A new fitting goal for the bidirectional deconvolution

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ABSTRACT

In this paper,we introduce ~~another~~ fitting goal for ~~the~~ bidirectional ~~devonvolution.~~ In our new method, we estimate the causal filters and anti-causal filters simultaneously instead of separately. ~~Three~~ data expamples ~~have been tested~~ and the effectiveness and limitations of the method ~~have been discussed here.~~ The result shows that the wavelet can be ~~almost~~ compressed into a spike by our method. The two filters can be estimated equally when we are dealing with the zero phase wavelet. In addition, our new method has a lower computational cost and faster convergence rate ~~when compared with the~~ previous method.

INTRODUCTION

In ~~the~~ previous report ~~(Zhang and Claerbout (2010)), Yang~~ and Claerbout introduced a new bidirectional deconvolution method that overcomes the minimum phase assumption. They factored the mixed-phase wavelet into two parts, the minimum phase part and the maximum phase part, which can be estimated by a causal filter and an anti-causal filter respectively. Since such deconvolution is a non-linear problem, a pair of conventional linear deconvolutions were utilized to invert these two filters alternatively and iteratively. In their paper, both theory and data examples ~~shows~~ that the mixed-phase wavelet can be perfectly inverted ~~by~~ using this bidirectional deconvolution.

However, there are some ~~problems in the way of~~ inverting ~~this~~ two filters separately. ~~Firstly, they will have enough linear iterations for solving one inversion until another filter gets inverted. And they also need~~ non-linear iterations to solve the two inversions iteratively. These two kinds of iterations may ~~leads~~ to ~~a low~~ convergence ~~rate.~~ ~~Secondly, they have to reverse the filters~~ during each non-linear ~~iterations~~. Thus ~~when they~~ estimate one filter ~~they also need its~~ reverse in the inversion. In addition, ~~another issue has been raised which is also mentioned in their paper. When they inverted~~ a zero-phase wavelet~~, they produced~~ two different filters; that is, the causal part and the anti-causal part are different, which is against the nature of the zero phase wavelet.

~~Therefore,~~ we invert these two filters simultaneously instead of separately ~~to solve the above problems in this paper~~.

THEORY

In this paper, we still rely on the idea of bidirectional deconvolution ~~method~~ to deal with the mixed-phase wavelet. The wavelet can be factored ~~in to the~~ minimum-phase part and ~~the~~ non-minimum-phase part. The deconvolution problem can be defined as ~~below.~~

$$d * a * b^r = r, \tag{1}$$

where d is the data, a is the unknown causal filter and b is the unknown anti-causal filter. Again the hybrid norm is applied to r and the reflectivity model is ~~symply~~ r plus a time shift. Now think of perturbations Δa and Δb:

$$d * (a + \Delta a) * (b^r + \Delta b^r) = r. \tag{2}$$

If we assume the the nonlinear part $\Delta a \Delta b$ is relatively small, we can neglect this term as follows:

$$d * a * b^r + d * a * \Delta b^r + d * b^r * \Delta a \approx r. \tag{3}$$

~~Let's~~ use matrix algebraic notation to rewrite the fitting goal. We also want to guarantee filter a to be causal and filter b to be anti-causal during the iterations. For this we need mask matrices (diagonal ~~matricess~~ with ones on the diagonal where variables are free and zeros where they are constrained). The free-mask matrix for Δa is denoted $\mathbf{K}$ and that for the Δb^r is denoted $\mathbf{Y}$.

$$\begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b^r} \end{bmatrix} \begin{bmatrix} \mathbf{Y} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{b^r} \\ \Delta \mathbf{a} \end{bmatrix} + \mathbf{d} * \mathbf{a} * \mathbf{b^r} \approx \mathbf{0}. \tag{4}$$

From equation (4), we have our new model $\mathbf{m} = \begin{bmatrix} \Delta \mathbf{b^r} & \Delta \mathbf{a} \end{bmatrix}^T$ and new operater $\mathbf{F} = \begin{bmatrix} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b^r} \end{bmatrix}$. Now we can acquire these two filters only by applying the conventional inversion method and hybrid norm solver. The pseudocode for minimizing this new objective function by ~~H~~yperbolic ~~C~~onjugate ~~D~~irection method developed by ~~Jon (Claerbout (2010))~~ is:

non - linear iteration
{
$\quad \mathbf{r} = -\mathbf{d} * \mathbf{a} * \mathbf{b^r}$
$\quad \mathbf{F} = \left[\begin{array}{cc} \mathbf{d} * \mathbf{a} & \mathbf{d} * \mathbf{b^r} \end{array} \right]$
$\quad$ linear iteration
$\quad$ {
$\qquad \Delta\mathbf{m} = (\mathbf{FJ})^T C'(\mathbf{r})$
$\qquad \Delta\mathbf{r} = \mathbf{FJ}\Delta\mathbf{m}$
$\qquad \mathbf{m} \leftarrow \text{Hyperbolic\_cgstep}(\Delta\mathbf{m}, \mathbf{m}, \Delta\mathbf{r}, \mathbf{r})$
$\quad$ }
$\quad \mathbf{a} \leftarrow \mathbf{a} + \Delta\mathbf{a}$
$\quad \mathbf{b^r} \leftarrow \mathbf{b^r} + \Delta\mathbf{b^r}$
}

where $C'(\mathbf{r})$ is the first derivative of the hybrid norm.

From the template we notice that both linear and non-linear iterations are needed. $\Delta\mathbf{a}$ and $\Delta\mathbf{b^r}$ are inverted by ~~H~~yperbolic ~~C~~onjugate ~~D~~irection method in each linear iteration. Filter $\mathbf{a}$ and $\mathbf{b^r}$ are updated in the non-linear iteration, which generate a new operator $\mathbf{F}$ to update the model. However only 2 linear iterations ~~are applied in the method~~ instead of 100 linear interations ~~in~~ the previous method~~. Therefore, the convergence rate becomes fast~~. In addition, ~~we have~~ no need to reverse the filters in the non-linear iteration, which makes our coding more convenient.

