A new fitting goal for bidirectional deconvolution

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ABSTRACT

In this paper, we introduce a new fitting goal for bidirectional deconvolution. In our new method, we estimate the causal filters and anti-causal filters simultaneously instead of separately. We test three data examples and discuss the effectiveness and limitations of the method. The result shows that the wavelet can be compressed almost into a spike using our method. The two filters can be estimated equally when we are dealing with the zero-phase wavelet. In addition, our new method has a lower computational cost and faster convergence rate than previous methods.

INTRODUCTION

In a previous report, Zhang and Claerbout (2010) introduced a new bidirectional deconvolution method that overcomes the minimum-phase assumption. They factored the mixed-phase wavelet into two parts, the minimum-phase part and the maximum-phase part, which can be estimated by a causal filter and an anti-causal filter, respectively. Since such deconvolution is a non-linear problem, a pair of conventional linear deconvolutions were utilized to invert these two filters alternately and iteratively. In their paper, both theory and data examples show that the mixed-phase wavelet can be perfectly inverted using this bidirectional deconvolution.

However, there are some obstacles to inverting these two filters separately. First, inverting two filters takes twice as many linear iterations as inverting one, and non-linear iterations are also necessary to solve the two inversions iteratively. These two kinds of iterations may lead to slow convergence. Second, the filters must be reversed during each non-linear iteration. Thus to estimate one filter also requires its reverse in the inversion. In addition, inverting a zero-phase wavelet produces two different filters; that is, the causal part and the anti-causal part are different, which is contrary to the nature of the zero-phase wavelet.

To avoid these problems, we invert these two filters simultaneously instead of separately.

THEORY

In this paper, we still rely on the idea of bidirectional deconvolution to deal with the mixed-phase wavelet. The wavelet can be factored into a minimum-phase part and a non-minimum-phase part. The deconvolution problem can be defined as follows:

$$d * a * b^r = r, \tag{1}$$

where d is the data, a is the unknown causal filter, and b is the unknown anti-causal filter. Again the hybrid norm is applied to r, and the reflectivity model is simply r plus a time shift. Now think of perturbations Δa and Δb:

$$d * (a + \Delta a) * (b^r + \Delta b^r) = r. \tag{2}$$

If we assume the the nonlinear part $\Delta a \Delta b$ is relatively small, we can neglect this term as follows:

$$d * a * b^r + d * a * \Delta b^r + d * b^r * \Delta a \approx r. \tag{3}$$

We use matrix algebraic notation to rewrite the fitting goal. We also want to guarantee filter a to be causal and filter b to be anti-causal during the iterations. For this we need mask matrices (diagonal matrices with ones on the diagonal where variables are free and zeros where they are constrained). The free-mask matrix for Δa is denoted $\mathbf{K}$, and that for Δb^r is denoted $\mathbf{Y}$.

$$\begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b^r} \end{bmatrix} \begin{bmatrix} \mathbf{Y} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{b^r} \\ \Delta \mathbf{a} \end{bmatrix} + \mathbf{d} * \mathbf{a} * \mathbf{b^r} \approx \mathbf{0}. \tag{4}$$

From equation (4), we have our new model $\mathbf{m} = \begin{bmatrix} \Delta \mathbf{b^r} & \Delta \mathbf{a} \end{bmatrix}^T$ and new operater $\mathbf{F} = \begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b^r} \end{bmatrix}$. Now we can acquire these two filters only by applying the conventional inversion method and hybrid norm solver. The pseudocode for minimizing this new objective function by the hyperbolic conjugate-direction method developed by Claerbout (2010) is:

$$
\begin{array}{l}
\text{non - linear iteration} \\
\{ \\
\quad \mathbf{r} = -\mathbf{d} * \mathbf{a} * \mathbf{b^r} \\
\quad \mathbf{F} = \begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b^r} \end{bmatrix} \\
\quad \text{linear iteration} \\
\quad \{ \\
\qquad \Delta \mathbf{m} = (\mathbf{FJ})^T C'(\mathbf{r}) \\
\qquad \Delta \mathbf{r} = \mathbf{FJ} \Delta \mathbf{m} \\
\qquad \mathbf{m} \leftarrow \text{Hyperbolic\_cgstep}(\Delta \mathbf{m}, \mathbf{m}, \Delta \mathbf{r}, \mathbf{r}) \\
\quad \} \\
\quad \mathbf{a} \leftarrow \mathbf{a} + \Delta \mathbf{a} \\
\quad \mathbf{b^r} \leftarrow \mathbf{b^r} + \Delta \mathbf{b^r} \\
\}
\end{array}
$$

where $C'(\mathbf{r})$ is the first derivative of the hybrid norm.

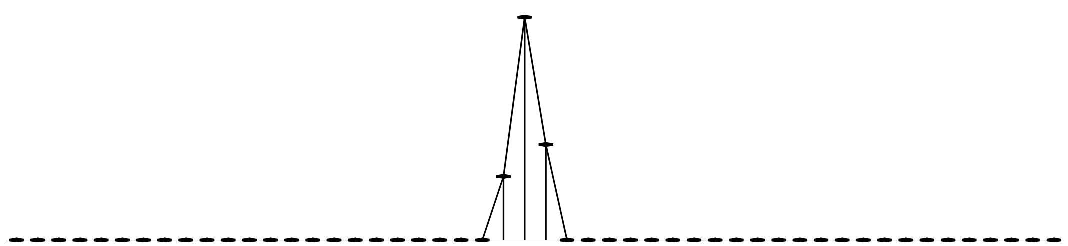
From the template we notice that both linear and non-linear iterations are needed. $\Delta\mathbf{a}$ and $\Delta\mathbf{b^r}$ are inverted by the hyperbolic conjugate-direction method in each linear iteration. Filters $\mathbf{a}$ and $\mathbf{b^r}$ are updated in the non-linear iteration, which generates a new operator $\mathbf{F}$ to update the model. However, this method requires only 2 linear iterations, instead of the 100 linear interations required by the previous method, greatly speeding convergence. In addition, there is no need to reverse the filters in the non-linear iteration, which makes our coding more convenient.

