

# A new algorithm for bidirectional deconvolution

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## ABSTRACT

We introduce a new algorithm for bidirectional deconvolution. In our method, we estimate the causal filters and anti-causal filters simultaneously instead of alternately. We test three data examples (1D synthetic, 2D synthetic and 2D field data). The results show that the wavelet can be compressed almost into a spike using our method. The two filters can be estimated equally when we are dealing with a zero-phase wavelet. In addition, our method has a lower computational cost and faster convergence rate than the method discussed by Zhang and Claerbout (2010).

## INTRODUCTION

In a previous report, Zhang and Claerbout (2010) introduced a bidirectional deconvolution method that overcomes the minimum-phase assumption of the conventional deconvolution. They factored the mixed-phase wavelet into two parts, the minimum-phase part and the maximum-phase part, which can be estimated by a causal filter and an anti-causal filter, respectively. Since such deconvolution is a non-linear problem, a pair of conventional linear deconvolutions were utilized to invert these two filters alternately and iteratively. In their paper, both theory and data examples showed that the mixed-phase wavelet can be accurately inverted using this bidirectional deconvolution.

However, there are some obstacles to inverting these two filters sequentially. There is a battle between these two filters competing for the spectrum. This competition makes the solution jump back and forth between the causal filter and the anti-causal filter, which may lead to a low convergence rate and an unstable deconvolution result. In addition, when Zhang and Claerbout (2010) inverted a zero-phase wavelet by this method, they produced two different filters; that is, the causal part and the anti-causal part are different, which is contrary to the nature of the zero-phase wavelet.

To avoid these problems, we invert these two filters at the same time instead of sequentially, hoping this simultaneous inversion will lead to a faster convergence rate and more stable solutions.

## THEORY

In this paper, we still rely on the idea of bidirectional deconvolution to deal with the mixed-phase wavelet. The wavelet can be factored into a minimum-phase part and a non-minimum-phase part. The deconvolution problem can be defined as follows:

$$d * a * b^r = r, \quad (1)$$

where  $d$  is the data,  $a$  and  $b$  are the unknown causal filters, and superscript  $r$  denotes the time reverse of filter  $b$ . Again, the hybrid norm is applied to  $r$ , and the reflectivity model is simply  $r$  plus a time shift. Now consider perturbations  $\Delta a$  and  $\Delta b$ :

$$d * (a + \Delta a) * (b^r + \Delta b^r) = r. \quad (2)$$

If we assume the the nonlinear part  $\Delta a \Delta b$  is relatively small, we can neglect this term:

$$d * a * b^r + d * a * \Delta b^r + d * b^r * \Delta a \approx r. \quad (3)$$

We use matrix algebraic notation to rewrite the fitting goal. We also want to guarantee filter  $a$  to be causal and filter  $b^r$  to be anti-causal during the iterations. For this we need mask matrices (diagonal matrices with ones on the diagonal where variables are free and zeros where they are constrained). The free-mask matrix for  $\Delta a$  is denoted  $\mathbf{K}$ , whose first diagonal element is zero, and that for  $\Delta b^r$  is denoted  $\mathbf{Y}$ , whose last diagonal element is zero:

$$\begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b}^r \end{bmatrix} \begin{bmatrix} \mathbf{Y} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{b}^r \\ \Delta \mathbf{a} \end{bmatrix} + \mathbf{d} * \mathbf{a} * \mathbf{b}^r \approx \mathbf{0}. \quad (4)$$

From equation (4), we have our new model  $\mathbf{m} = \begin{bmatrix} \Delta \mathbf{b}^r & \Delta \mathbf{a} \end{bmatrix}^T$  and new operator  $\mathbf{F} = \begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b}^r \end{bmatrix}$ . Now we can acquire these two filters only by applying the conventional inversion method and hybrid norm solver. The pseudocode for minimizing this new objective function by the hyperbolic conjugate-direction method developed by Claerbout (2010) is:

