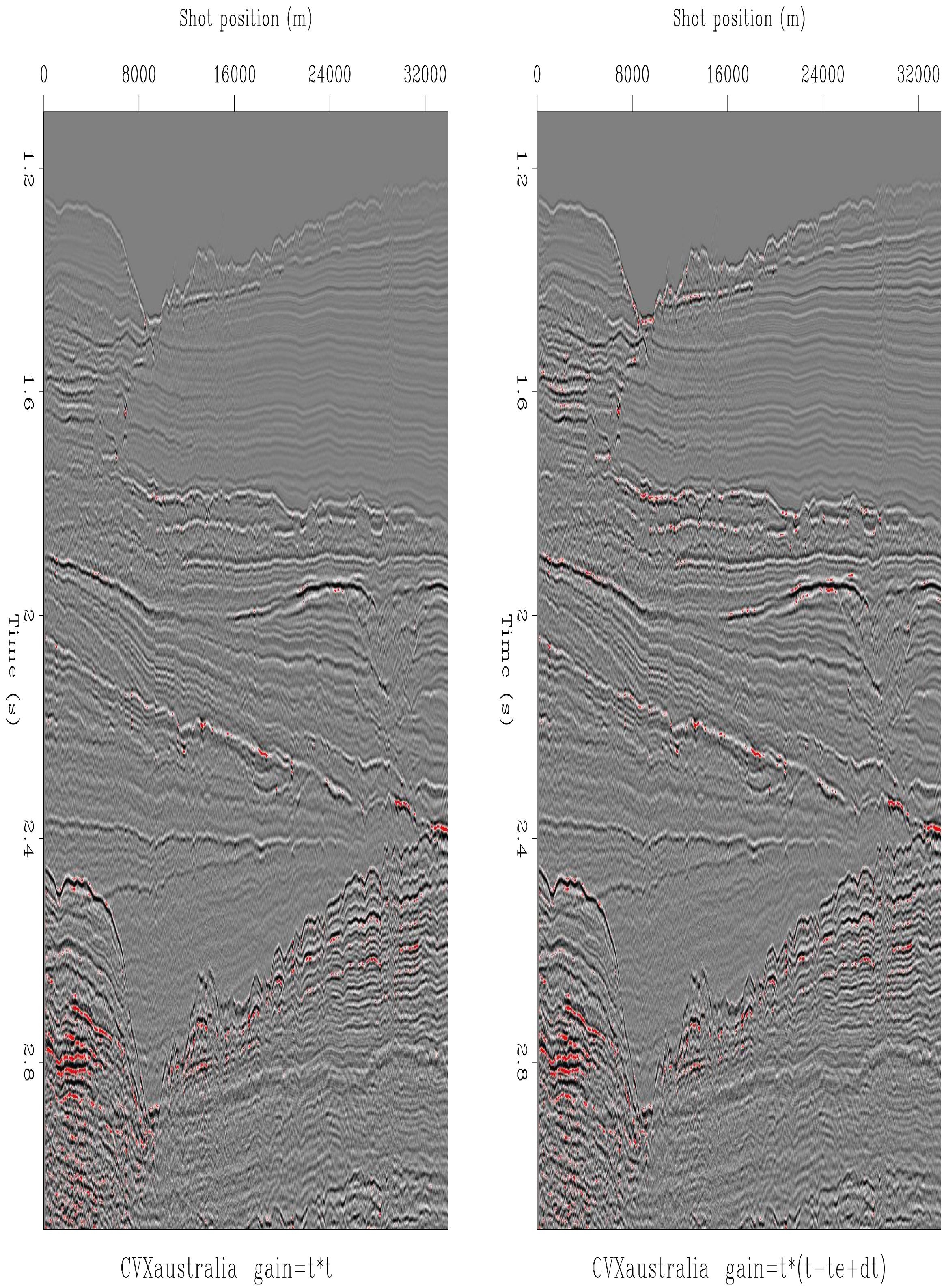

Figure 2: Chevron Australia near-trace section gained by t^2 (left) and by $(t - \sqrt{1.2^2 + x^2/v^2} + .35)t$ (right). The new deep-water gain is better at early time. [**ER**]

