# Predicting Rugged water-bottom multiples through wavefield extrapolation with rejection and injection

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## ABSTRACT

Although convolution-based and WEM-modeling-based surface-related multiple prediction methods are well-recognized in marine seismic data processing today, the effectiveness and efficiency of these methods are still a challenge in practice. In this paper, we present a WEM-modeling-based approach to multiple prediction. When wave-field rejection and injection are used during wave-field extrapolation, rugged water-bottom multiples can be well predicted when the water-bottom elevation and water velocity are known

## INTRODUCTION

Seismic multiples can directly contaminate the stacking velocity and obscure the primary images in stack sections. Even worse, the energy of seismic multiples can smear into the target zones during seismic migration and result in a suboptimal result during seismic inversion. Therefore, seismic multiple attenuation has been a primary concern in marine seismic data processing.

In recent years, both the geophysical industry and academia have made significant progress in seismic multiple attenuation, including multiple prediction and adaptive multiple subtraction. As a result, a diverse toolkit for multiple attenuation is available in marine seismic data processing today. However, the effectiveness and efficiency of some multiple attenuation methods are still a challenge in practice.

In theory, 3D surface-related multiple elimination, or SRME, can predict surface-related multiples accurately from the seismic data itself without a priori knowledge of the subsurface (Verschuur and Berkhout, 1997; van Dedem and Verschuur, 2001). For a trace of a given shot-receiver pair, convolution-based SRME predicts the multiple model trace by convolving the common-shot gather related to the given shot point with the common-receiver gather related to the given receiver point. Ideally this method requires that the source and receiver are co-located, the source signatures are consistent, and the seismic data are completely acquired and well sampled. However, for most 2D and 3D marine streamer data, this is not the case, because of boat steering that deviates from the designed sail line, cable feathering and near offset gap, among other causes. Therefore, either data pre-processing for SRME or built-in processing during SRME is required, either of which reduces the computational efficiency.

As a complementary approach to 3D convolution-based multiple prediction, WEM-modeling-based multiple prediction employs downward and upward wavefield extrapolation between the surface and the water bottom to predict water-bottom multiples and peg-legs based on a previously produced near-surface model (Wiggins, 1988; T. Weisser, 2006; Stork, 2006). This method theoretically is independent of seismic acquisition geometry and ideally requires a near-surface velocity model with water velocity, water-bottom topography and subsurface velocities. Unfortunately, in practice, it is not easy to estimate the subsurface velocities. Therefore, to allow wavefield extrapolation using only the water velocity and the water bottom topography, conventional wave equation modeling generally performs an approximation to the sea floor surface prior to wavefield continuation. In general, the Kirchhoff integral and finite-difference methods require dipping and horizontal flat sea floor surfaces respectively. Obviously, in the case of a structured or rugged sea floor, this approximation will result in suboptimal multiple modeling.

With the aim of improving the effectiveness and efficiency of wave-equation multiple modeling, we employ the so-called wavefield rejection and injection technique to perform wavefield extrapolation, so that we can use only the water velocity and the water-bottom topography to predict rugged water-bottom multiples and peg-legs. To best match this technique, we also present an algorithm for recursive Kirchhoff wavefield extrapolation in the space-frequency domain.

## PREDICTING RUGGED WATER-BOTTOM MULTIPLES AND PEG-LEGS

Wave equation multiple modeling consists of downward wavefield extrapolation from the sea surface to the sea floor and upward wavefield extrapolation from the sea floor to the sea surface. In general, the water velocity and the water-bottom elevation can be measured with relative ease by field survey or estimated from seismic data. It is relatively difficult or expensive to get an accurate interval velocity of the subsurface by seismic velocity analysis when the lateral velocity varies strongly. Therefore, if we can use only the water velocity and water-bottom elevation to predict the water-bottom multiples and peg-legs, the efficiency will be improved.

One of the solutions to the above-mentioned issue is to make an approximation to the rugged water bottom. The approximation depends on which algorithm we choose. For example, if the wavefield extrapolation is based on the Kirchhoff integral algorithm, a dipping flat surface can be selected to approximate the rugged water bottom, so that the calculation of the Greens function does not involve the subsurface velocity. Similarly, if the wavefield extrapolation is based on the phase shift or finite difference algorithm, a horizontal flat surface can be chosen. Obviously, these approximations to the real water bottom will sacrifice effectiveness for efficiency.