Although the fitting goal is linearized, we still need the initial model to be close enough to get a good result. Here we expect an impulse function for both filters a and b. The following sections will show the application of this new method and demonstrate its effectiveness and limitations.

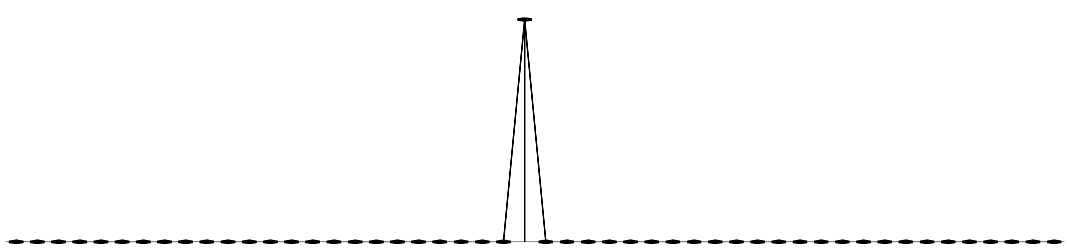
APPLICATION

Single wavelet

The first data example is the simplest mixed-phase wavelet, which only has three points [2,7,3]. We use it to verify the ability of our method to deal with the mixed-phase wavelet. The input data and its bidirectional result are shown in figure 1(a) and figure 1(b). In this case, our new method is able to compress the simple mixed phase wavelet into a spike.



(a)



(b)

Figure 1: (a) The three data points [2,7,3]; (b) the result of our deconvolution method. [**ER**]

To illustrate the capabilities and limitations of our new method, we analyze the results obtained by inverting the zero-phase wavelet. This wavelet is created by convolving the minimum-phase with its own time-reverse wavelet.

Figure 2(a) and figure 2(b) show the filters estimated by our method. Figure 2(c) and figure 2(d) show the filters inverted by the previous method of deconvolution. Here we time-reverse the anti-causal filter b^r into a causal filter b for easier comparison. Ideally, filter a and filter b should be identical, because the zero-phase wavelet is symmetric, with its minimum-phase part the same as its maximum-phase part, but time-reversed. The results of our method perfectly satisfy the theory, which shows the extreme similarity between filter a and filter b. When we invert the filters, the update direction is the same for both filters, because the searching gradients are equal. However, using the method from Zhang and Claerbout (2010) yields a filter a that is quite different from filter b, because they are inverted separately. By this measure, our new deconvolution is a big improvement.

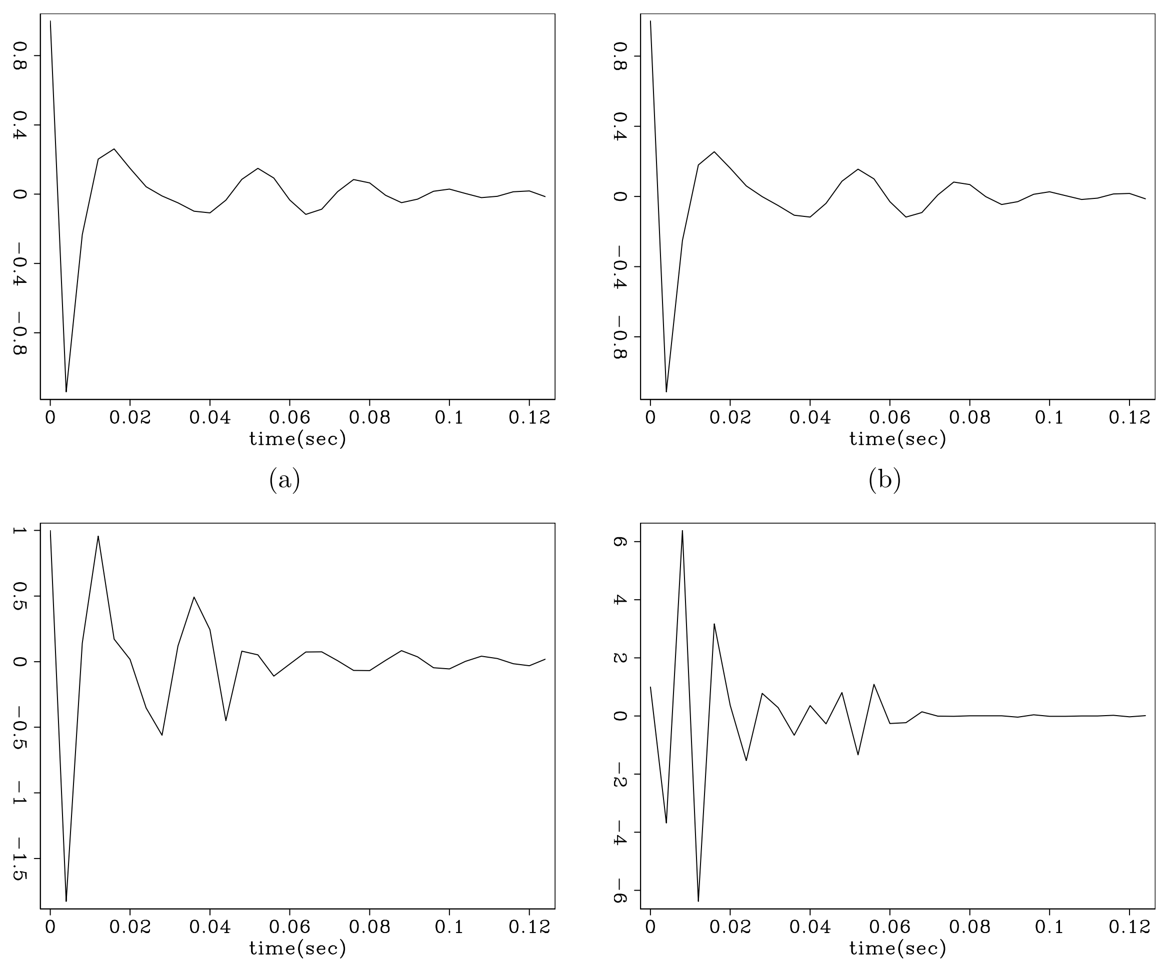


Figure 2: For zero-phase wavelet inversion, (a) filter a estimated by our method; (b) filter b estimated by our method; (c) filter a estimated by the previous method; (d) filter b estimated by the previous method. [**ER**]