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non - linear iteration
{
   $\mathbf{r} = -\mathbf{d} * \mathbf{a} * \mathbf{b}^r$ 
   $\mathbf{F} = \begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b}^r \end{bmatrix}$ 
  linear iteration
  {
     $\mathbf{g} = (\mathbf{FJ})^T H'(\mathbf{r})$ 
     $\Delta \mathbf{r} = \mathbf{FJg}$ 
     $\mathbf{m} \leftarrow \text{Hyperbolic\_cgstep}(\mathbf{g}, \mathbf{m}, \Delta \mathbf{r}, \mathbf{r})$ 
  }
   $\mathbf{a} \leftarrow \mathbf{a} + \Delta \mathbf{a}$ 
   $\mathbf{b}^r \leftarrow \mathbf{b}^r + \Delta \mathbf{b}^r$ 
}

```

where  $H'(\mathbf{r})$  is defined as the first derivative of the hybrid norm  $\sqrt{R^2 + \mathbf{r}^2} - R$ , where  $R$  is the  $l_1/l_2$  threshold parameter,  $\mathbf{J}$  is the mask matrix  $\begin{bmatrix} \mathbf{Y} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}$ , and  $\mathbf{g}$  is the gradient.

From the template we notice that both linear and non-linear iterations are needed. Perturbations  $\Delta \mathbf{a}$  and  $\Delta \mathbf{b}^r$  are inverted by the hyperbolic conjugate-direction method in each linear iteration. Filters  $\mathbf{a}$  and  $\mathbf{b}^r$  are updated in the non-linear iteration, which generates a new operator  $\mathbf{F}$  to update the model. However, this method requires only 2 linear iterations to reach convergence, instead of the 100 linear iterations required by the previous method, greatly speeding convergence. In addition, there is no need to reverse the filters in the non-linear iteration, which makes our implementation more convenient.