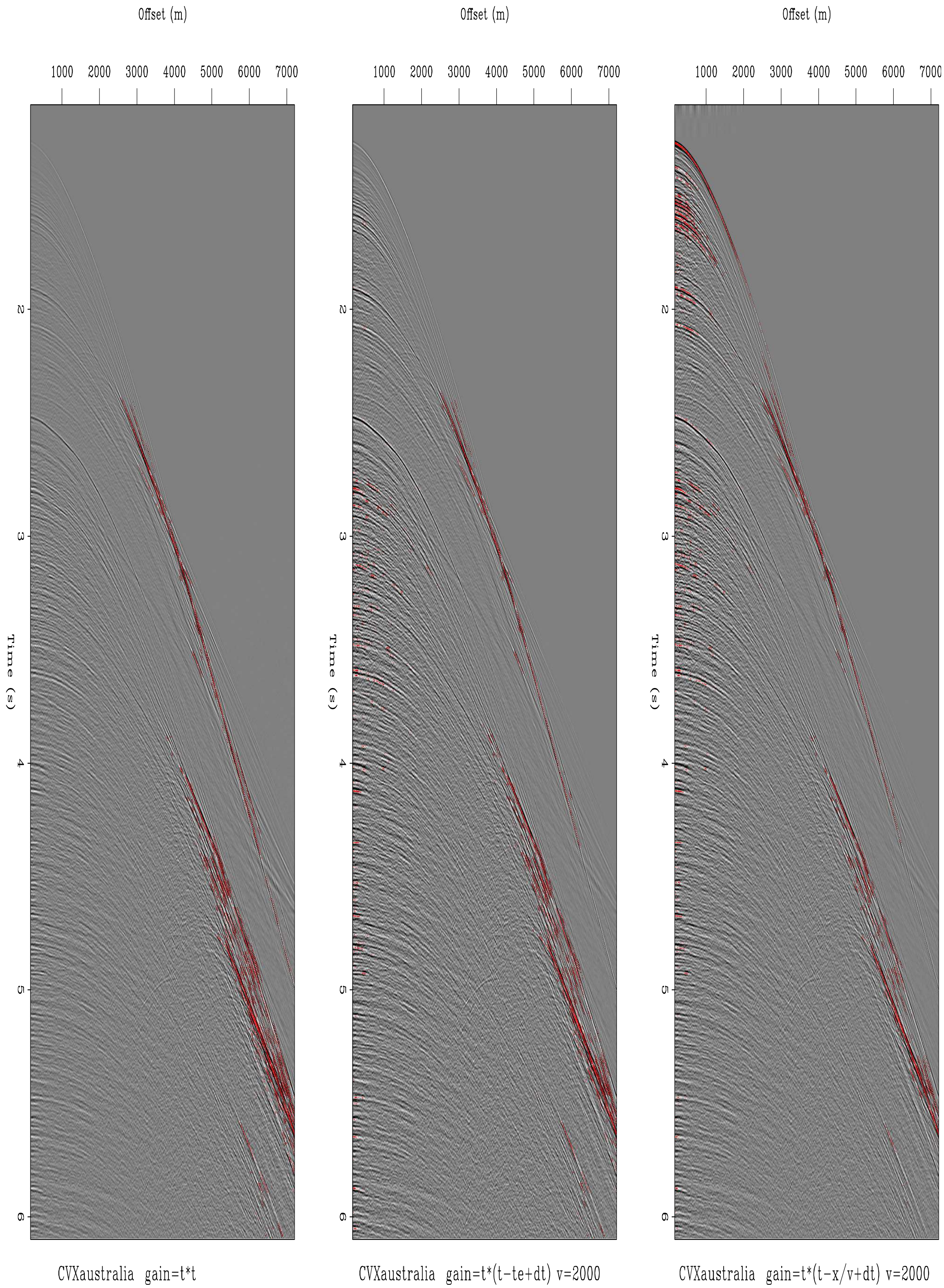

Figure 3: Test of expeditious gain: Kjartansson t^2 (left); Deep-water $(t - \sqrt{1.2^2 + x^2/v^2} + \Delta t)t$ (center); and Expeditious $(t - |x|/v + \Delta t)t$ (right). [**ER**]