Figure 3(a), figure 3(b) and figure 3(c) show the zero-phase wavelet and its birectional deconvolution, using our new fitting goal and the method of Zhang and Claerbout (2010). The results show that the wavelet is almost compressed into a spike by our method, but it is not as spiky as the result of the previous method. One possible reason may be the nature of the non-linear problem. There may be multiple minima in this problem, and due to our additional condition that filter a and filter b should be the same in this case, we find a different minimum, which leads to a different result. Thus a good starting guess may help us to get a better result.

2D synthetic data

After applying deconvolution on the simple 1D case, we test the Zhang and Claerbout (2010) method and our new method on a more complicated 2D synthetic data. Figure 4(a) shows the starting reflectivity model. Figure 4(b) shows the data generated by convolving the reflectivity model with the zero-phase wavelet in the previous section. All traces use the same wavelet when generating the data, and all traces share the same wavelet when we are doing the deconvolution.

Figure 5(a) and figure 5(b) show the filters estimated by our method. Figure 5(c) and figure 5(d) show the filters inverted by the older method of deconvolution. As we expect, our estimated filters are identical. However, the filters inverted by the previous method have slight differences.

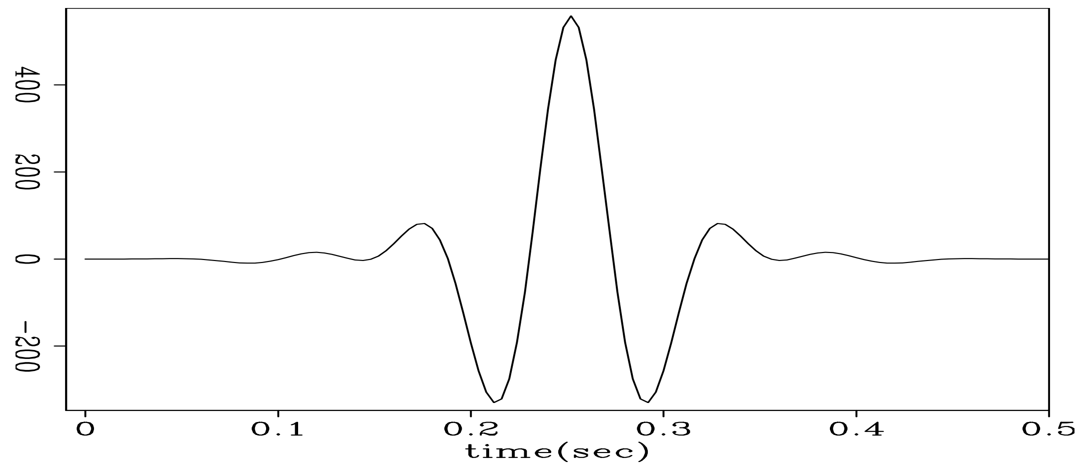
Figure 6(a) and figure 6(b) show the birectional deconvolution using our method and the older method. Both methods retrieve the sparse reflectivity model and compress the wavelet into a spike, but the deconvolution model produced by the previous method is more spiky than ours, just as in the previous section, because of the wavelet we use.

We also compare the computational costs of these two methods. To deal with this synthetic data, we use 2 linear iterations and 280 non-linear iterations to reach convergence, whereas Zhang and Claerbout (2010) used 100 linear iterations and 20 non-linear iterations, which amounts to 2000 iterations. In fact, our code is almost 6 times faster than the previous method. Although our result is not as spiky as the previous result, the computational advantage is great enough to make the compromise worthwhile.

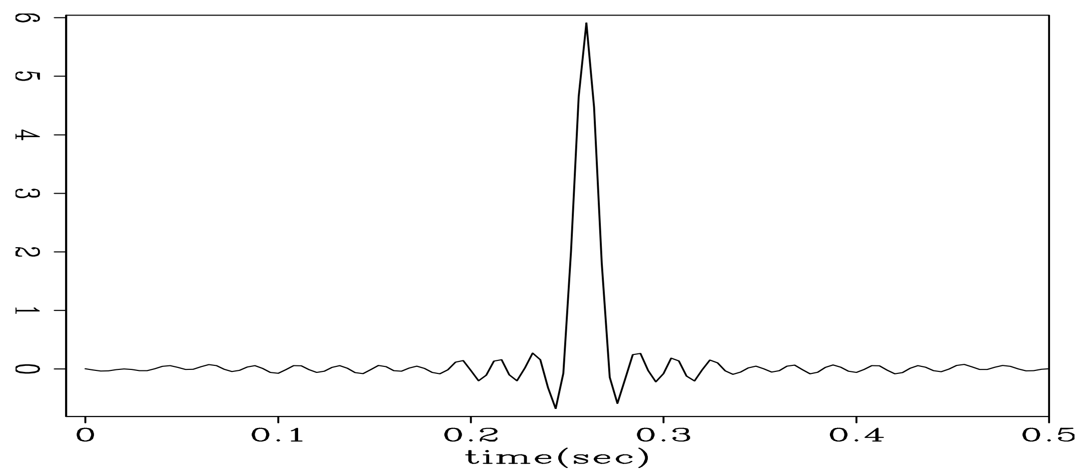
2D field data

The field data we use in this example is a common-offest section of marine field data. Figure 7 shows the input data. Figure 8(a) and figure 8(b) show the birectional deconvolution using our method and the previous method. Both perform well to retrieve the sparse reflectivity in this field data.