Although the fitting goal is linearized, we still need the initial model to be close enough to get a good result. Here we expect an impulse function for both filters  $a$  and  $b$ . The following sections will show the application of this new method and demonstrate its effectiveness and limitations, when compared with the previous method discussed by Zhang and Claerbout (2010).

## APPLICATION

### Single wavelet

The first data example is the simplest mixed-phase wavelet, which only has three points [3,7,2]. We use it to verify the ability of our method to deal with the mixed-phase wavelet. The input data and its bidirectional result are shown in Figures 1 and

2. In this case, our new method is able to compress the simple mixed phase wavelet into a spike.

Figure 1: The three data points [3,7,2]. [ER]

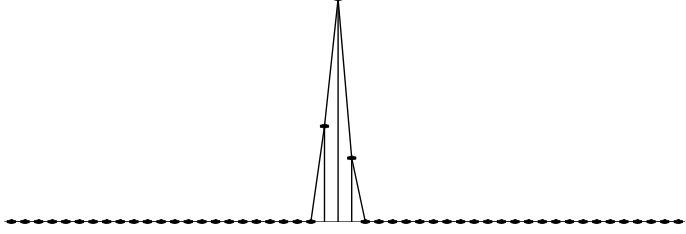
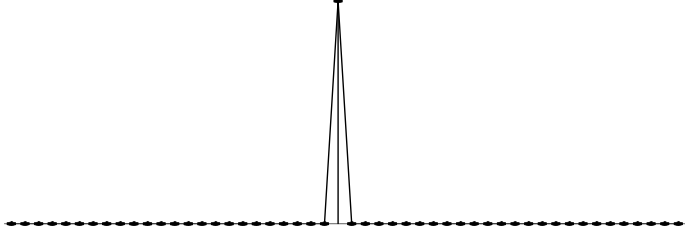


Figure 2: The result of our deconvolution method. [ER]



To illustrate the capabilities and limitations of our new method, we analyze the results obtained by inverting the zero-phase wavelet. This wavelet is created by convolving the minimum-phase with its own time-reversed wavelet.

Figures 3(a) and 3(b) show the filters estimated by our method. Figures 3(c) and 3(d) show the filters inverted by the previous method of bidirectional deconvolution. Here we time-reverse the anti-causal filter  $b^r$  into a causal filter  $b$  for easier comparison. Ideally, filter  $a$  and filter  $b$  should be identical, because the zero-phase wavelet is symmetric, with its minimum-phase part the same as its maximum-phase part, but time-reversed. The results