

Short note: Three dimensional deconvolution of helioseismic data

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ABSTRACT

This is a short note on helioseismic deconvolution. Herein results are presented by deconvolving helioseismic data with a calculated impulse response in 3D to help determine source information in the shallow solar interior. Tentatively it can be concluded that there the solar acoustic energy is close to uniformly distributed throughout the convective envelope.

INTRODUCTION

Deconvolution is a technique that the majority of geophysicists are very familiar with, since the presence of a source function inherent within recorded seismic data is an issue that permeates all areas of seismology. Deconvolution simply seeks to remove this source function from the recorded data, leaving the response of the Earth (and any other instrument responses from the recording system.) The main problem within deconvolution is the estimation of the source function, since often these are non-stationary and non-minimum phase, and as such typical 'batch processing' methods such as spiking and predictive deconvolution will make these assumptions, along with several others.

Helioseismology attempts to determine certain aspects of the solar interior by studying acoustic oscillations and disturbances on the surface of the sun. The source function for these oscillations is attributed to turbulence within the convective envelope of the outer sun, and as such is stochastic. Generally helioseismologists work with these source functions by cross-correlating passively observed seismograms and calculating time-distance curves by picking lags.

Rickett and Claerbout (2001) showed that is possible to estimate the solar impulse response by using spectral factorisation techniques, notably by applying one-dimensional Kolmogorov spectral factorisation theory to the 3D observed data by using helical boundary conditions (Claerbout, 1998). This short note is an extension of Rickett's work, whereby the postulation is that if we deconvolve the raw data with the three dimensional solar impulse response information about the location and separation of these solar source regions may be revealed.

The helioseismic data used was the SOHO/MDI dataset. The data was transformed to Cartesian coordinates by projecting the high resolution data from an approximate

18 degree square onto a tangent plane. Time sampling for this survey is 60 seconds, and after pre-processing the sampling in space over the solar surface is largely regular and set at 824,800 m.

INTRODUCTION TO HELIOSEISMOLOGY

Helioseismology is a subset of seismology and is unique in the fact that the recorded data is not terrestrial. By recording surface solar oscillations and accounting for Doppler shifts it is possible to constrain information - such as velocities and distances - about the shallow solar interior. This problem is more complex than most terrestrial surveys since there are now problems such as a spherical target, unknown source function and location, and the fact the sun is moving at significant rate (giving a Doppler shift.) However, by using passive data techniques such as cross-correlation and spectral factorisation it is possible to gain valuable insight into solar properties.

Often large scale trends are delineated by decomposing the stochastic wavefields into spherical harmonics (Harvey, 1995), which works well for studying macro solar trends. However to describe small scale events, harmonics of a high order need to be computed, such as is done with cosmic microwave background (CMB) studies, however focusing on smaller areas using this technique is inefficient. What is often done instead is to cross correlate oscillatory dopplergram traces, since the lags acquired from doing this can give information, such as velocity, about ray paths travelling between the two trace locations (Duvall, 1993).

ACQUIRING THE SOLAR IMPULSE RESPONSE

The method used to acquire the solar response is described in detail in Rickett and Claerbout (2001). The premise is that Kolmogorov spectral factorisation is used since this is an efficient method of constructing a minimum phase time domain function from a given power spectrum (Kolmogorov, 1939). The theory is 1D, however as shown in Claerbout (2001) by applying helical boundary conditions it is possible to model the dataset as a long 1D trace by applying a sequence of lags, and then the factorisation can be applied and the 3D data reconstructed. The raw data and the factorised impulse response can be seen in Figure 1 and Figure 2 respectively.

SOLAR DECONVOLUTION IN 3D

Deconvolution in seismology typically seeks to remove the source signature from the recorded data. For 3D data this can be done in multiple ways: as a three way convo-