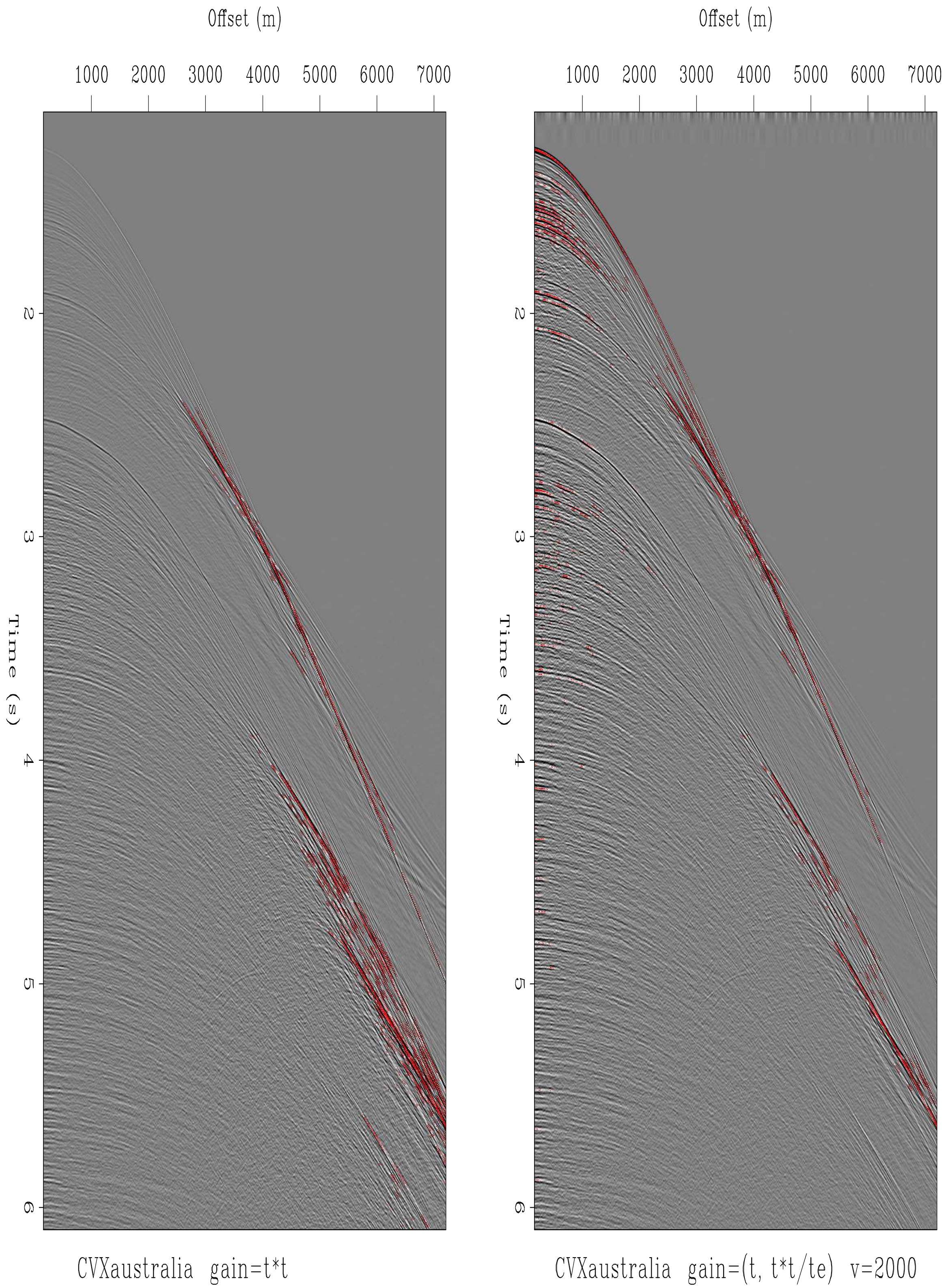

Figure 4: Chevron Australia shot gather gained by first guess weighting, t in the water path, t^2/t_e later. Early arrivals are too strong. [**ER**]

```
#       tau is travel time to water bottom
#       velocitymute is used to estimate travel time to water bottom
#       tspec=.350 = time before water bottom where mute ends and gain begins.
#       2-D data with offsetaxis=3 (section) requires n3=1 and o3 to be present.
```

CONCLUSIONS

1. The code is 35 lines long plus the above documentation. Located in /homes/sep/jon/res/futterman/Dmgain.rst After I get some user feedback, we can think about installing it.

2. Testing should be done on data less dominated by multiples. The same makefile can be used.

3. If someone codes identification of t_e from the data itself, results should be better from several points of view. Then the code would work equally well on shot gathers as midpoint gathers.

4. The gain function should have an optional radial x/t dependence to account for critical angle strength at water velocity. Need a wave propagation theorist to tell me the equation to use!

5. Adam's good idea is that the required parameters might be estimatable from the scaled data by some optimization code. This could be our default, as `Tpow` is supposed to compute a `tpow` if you don't specify one.

REFERENCES

Kjartansson, E., 1979, Constant Q-wave propagation and attenuation: J. Geophys. Res., **84**, 4737–4748.