of our method perfectly satisfy the theory, which shows the extreme similarity between filter  $a$  and filter  $b$ . When we invert the filters, the update direction is the same for both filters, because the searching gradients are equal. However, using the method from Zhang and Claerbout (2010) yields a filter  $a$  that is quite different from filter  $b$ , because they are inverted separately.

Figures 4, 5 and 6 show the zero-phase wavelet and its bidirectional deconvolution, using our new algorithm and the method of Zhang and Claerbout (2010). The results show that the wavelet is almost compressed into a spike by our method, but it is not as spiky as the result of the previous method. One possible reason may be the nature of the non-linear problem. There may be multiple minima in this problem, and due to our additional condition that filter  $a$  and filter  $b$  should be the same in this case, we find a different minimum, which leads to a different result.

Thus a good starting guess may help us to get a better result. Or perhaps the preconditioning can also make the solution fast converge to the global minima by utilizing prior information.

Figures 7 and 8 show the wavelet estimated by our method and the previous method. We notice that both of the wavelets approximate the input zero-phase wavelet, which is shown in Figure 4. However our estimated wavelet looks more

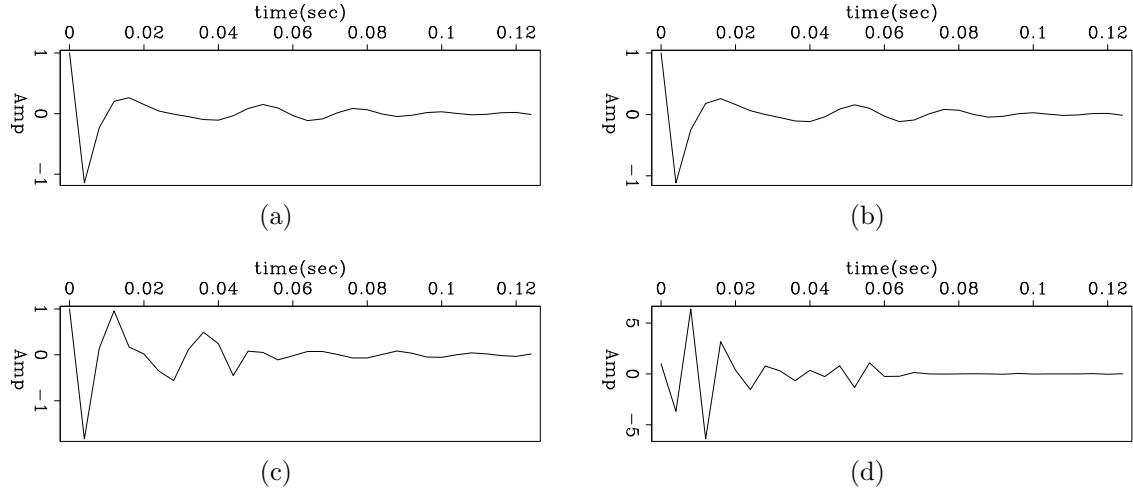


Figure 3: For zero-phase wavelet inversion, (a) filter  $a$  estimated by our method; (b) filter  $b$  estimated by our method; (c) filter  $a$  estimated by the previous method; (d) filter  $b$  estimated by the previous method. [ER]