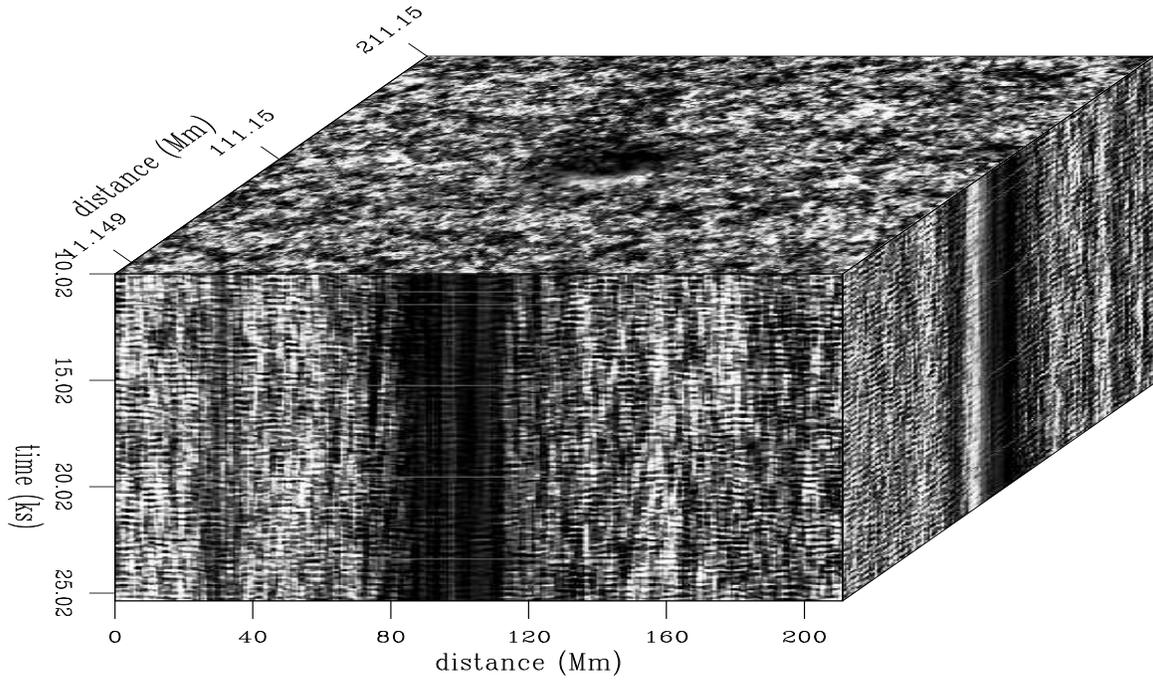


Figure 1: The raw helioseismic data, with a sun spot in the centre. [ER]

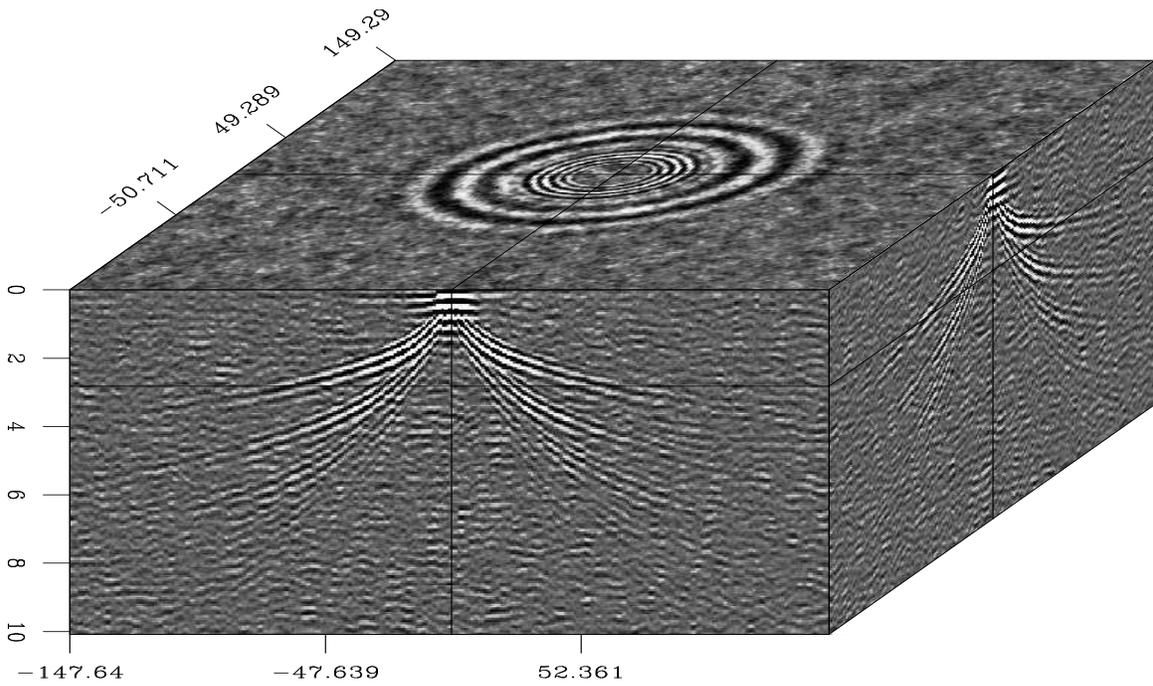


Figure 2: The solar impulse response. [ER]

lution integral in time, as a three way multiplication in the Fourier domain, or as 1D Fourier multiplication in helical coordinates. In this case the solar impulse response has been estimated using spectral factorisation, and by deconvolving the raw data with this response we can find information about the source - namely signature and location.