Although the fitting goal is linearized, we still need the ~~starting sultion~~ to be close enough to get a good result. Here we expect an impulse function for both filter a and b. The following sections will show the application of this new method and demonstrate its effectiveness and limitations.

APPLICATION

Single wavelet

The first data example is the most simple mixed-phase wavelet, which only has three points [2,7,3]. We use it to verify the ability of our method to deal with the mixed-phase wavelet. The input data and its bidirectional result ~~is~~ shown in figure 1(a) and figure 1(b). In this case, our new method is able to compress the simple mixed phase wavelet into a spike.

To illustrate the capabilities and limitation of our new method, we analyze the results obtained by inverting the zero-phase wavelet. This wavelet is created by convolving the minimum-phase with its own time-reverse wavelet.

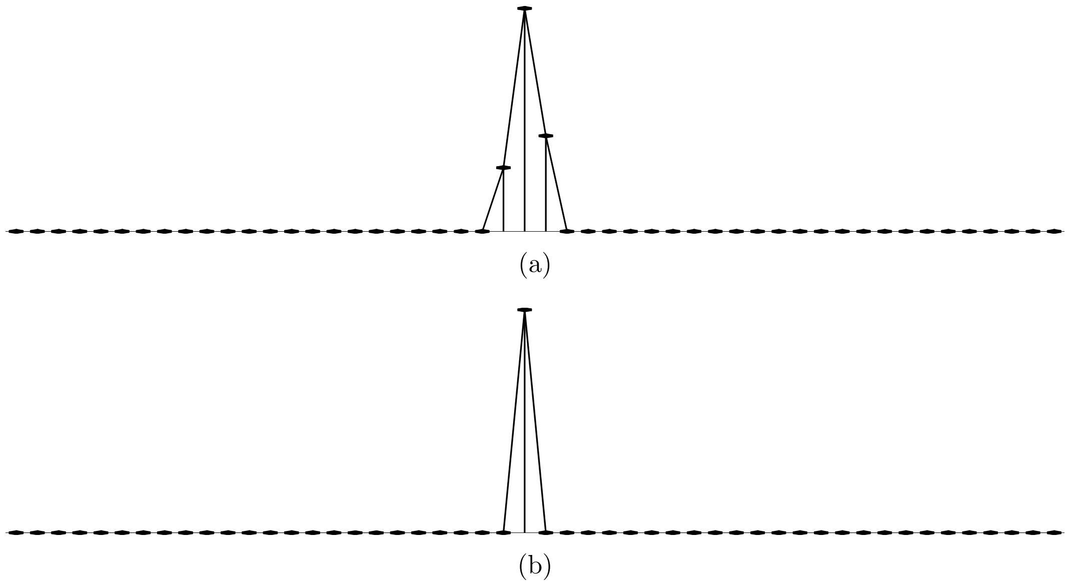


Figure 1: (a) The three points ~~data~~ [2,7,3]; (b) the ~~deconvolution~~ result ~~by~~ our method. [**ER**]

Figure 2(a) and figure 2(b) show the filters estimated by our method. Figure 2(c) and figure 2(d) show the filters inverted by the previous ~~way~~ of deconvolution. Here we time-reverse the anti-causal filter b^r into a causal filter b for ~~the convenience of comaprison.~~ Ideally filter a and filter b should be the same because the zero-phase wavelet is symmetric ~~whose~~ minimum part and maximum part ~~are~~ the same but time-reversed. The results ~~by~~ our method perfectly ~~satisfies~~ the theory, which shows the extreme similarity between filter a and filter b. ~~The reason is that when~~ we ~~are inverting~~ filters, ~~our~~ update direction for both filters ~~are the same~~ because the searching gradient are equal. However filter a is quite different from filter b ~~in the result by the previous method~~, because they are inverted separately. ~~From this point of view,~~ our new deconvolution is a big improvement.

Figure 3(a), figure 3(b) and figure 3(c) show the zero-phase wavelet and its birectional deconvolution ~~by~~ our new fitting goal and ~~the previous way of inverting~~. The results show that the wavelet is almost compressed into a spike by our method, but not as spiky as the result by the previous method. One possible reason may ~~own to~~ the nature of the non-linear problem. There may be multiple minima in this problem, and due to our additional condition that filter a and filter b should be the same in this case, we find a different ~~minima~~ which leads to a different result. Thus a good starting guess may help us to get a better result.