Figure 4: Zero-phase wavelet. [ER]

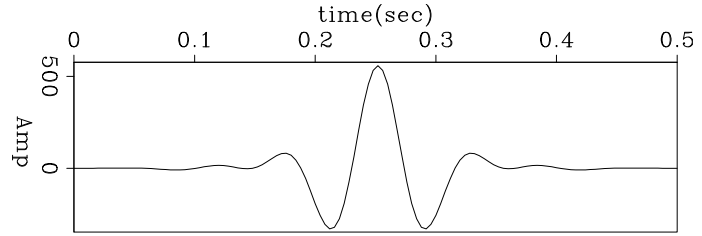


Figure 5: Deconvolution result by our method. [ER]

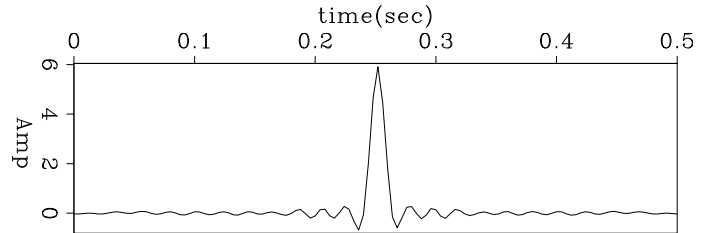
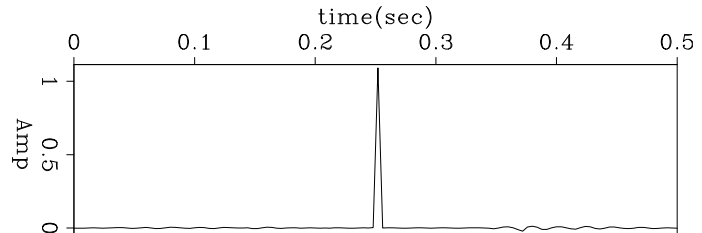


Figure 6: Deconvolution result by the previous method. [ER]



symmetric and cleaner with less side lobes than the one estimated by the previous method. The reason is that our estimated filters are identical, which make the causal part and anti-causal part of the inverted wavelet the same.

Figure 7: Shot wavelet estimated by our method

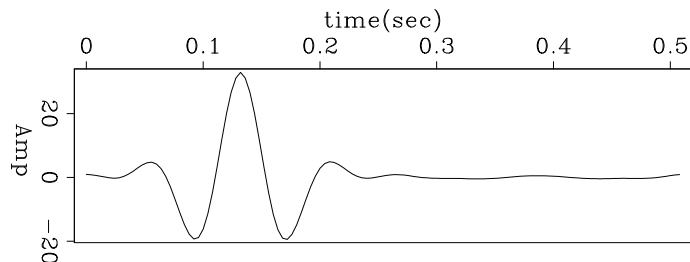
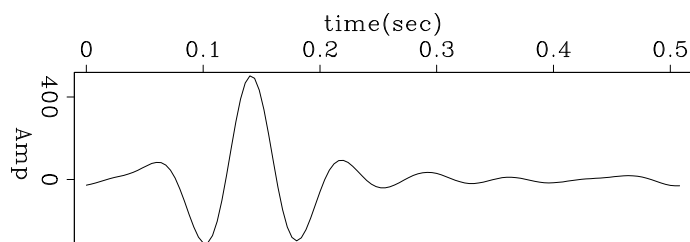


Figure 8: Shot wavelet estimated by the previous method



## 2D synthetic data

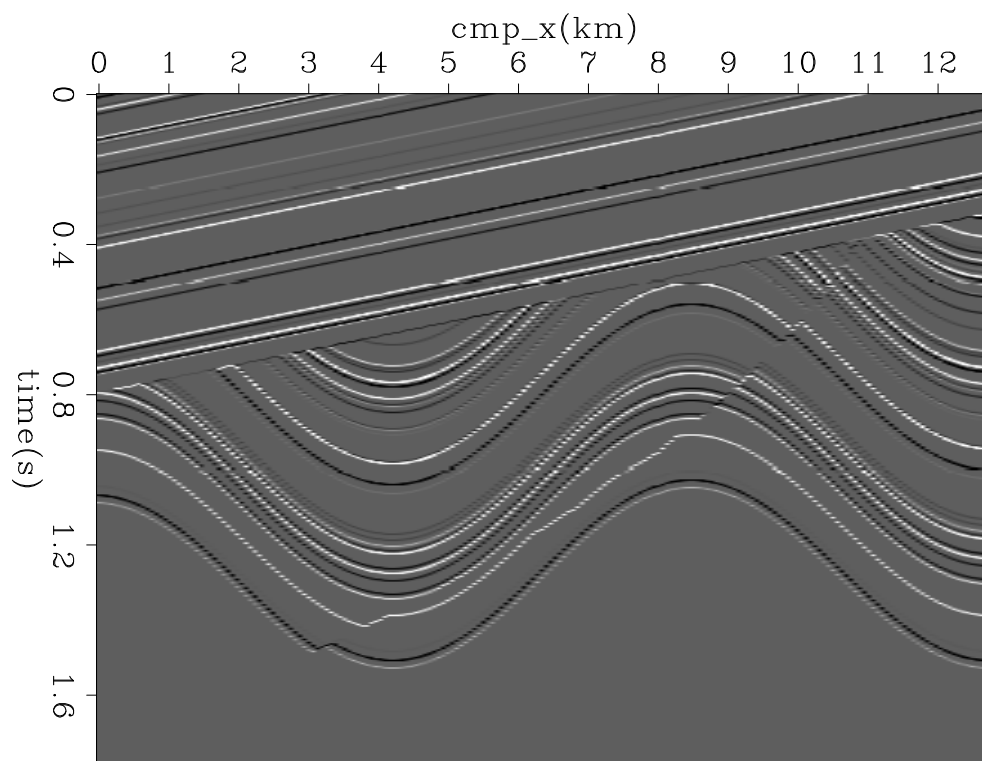
After applying deconvolution on the simple 1D case, we test the Zhang and Claerbout (2010) method and our new method on more complicated 2D synthetic data. Figure 9(a) shows the starting reflectivity model. Figure 9(b) shows the data generated by convolving the reflectivity model with the zero-phase wavelet in the previous section. All traces use the same wavelet when generating the data, and all traces share the same wavelet when we are doing the deconvolution.

Figures 10(a) and 10(b) show the bidirectional deconvolution using our method and the older method. Both methods retrieve the sparse reflectivity model and compress the wavelet into a spike, but the deconvolution model produced by the previous method is more spiky than ours, just as in the previous section, because of the wavelet we use.