In frequency space the stochastic oscillation model can be described as the following multiplication

$$D(k_x, k_y, \omega) = S(k_x, k_y, \omega)G(k_x, k_y, \omega), \quad (1)$$

where D is the raw data, S the source function and G the impulse response. The 3D deconvolution can then be applied as a division in frequency, and then transformed to time (Rickett, 2001).

This method suffers from the fact that by dividing the input data D by G , any small or zero values in G will cause large perturbations in the solution for S . This is a problem addressed many, many times in geophysics, and one solution to helping to constrain the estimation is to add a small amount of white noise (a constant in frequency space) to the denominator, ensuring a maximum possible value in the output (Claerbout, 2001).

$$S(k_x, k_y, \omega) = G(k_x, k_y, \omega)/(D(k_x, k_y, \omega) + \epsilon), \quad (2)$$

Part of the usual challenge of deconvolution is choosing an appropriate value for ϵ such that the final image has not been overly steered (Claerbout, 2001). To ensure the 3D Fourier deconvolution was working correctly a synthetic model was produced, convolved with the impulse response and transformed to the time domain. This was then deconvolved with the impulse response, and the initial model was recovered clearly, with the exception of some Gibbs' artifacts due to the domain transformations and truncation of the impulse response. Subsequently the deconvolution part of this process was applied to the raw solar data, and a 3D volume acquired.

RESULTS AND FUTURE WORK

When using too low an ϵ value (or none at all) then a lot of low frequency noise was visible in the deconvolved image and this energy dominated any smaller events. When using $\epsilon = 0.01$ Figure 3 is produced. This initial deconvolved image is noisy and no discernible areas of high amplitude contrast are noticeable, with the exception of the sun spot, as was visible in the raw data. A potential conclusion from this is

that source regions are too poorly separated in time and space to be visible, or that all shallow source regions operate at a similar power giving no conclusive separations after deconvolution.

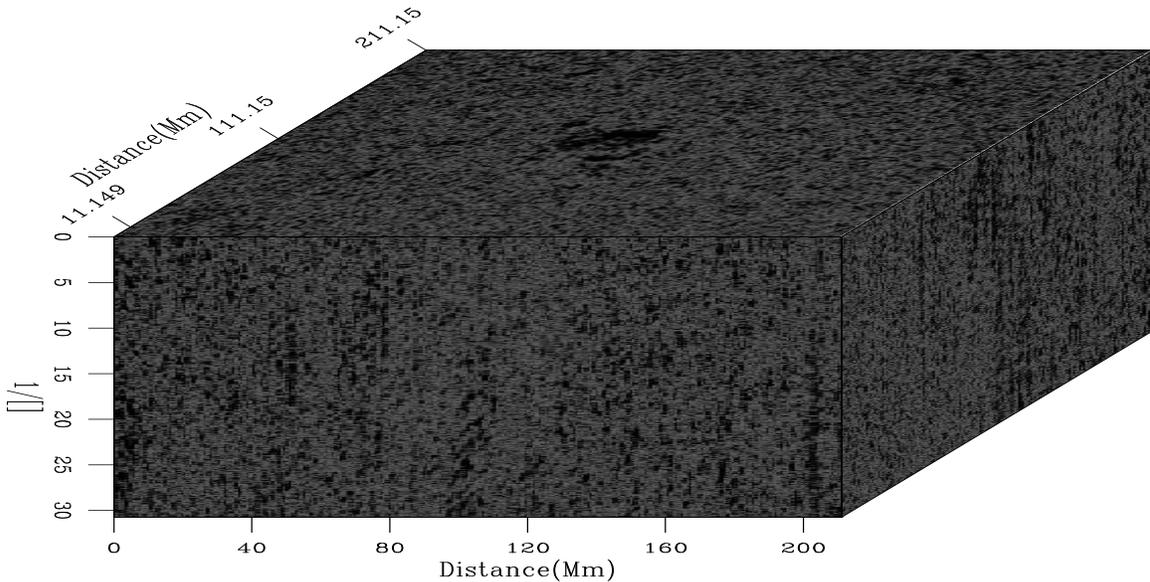


Figure 3: The deconvolved data. [ER]

The next stage for this concept will be to further tune ϵ and experiment with smaller sections of the data and also to test with balancing the data and the impulse response. One way could be to window the impulse response and use a smaller section for the deconvolution, and also to focus on smaller time windows.

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