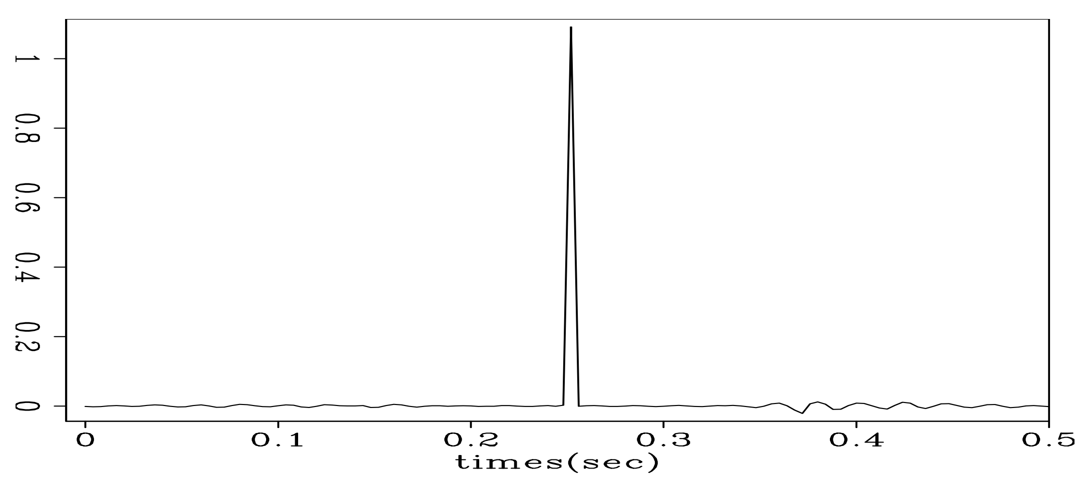
The filters estimated by our method are shown in figure 9(a), and figure 9(b),



(a)

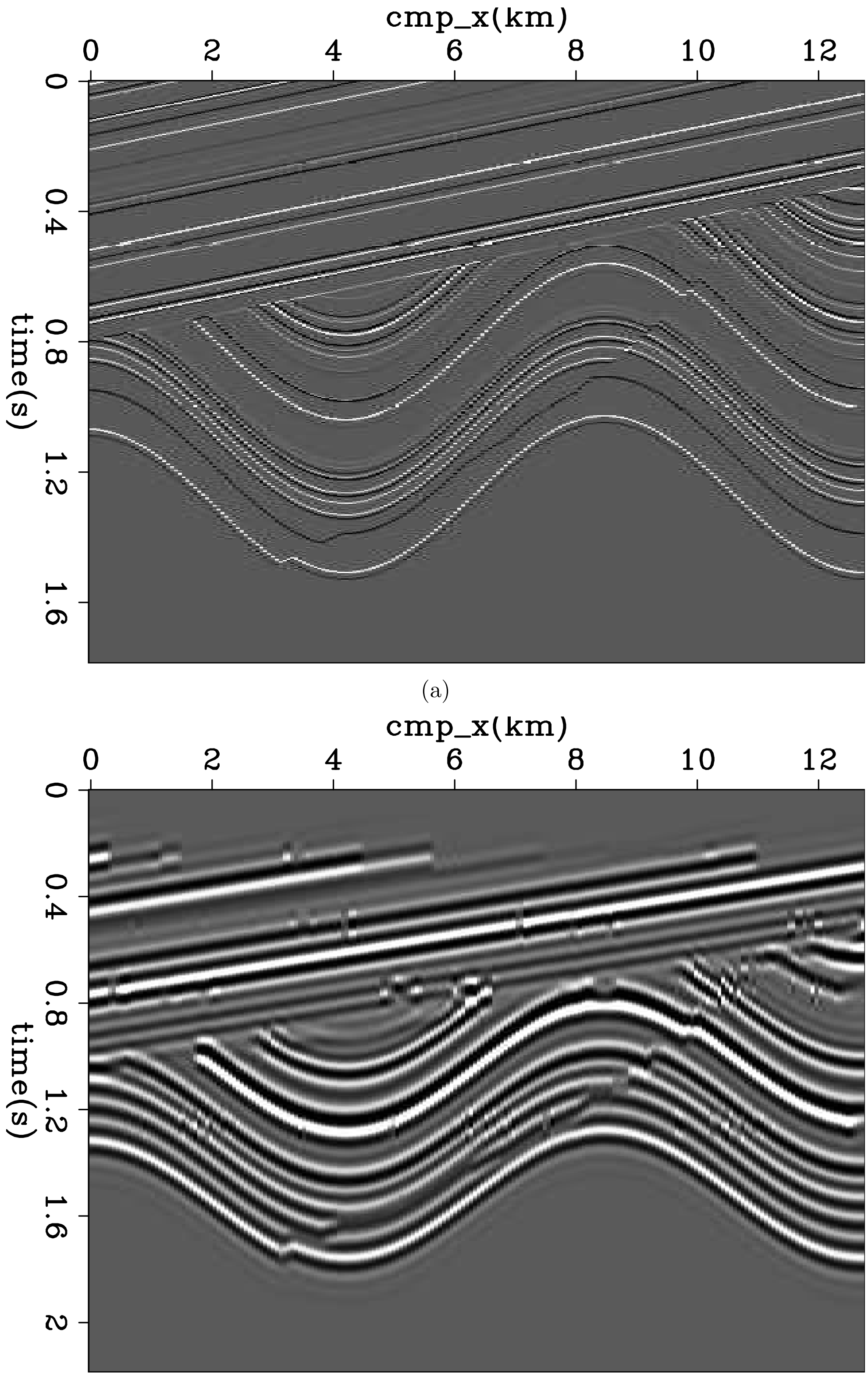


(b)