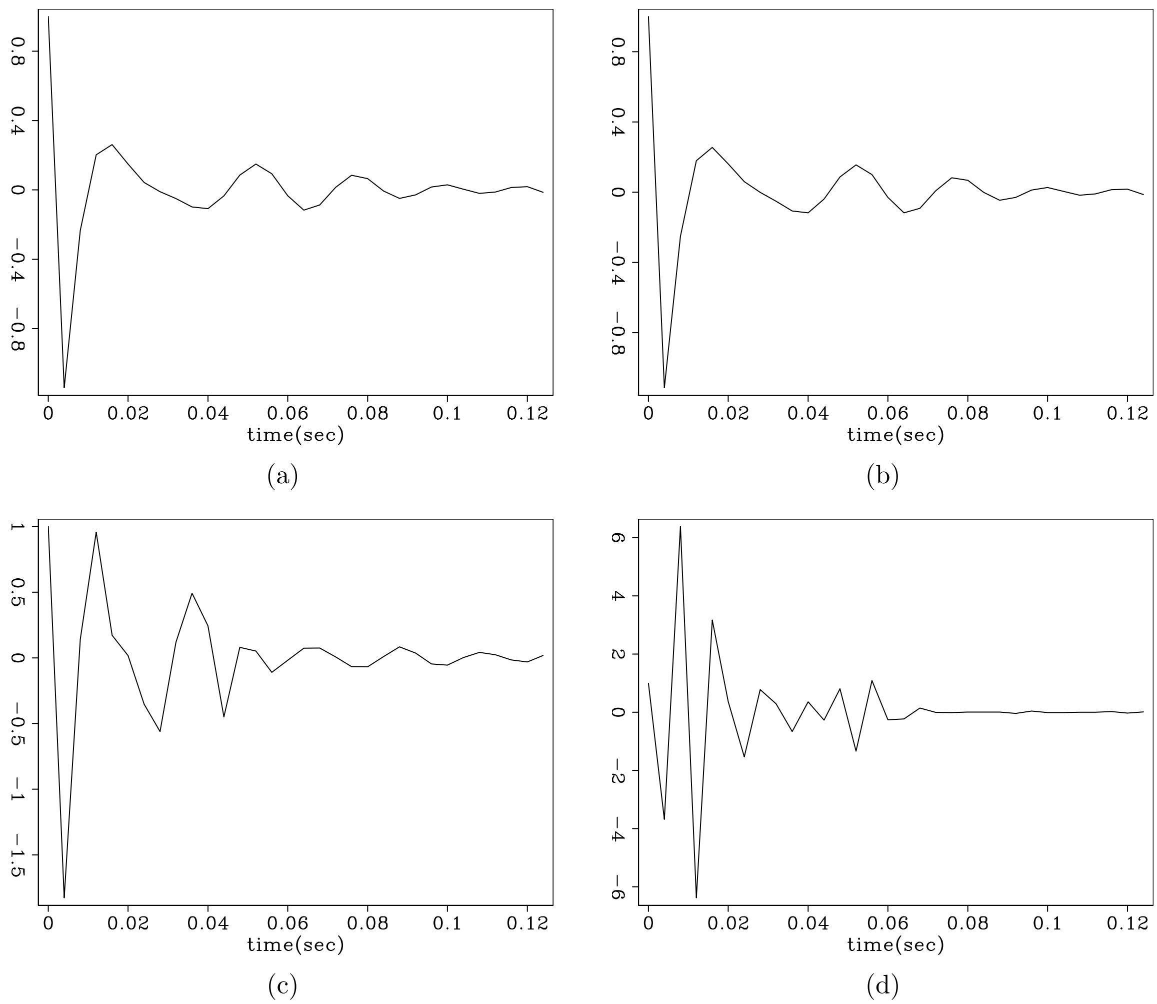


Figure 2: For zero phase wavelet inversion, (a) filter a estimated by our method; (b) filter b estimated by our method; (c) filter a estimated by the previous method; (d) filter b estimated by the previous method. [**ER**]

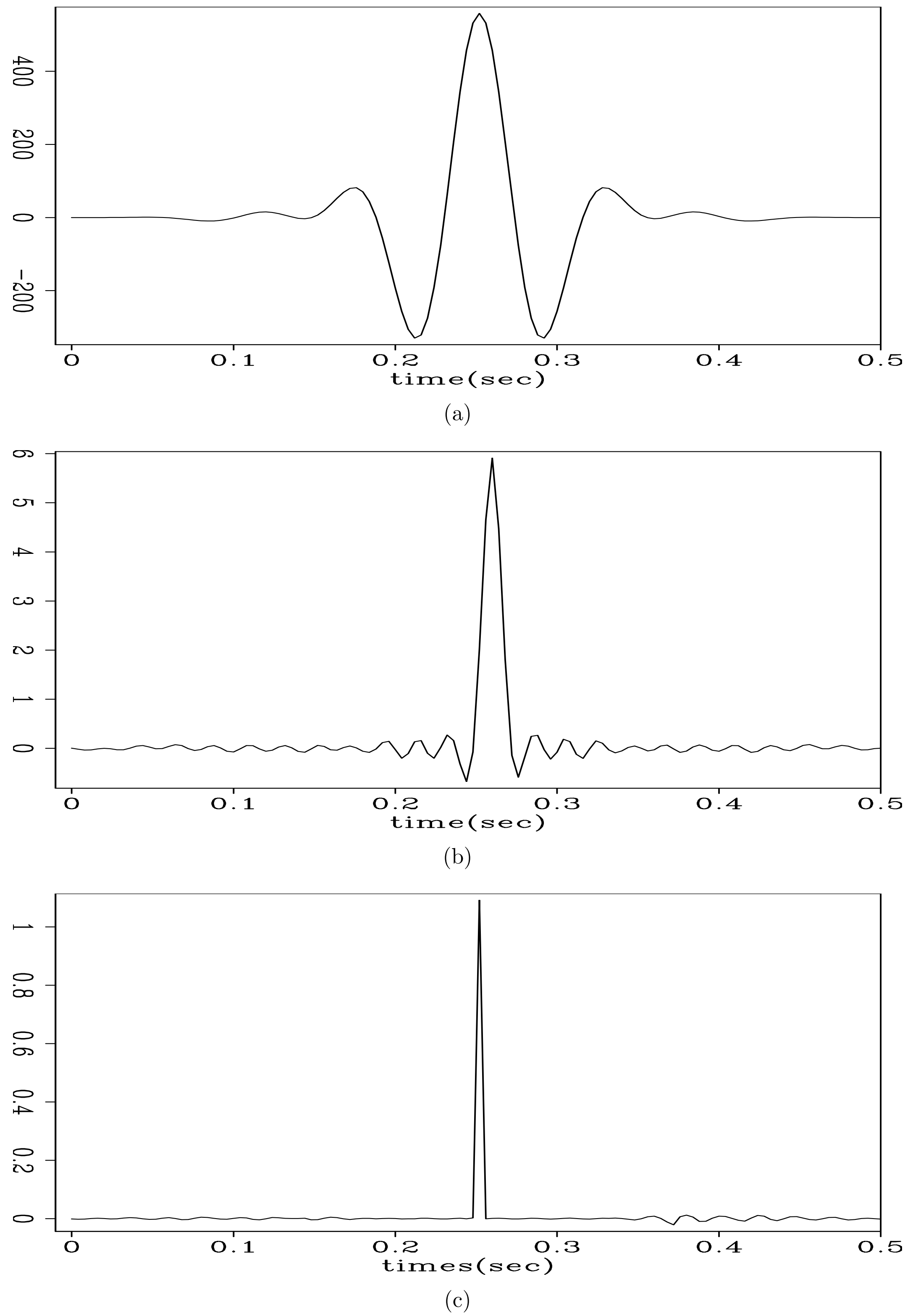


Figure 3: (a) Zero phase wavelet; (b) deconvolution result by our method; (c) deconvolution result by the previous method. [**ER**]