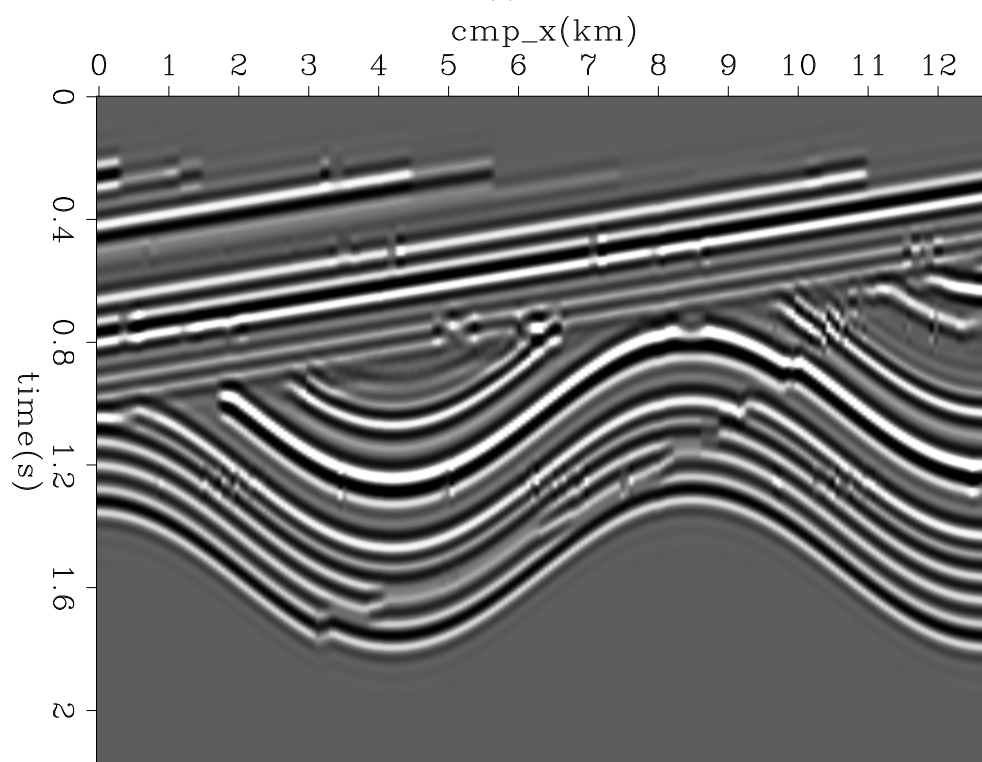
We also compare the computational costs of these two methods. To deal with this synthetic data, we use 2 linear iterations and 280 non-linear iterations to reach convergence, whereas Zhang and Claerbout (2010) used 100 linear iterations and 20 non-linear iterations, a total of 2000 iterations. In fact, our code is almost 6 times faster than the previous method.

## 2D field data

The field data we use in this example is a common-offest section of marine field data. Figure 11 shows the input data. Figures 12(a) and 12(b) show the bidirectional

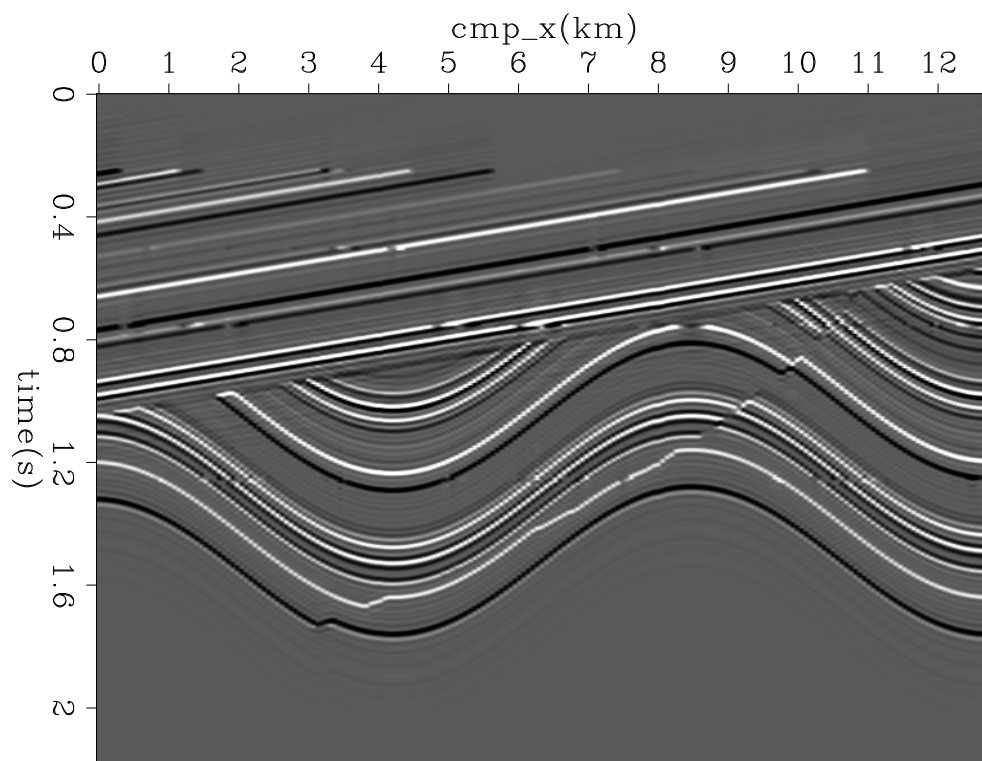


(a)

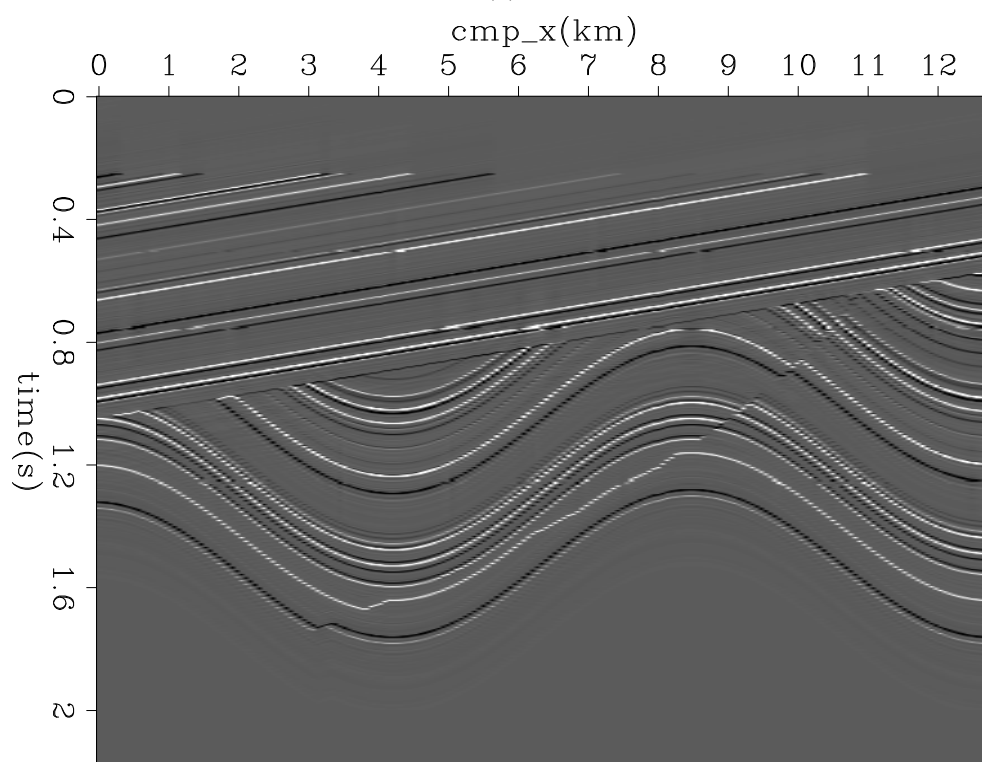


(b)

Figure 9: (a) The 2D synthetic reflectivity model; (b) the synthetic data generated using the zero-phase wavelet. [ER]



(a)



(b)

Figure 10: Given the 2D synthetic data in Figure 9(b), (a) reflectivity model retrieved using our method; (b) reflectivity model retrieved using the previous method. [ER]



deconvolution using our method and the previous method. Both perform well to retrieve the sparse reflectivity in this field data.

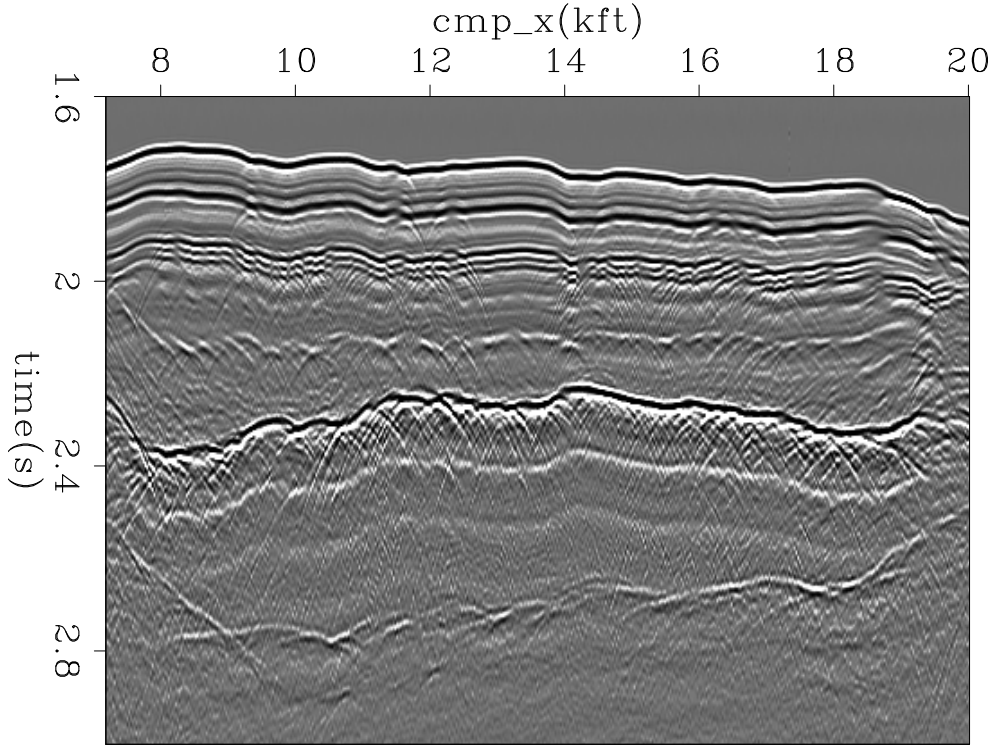
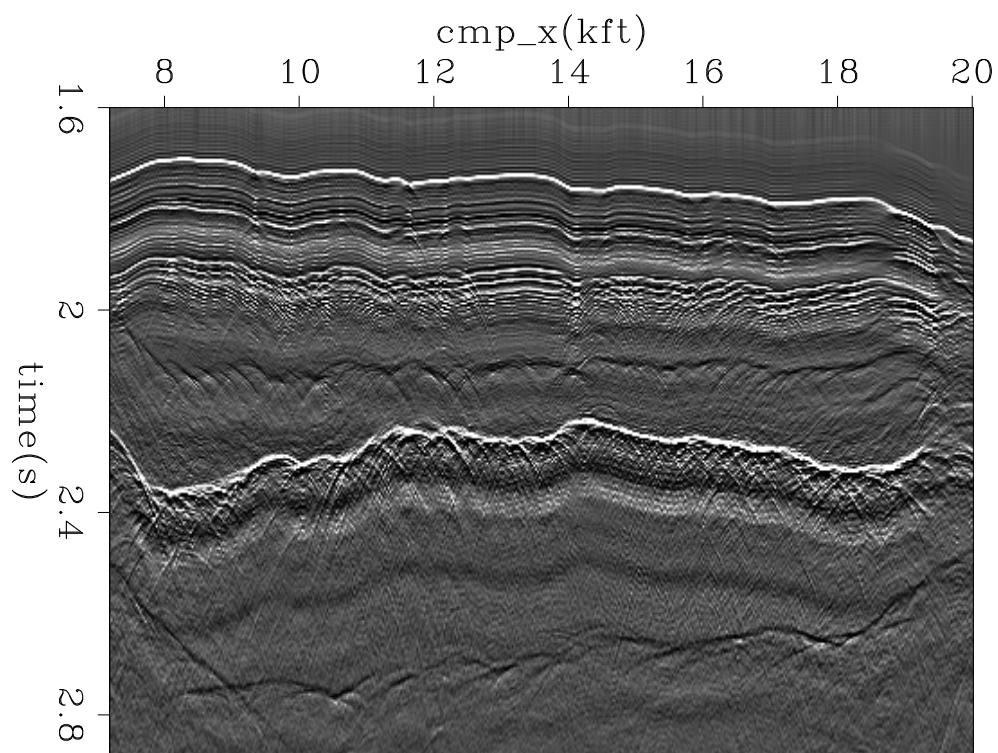


Figure 11: Input Common Offset data. [ER]

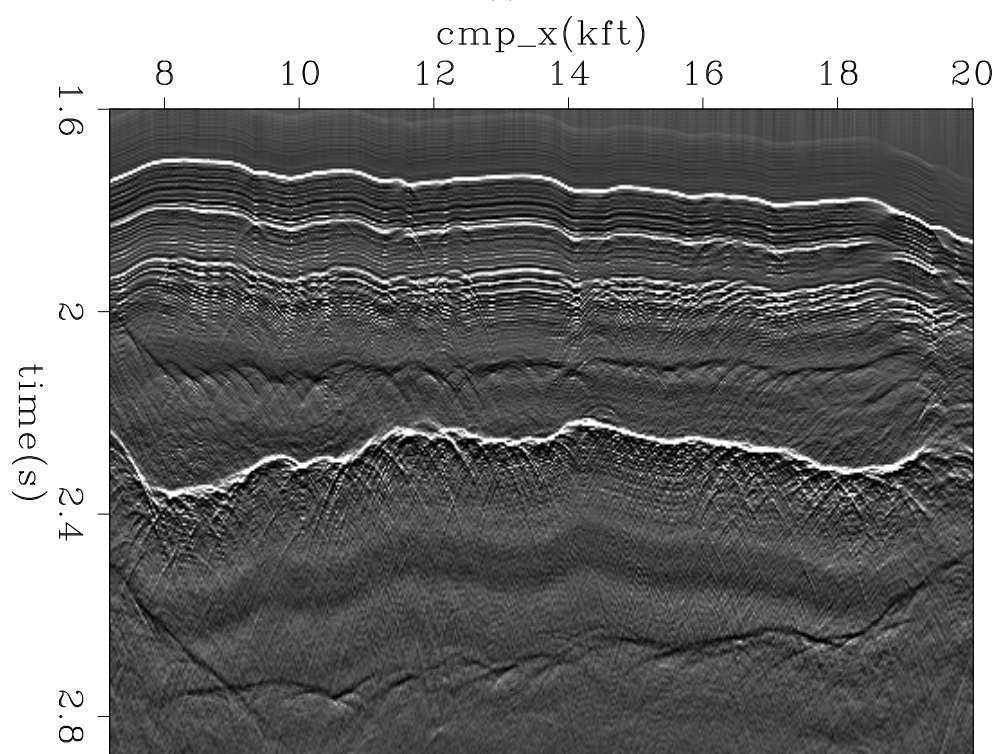
The filters estimated by our method are shown in Figures 14(a), and 14(b), and the filters estimated by the previous method are plotted in Figures 14(c) and 14(d). Because the wavelet we aim to invert is not symmetric, filter  $a$  and filter  $b$  are not equal. However, the strong events look like a double ghost (white, black, white), which approximates a symmetric wavelet. Thus we would like our filters to resemble each other. From the result, we notice that our filters satisfy these expectation.