(c)

Figure 3: (a) Zero-phase wavelet; (b) deconvolution result by our method; (c) deconvolution result by the previous method. [**ER**]



(b)

Figure 4: (a) The 2D synthetic reflectivity model; (b) the synthetic data generated using the zero-phase wavelet. [**ER**]

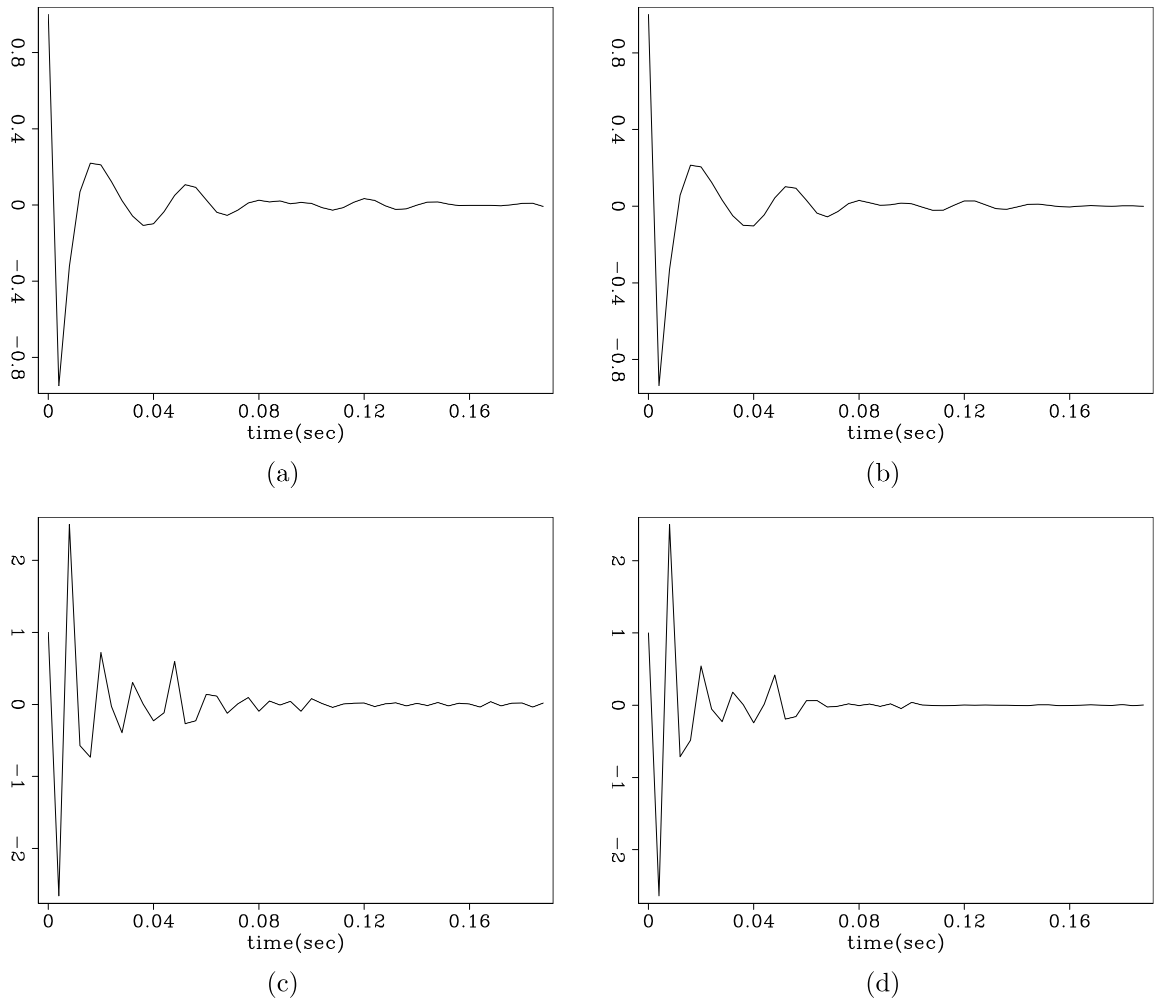
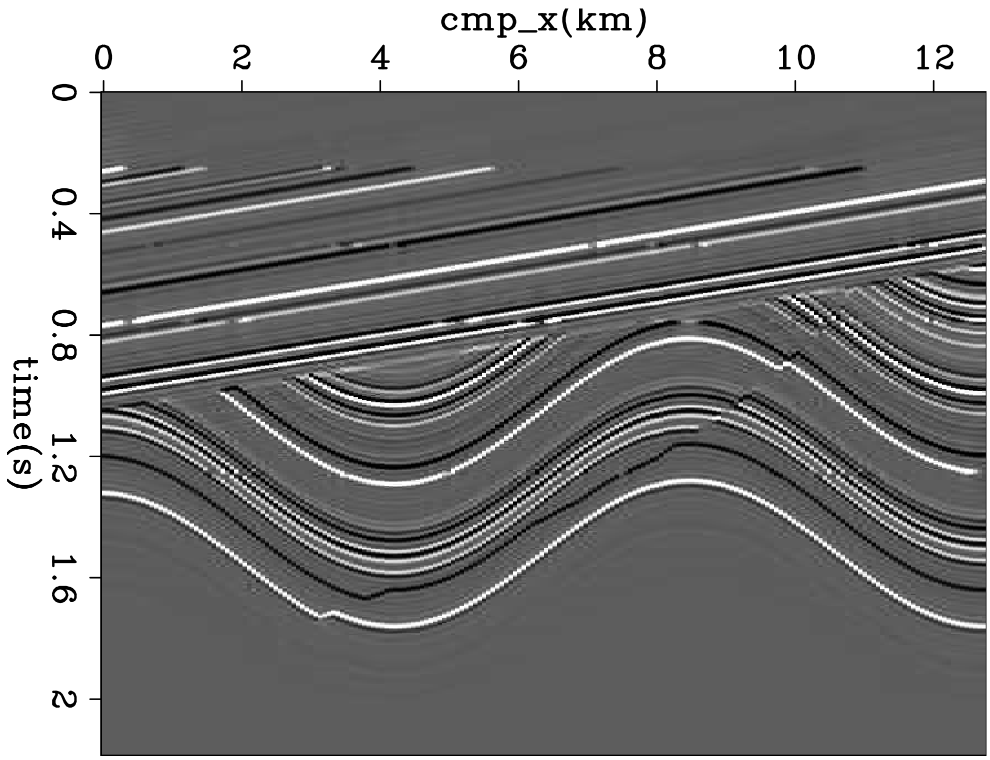
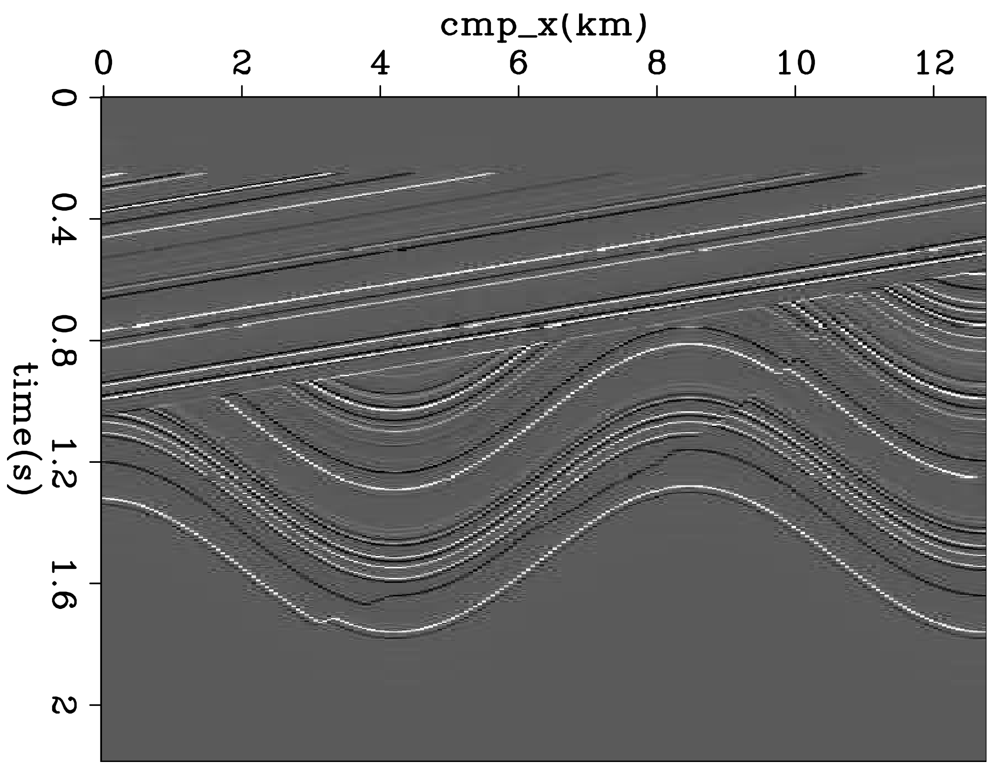