2D synthetic section

After applying deconvolution on the simple 1D case, we test the ~~previous~~ method and our new method on a more complicated 2D synthetic data. Figure 4(a) shows the starting reflectivity model. Figure 4(b) shows the data generated by convolving the reflectivity model with the zero~~-~~phase wavelet in the previous section. All traces use the same wavelet when generating the data, and all traces share the same wavelet when we are doing the deconvolution.

Figure 5(a) and figure 5(b) show the filters estimated by our method. Figure 5(c) and figure 5(d) show the filters inverted by the ~~previous way~~ of deconvolution. As we expect, our estimated filters are ~~totally the same.~~ However the filters inverted by the previous method have ~~slightly~~ differences.

Figure 6(a) and figure 6(b) show the birectional deconvolution ~~by our way of estimating and the previous way of inverting~~. Both ~~two~~ methods retrieve the sparse reflectivity model and compress the wavelet into a spike~~. But~~ deconvolution model by the previous method is more spiky than ours, just as ~~the same conclusion as~~ in the previous section because of the wavelet we use.

We also compare the computational cost between these two methods. To deal with this synthetic data, we use 2 linear iterations and 280 non-linear iterations ~~in our method to get it converged.~~ But Yang used 100 linear iterations and 20 non-linear iterations, which amounts to 2000 iterations. ~~The more iterations means more time consuming.~~ In fact, our code is almost 6 times faster than the previous. Although our result is not ~~that~~ spiky as the previous result, ~~it is good enough for the pratical application because of the computational advantage~~.

2D field data

The field data is a common-offest section of marine field data. Figure 7 shows the input data. Figure 8(a) and figure 8(b) show the birectional deconvolution ~~by our fashion~~ and the previous method. Both ~~of the method~~ perform ~~a good job~~ to retrieve the sparse reflectivity in this field data.

The ~~estimated~~ filters by our method are shown in figure 9(a), figure 9(b), and the ~~ones by~~ the previous ~~way~~ are plotted in figure 9(c), figure 9(d). Because the wavelet we aim to invert is not symmetric, ~~so~~ filter a and filter b are not equal. However, the strong events look like a double ghost (white, black,white), which approximate a symmetric wavelet. Thus we would like our filters ~~similar~~ each other. From the result we notice that our filters ~~are more~~ satisfy our expectation.

~~From the computational point of view, our~~ method requires 2 linear iterations and 100 non-linear iterations, ~~which only needs~~ 200 iteration in total. Yang used 100 iterations and 8 non-linear iterations, 800 iterations in total. ~~In the result,~~ our method is 4 times faster, which ~~reduce~~ ~~a large~~ computational cost.