Figures 13(a) and 13(b) show the estimated shot waveform. The data sampling is 4ms. We notice that both of our method estimate the bubbles and the double ghost. However, our inverted waveform is more like a double ghost, which can be noticed in the data. The reason is that our filters more resemble each other than the ones estimated by the previous method.

Our method requires 2 linear iterations and 100 non-linear iterations, or only 200 iterations in total. Zhang and Claerbout (2010) used 100 iterations and 8 non-linear iterations, or 800 iterations in total. Therefore, our method is four times faster, which is a large reduction in computational cost.



(a)



(b)

Figure 12: Given the common offset data in Figure 11, (a) reflectivity model retrieved using our method; (b) reflectivity model retrieved using the previous method. **[ER]**

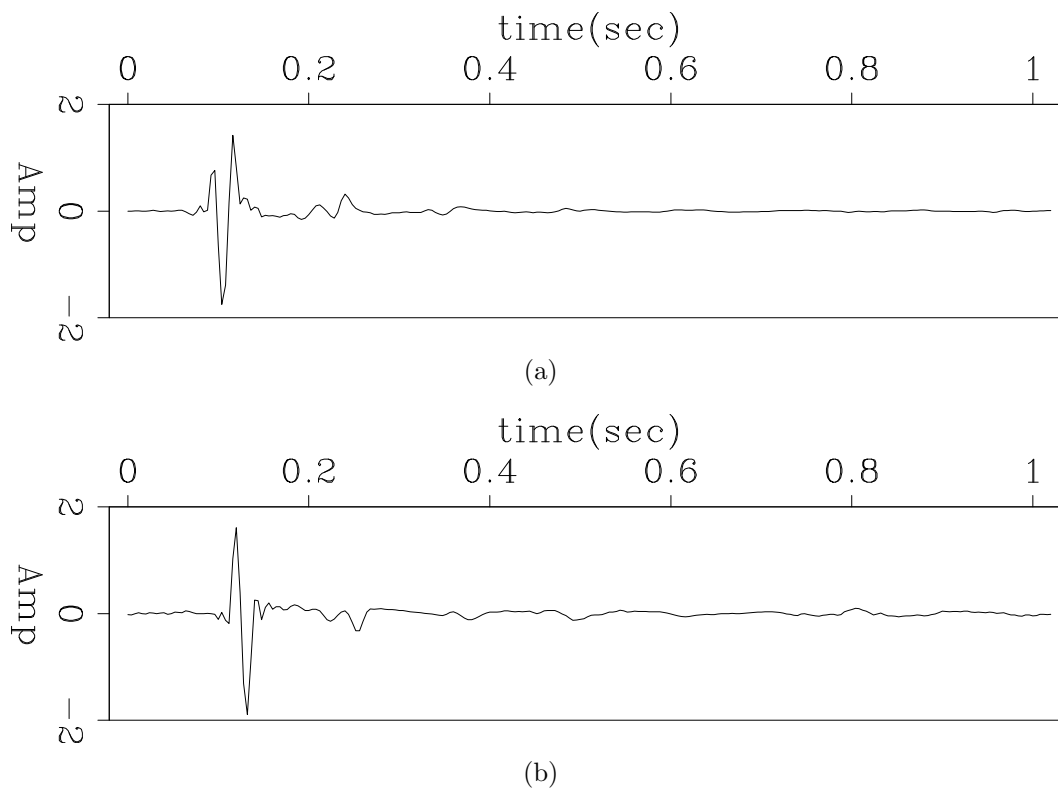


Figure 13: For 2D synthetic data, (a) shot wavelet estimated by our method; (b) shot wavelet estimated by the previous method. [ER]

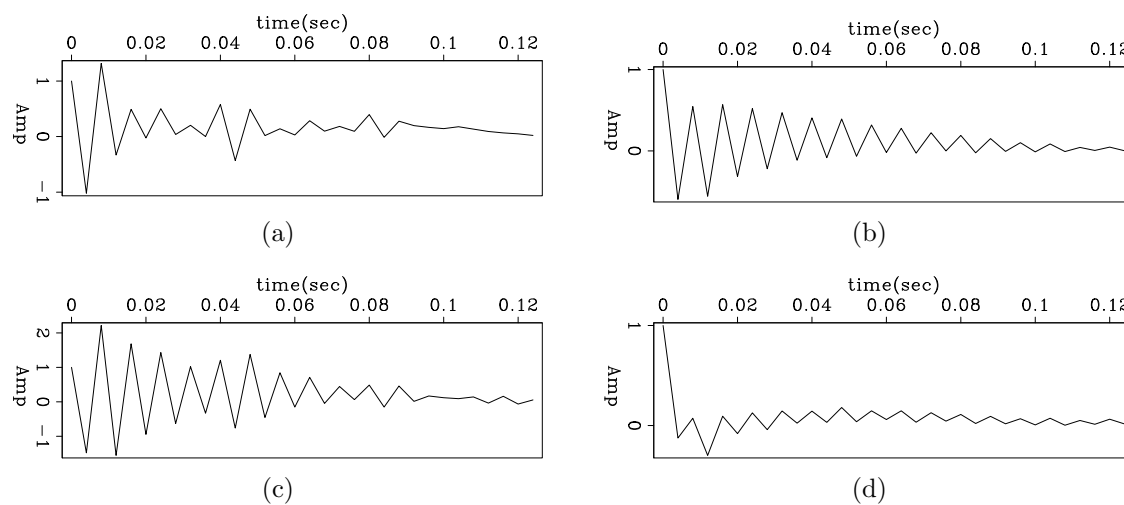


Figure 14: Given the common offset data in Figure 11, (a) filter *a* estimated by our method; (b) filter *b* estimated by our method; (c) filter *a* estimated by the previous method; (d) filter *b* estimated by the previous method. [ER]