Figure 5: For 2D synthetic data, (a) filter a estimated by our method; (b) filter b estimated by our method; (c) filter a estimated by the previous method; (d) filter b estimated by the previous method. [**ER**]



(a)



(b)

Figure 6: Given the 2D synthetic data in Figure 4(b), (a) reflectivity model retrieved using our method; (b) reflectivity model retrieved using the previous method. [**ER**]

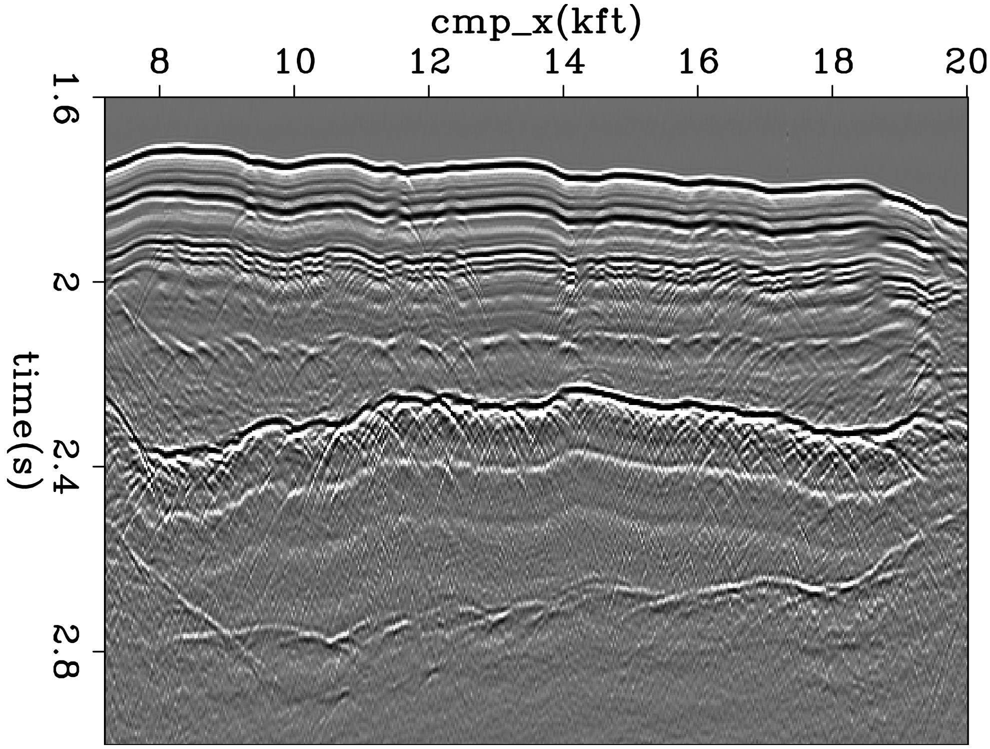


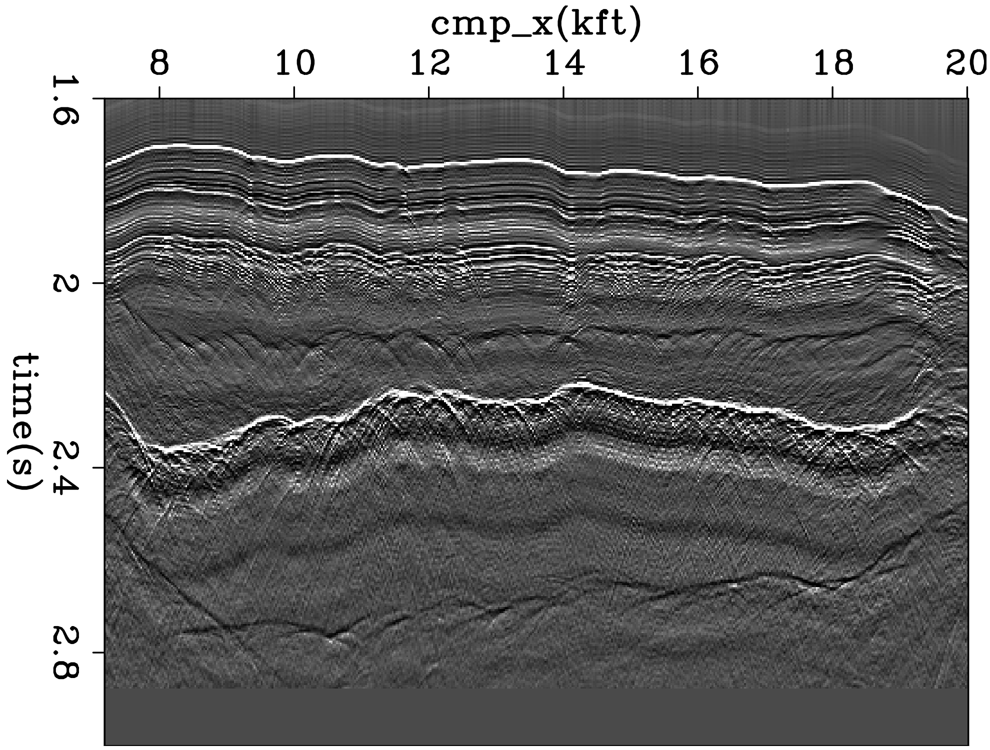
Figure 7: Input Common Offset data. [**ER**]

and the filters estimated by the previous method are plotted in figure 9(c), and figure 9(d). Because the wavelet we aim to invert is not symmetric, filter a and filter b are not equal. However, the strong events look like a double ghost (white, black,white), which approximates a symmetric wavelet. Thus we would like our filters to resemble each other. From the result, we notice that our filters satisfy our expectation.

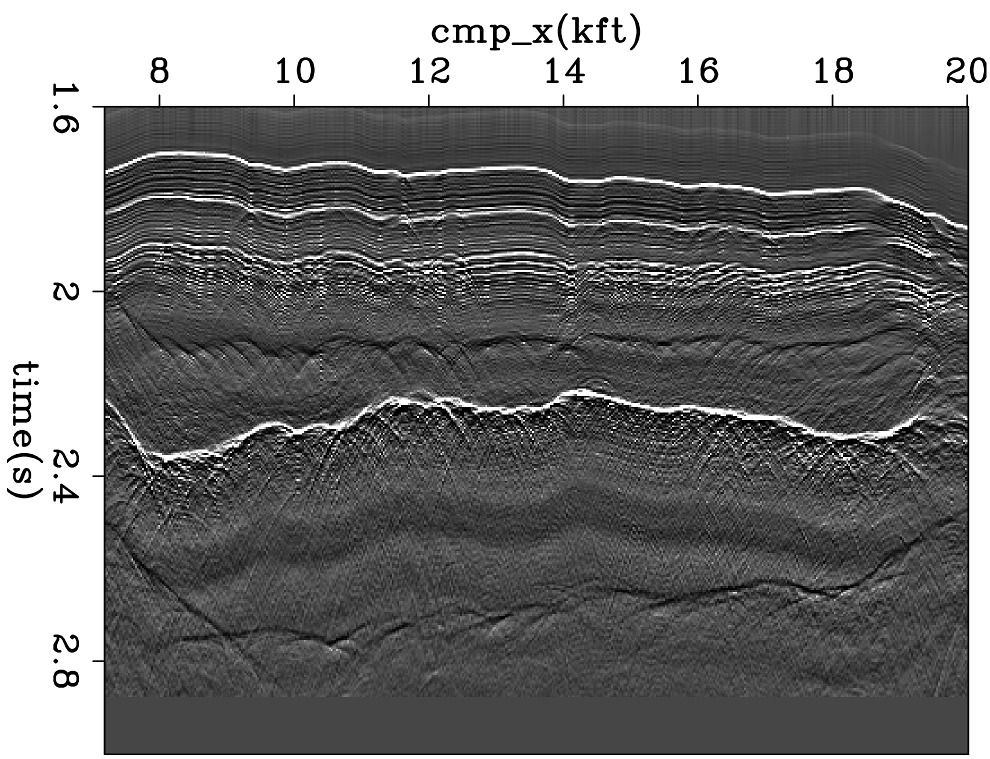
Our method requires 2 linear iterations and 100 non-linear iterations, or only 200 iteration in total. Zhang and Claerbout (2010) used 100 iterations and 8 non-linear iterations, or 800 iterations in total. Therefore, our method is 4 times faster, which is a large reduction in computational cost.