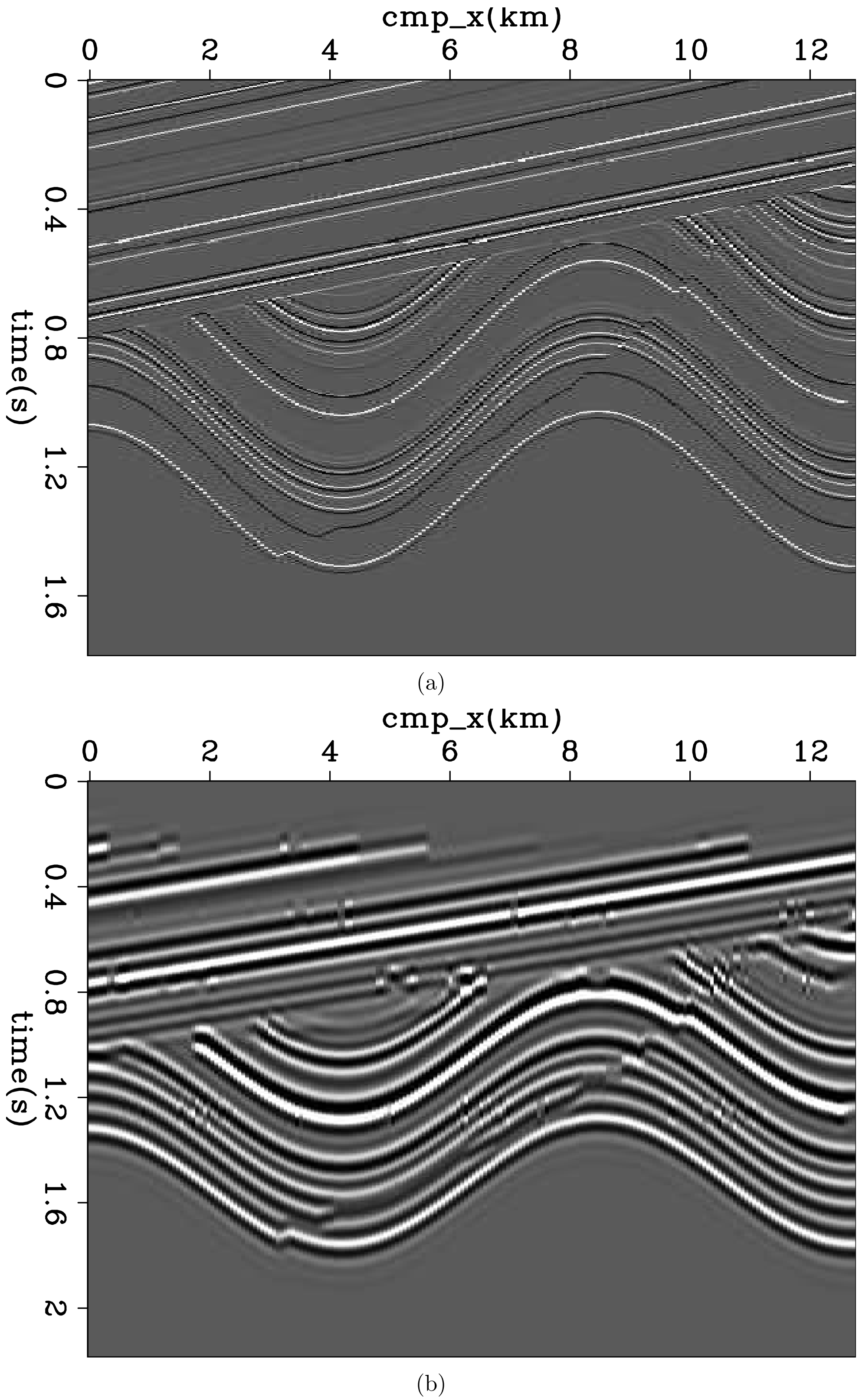


Figure 4: (a) The 2D synthetic reflectivity model; (b) the synthetic data generated using the zero-phase wavelet. [**ER**]