## CONCLUSION

In this paper, we introduce an algorithm for bidirectional deconvolution that estimates the two filters simultaneously. We test the new method on three data examples. The results show that the wavelet can be compressed almost into a spike. When we are dealing with the zero-phase wavelet, we obtain two identical filters, a major improvement compared with the previous bidirectional method. Another important advantage is the low computational cost and fast convergence rate due to the reduced number of linear iterations. However, we are surprised to see that our results are not as spiky as the ones produced by Zhang and Claerbout (2010). One possible reason may be the nature of the non-linear problem. Perhaps we need a good initial guess or preconditioning to achieve acceptable results.

## FUTURE WORK

As mentioned previously, the nature of the nonlinear problem strongly affects our results. Thus, a good initial guess is needed to obtain a better sparse reflectivity. In most cases, data will resemble the Ricker wavelet, as is true for the band-limited marine seismic data with ghosts and the for the land response of an accelerometer. For this situation, we can use the Ricker wavelet to approximate the data and derive the initial filter from this wavelet. Since the Ricker wavelet vanishes at zero frequency and at the Nyquist frequency, it has no stable inverse. Therefore, we use the approximated Ricker wavelet instead of the true one.

Another potential solution is to do the preconditioning, which utilizes prior information. In this non-linear problem, we hope it can guide the gradient along sensible pathways thus avoiding potential local minima.

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## REFERENCES

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