CONCLUSION

In this paper, we introduce a new fitting goal for bidirectional deconvolution to estimate the filters simultaneously. We test the new method on three data examples. The results show that the wavelet can be compressed almost into a spike. When we are dealing with the zero-phase wavelet, we obtain two identical filters, which is a big improvement compared with the previous bidirectional method. Another important advantage is the low computational cost and fast convergence rate due to the reduced number of linear iterations. Since our result is not perfectly spiky, we need a good initial guess to achieve acceptable results.



(a)



(b)

Figure 8: Given the common offset data in Figure 7, (a) reflectivity model retrieved using our method; (b) reflectivity model retrieved using the previous method. [**ER**]

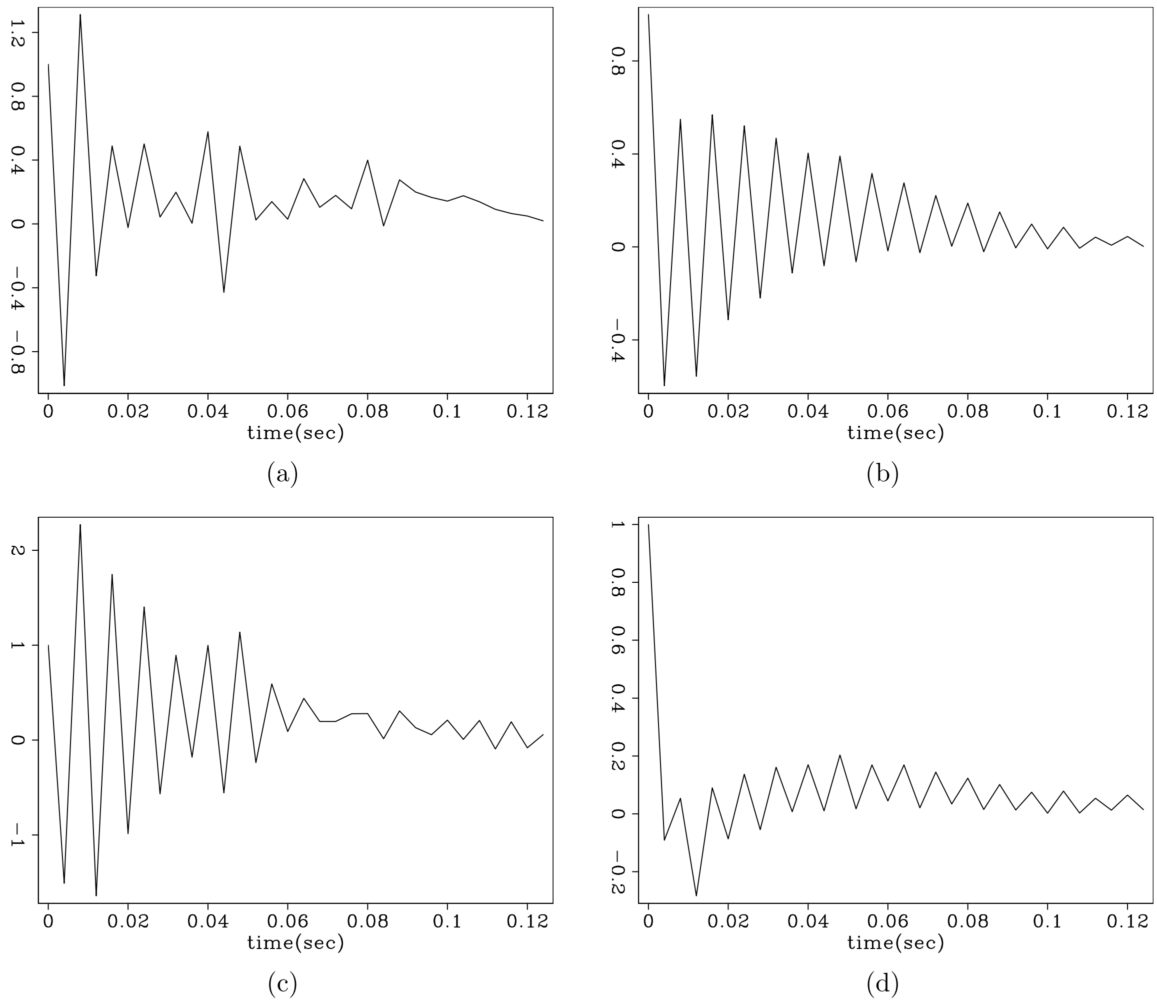


Figure 9: Given the common offset data in Figure 7, (a) filter a estimated by our method; (b) filter b estimated by our method; (c) filter a estimated by the previous method; (d) filter b estimated by the previous method. [**ER**]

FUTURE WORK

As we mention in this paper, the nature of the nonlinear problem strongly affects our result. Thus a good initial guess is needed to obtain a better sparse reflectivity. In most cases, data will resemble the Ricker wavelet, as is true for band-limited marine seismic data with ghosts and the for the land response of an accelerometer. For this situation, we can use the Ricker wavelet to approximate the data and derive the initial filter from this wavelet. Since the Ricker wavelet vanishes at zero frequency and at the Nyquist frequency, it has no stable inverse. Therefore, we use the approximated Ricker wavelet instead of the true one.

Another potential solution is to apply the blind deconvolution in the frequency domain instead of the time domain to make the process faster and more reliable.

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