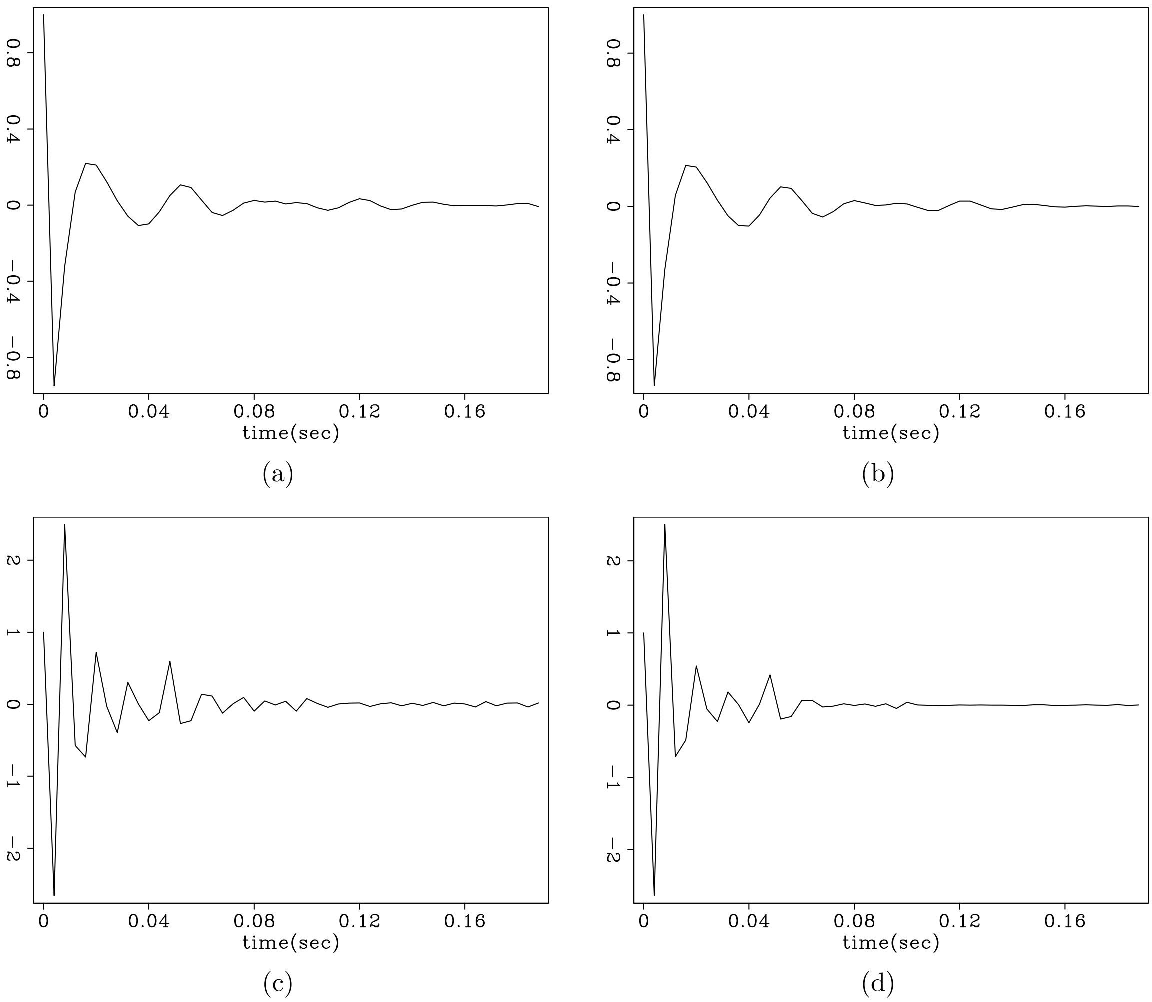
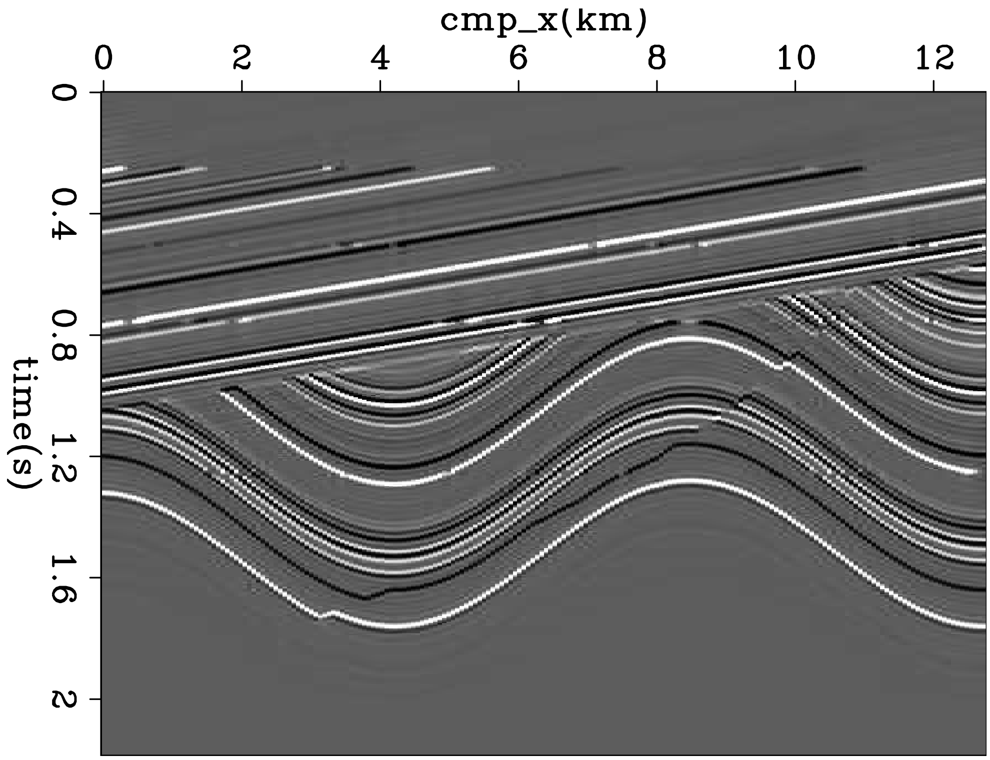
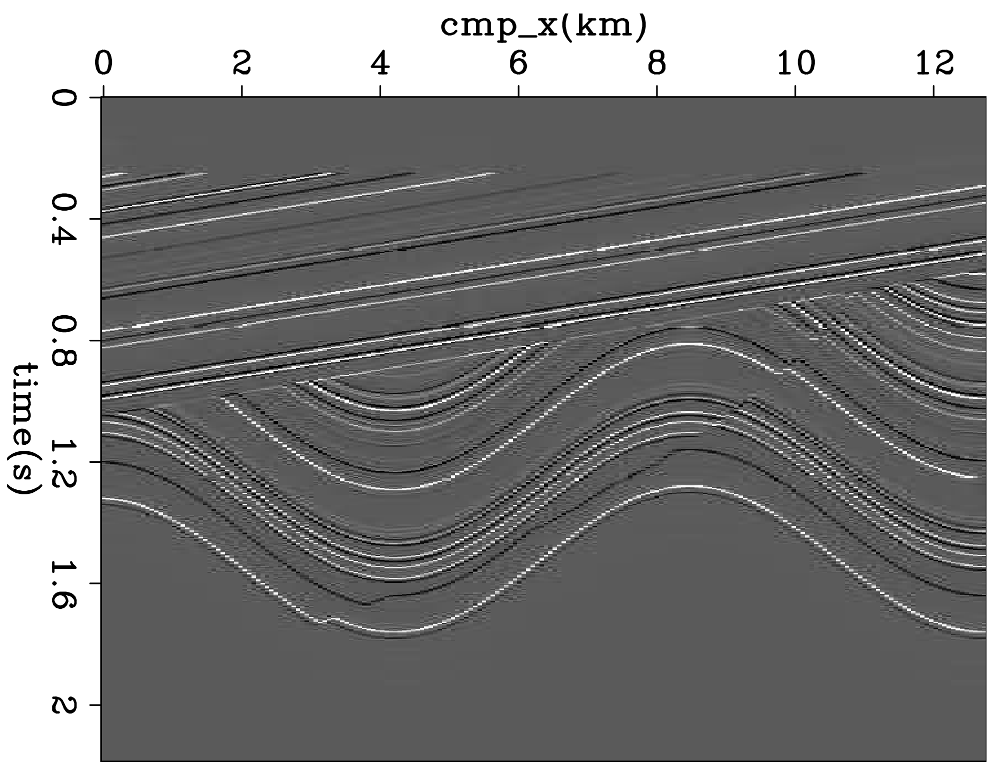


Figure 5: ~~Fot~~ 2D synthetic data, (a) filter a estimated by our method; (b) filter b estimated by our method; (c) filter a estimated by the previous method; (d) filter b estimated by the previous method. [**ER**]



(a)



(b)

Figure 6: Given the 2D synthetic data in Figure 4(b), (a) reflectivity model retrieved ~~from~~ our method; (b) reflectivity model retrieved ~~from~~ the previous method. [**ER**]

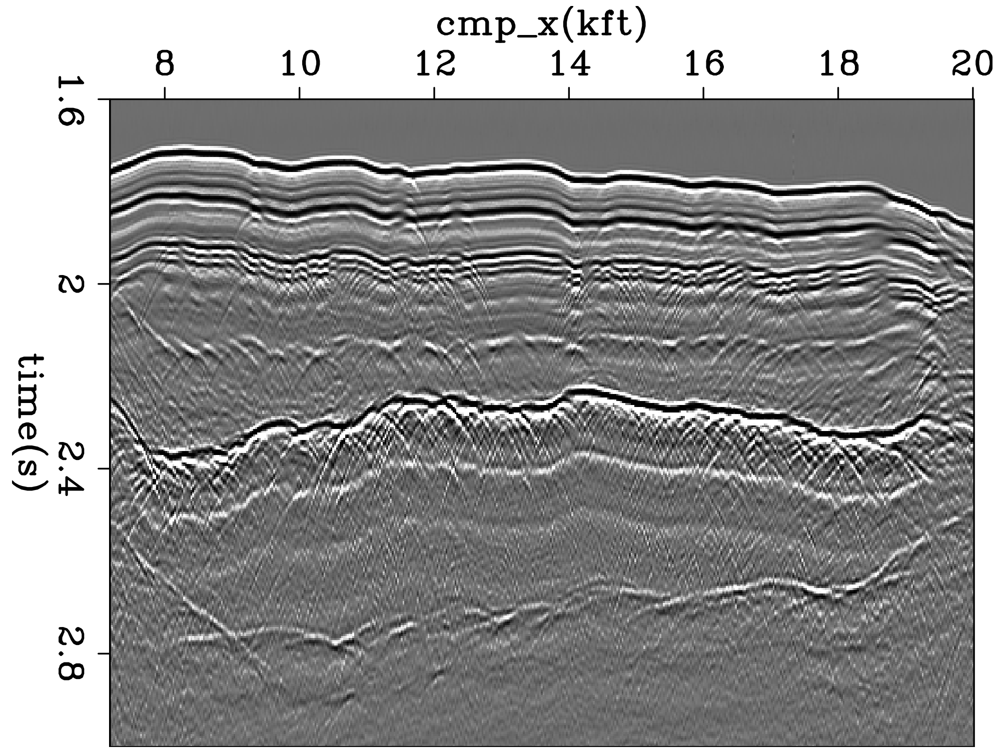


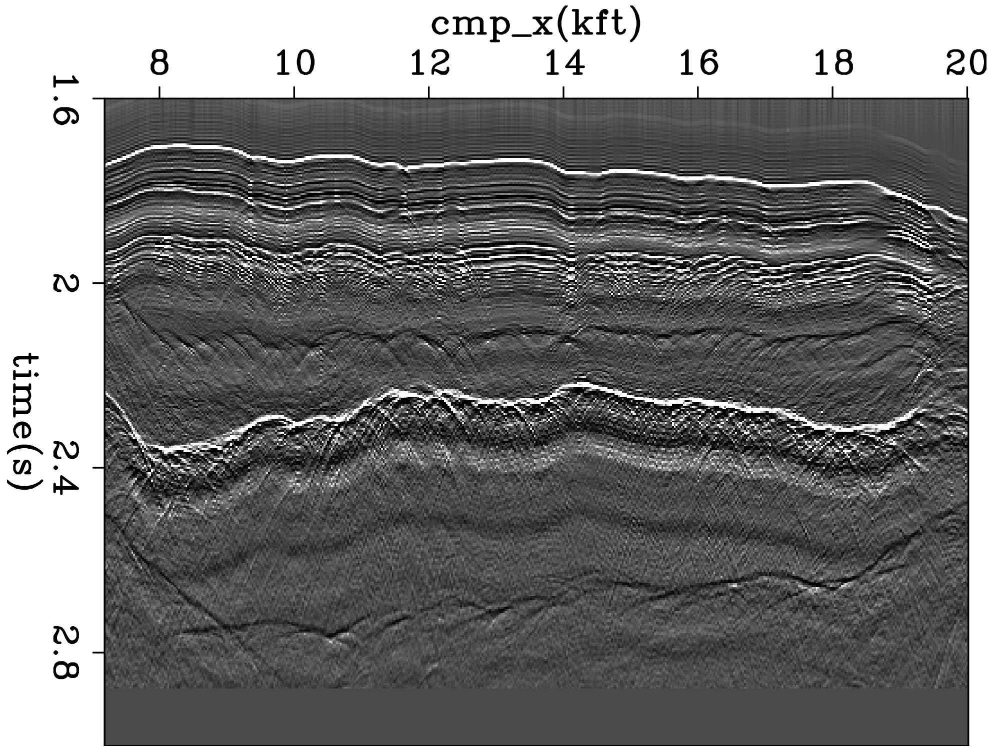
Figure 7: Input Common Offset data. [**ER**]

CONCLUSION

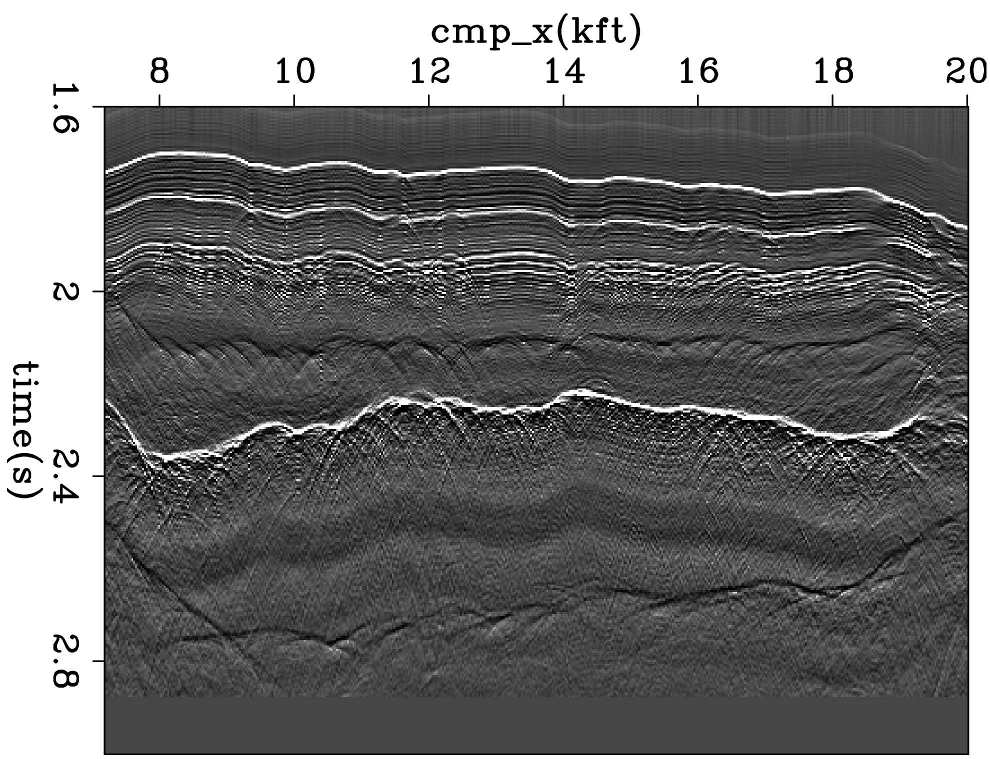
In this paper, we introduce ~~another~~ fitting goal for ~~the~~ bidirectional deconvolution to estimate the filters simultaneously. We test the new method on three data examples. The~~ result shows~~ that the wavelet can be ~~almost~~ compressed into a spike. When we are dealing with the zero~~-~~phase wavelet, we obtain two ~~same~~ filters, which is a big improvement compared with the previous bidirectional method. Another important advantage is the low computational cost and fast convergence rate due to the reduced linear iterations. Since our result is not perfectly spiky, we need a good initial guess to ~~make a better result~~.

FUTURE WORK

As we mention in this paper, the nature of the nonlinear problem strongly affect our result. Thus a good initial guess is needed to obtain a better sparse reflectivity. In most ~~of the~~ case, ~~we will handle the data which looks like~~ the Ricker wavelet, ~~for example, the~~ band-limited ~~mating~~ seismic data with ghosts and ~~the~~ land response of an accelerometer. For this situation, we can use Ricker wavelet to approximate the data and derive the initial filter from this wavelet. Since ~~ricker~~ wavelet vanishes at zero frequency and Nyquist frequency, it has no ~~bonded~~ inverse. ~~So~~ we ~~can~~ use the approximated Ricker wavelet instead of the true one.



(a)



(b)

Figure 8: Given the common offset data in Figure 7, (a) reflectivity model retrieved ~~from~~ our method; (b) reflectivity model retrieved ~~from~~ the previous method. [**ER**]

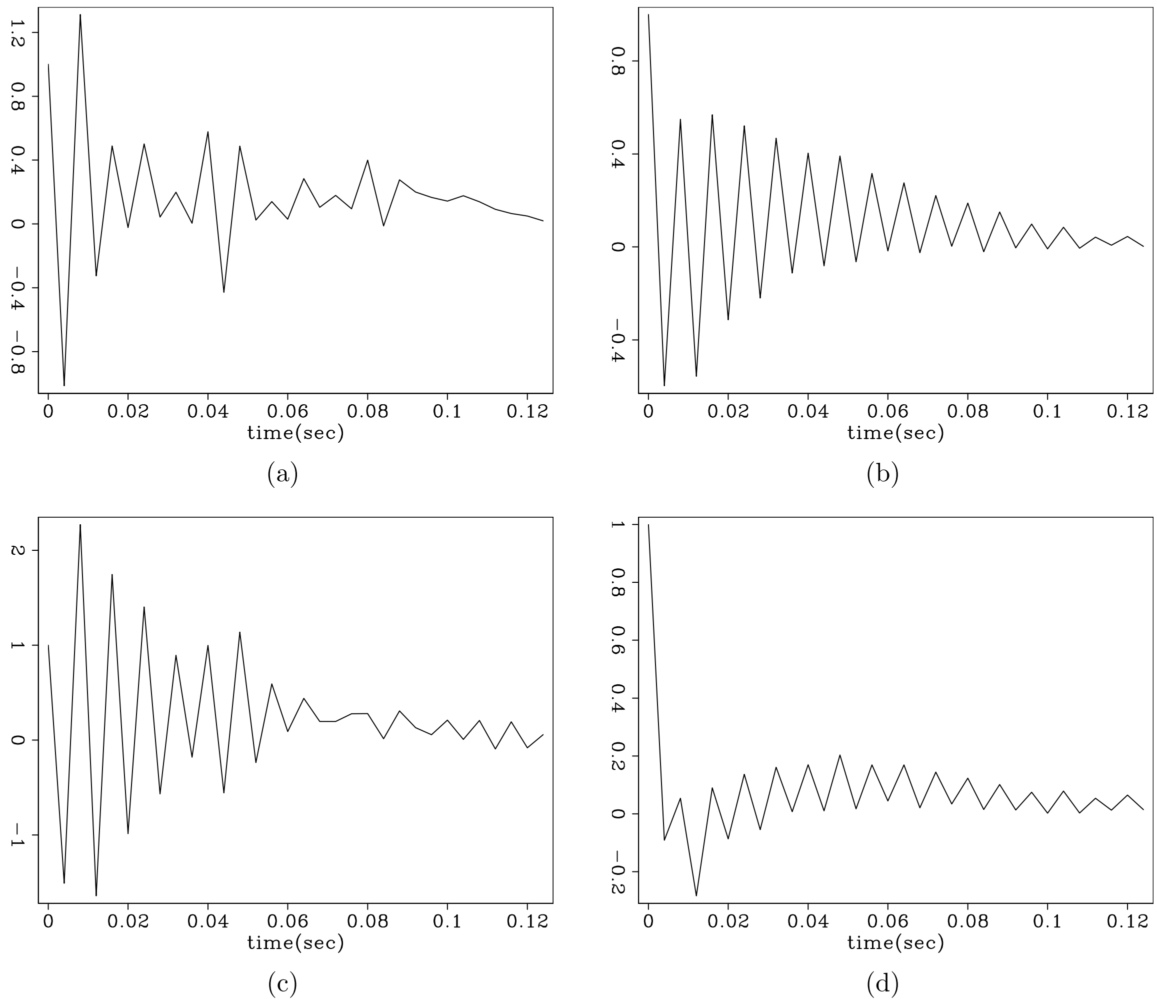


Figure 9: Given the common offset data in Figure 7, (a) filter a estimated by our method; (b) filter b estimated by our method; (c) filter a estimated by the previous method; (d) filter b estimated by the previous method. [**ER**]

Another potential solution is to apply the blind deconvolution in ~~the different domain; that is, to express the deconvolution in~~ the frequency domain instead of the time domain to make the process faster and more reliable.

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