To improve the efficiency without loss of effectiveness, we present a so-called wavefield extrapolation with rejection and injection.

## Wavefield extrapolation with rejection and injection

The idea of this method comes from the fact that water-bottom multiples and peglegs are trapped between the water bottom and sea surface, as shown in figure 1. Therefore, when we input the shot gather on the sea surface and perform downward wavefield extrapolation, the down-going wave below the water-bottom can be rejected. Similarly, when we inject the recorded down-going wavefield at the water bottom and perform the upward wavefield extrapolation, the up-going wave below the water bottom also can be rejected.

Obviously, based on the above-mentioned idea, an auxiliary model presentation should be given. As shown in figure 2, in this auxiliary model presentation, the solid circle, rectangle and star represent the nodes of computation in water, in the subsurface, and on the water bottom, respectively.

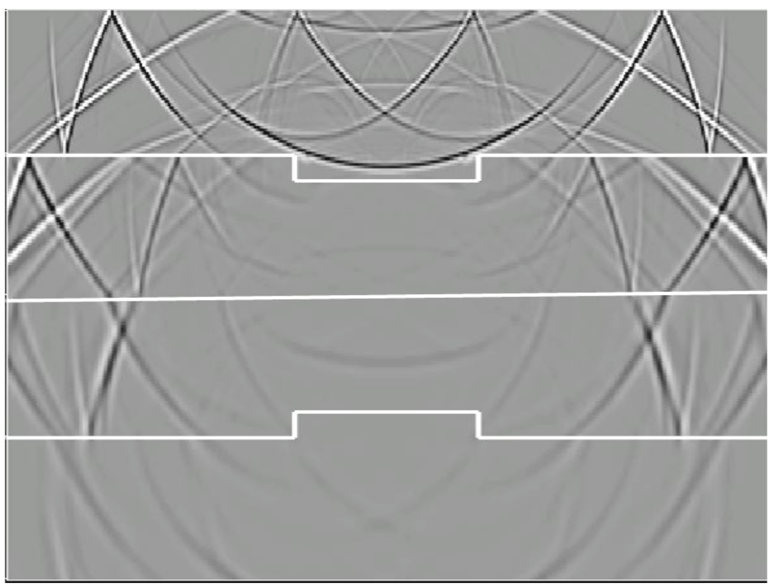

Figure 1: A snapshot of seismic modeling to show the water bottom multiples and illustrate wavefield rejection and injection. [**CR**]

## Recursive Kirchhoff wavefield extrapolation

Theoretically, wavefield rejection and injection can be carried out for any of the wavefield extrapolation algorithms. However, the computational efficiency can differ greatly. Recursive Kirchhoff wavefield extrapolation in both space-frequency and space-time domain is the most suitable algorithm.

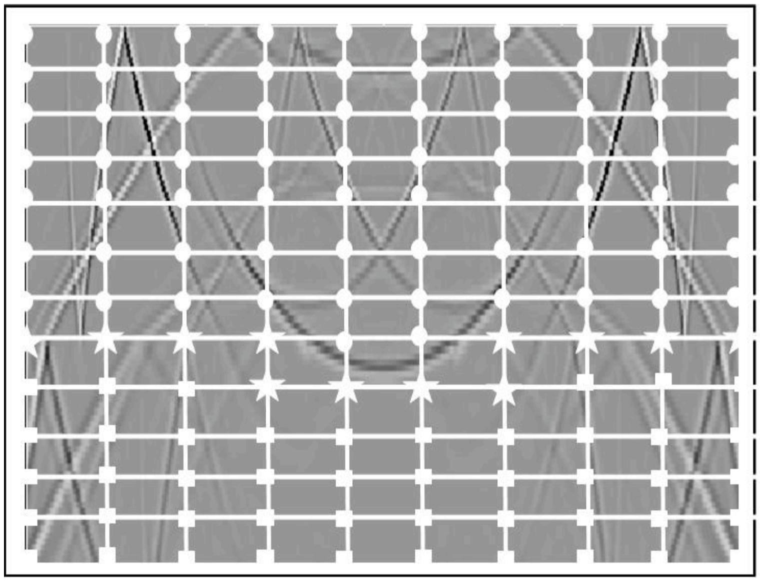

Figure 2: The auxiliary model presentation for wavefield extrapolation with wavefield rejection and injection. [**CR**]

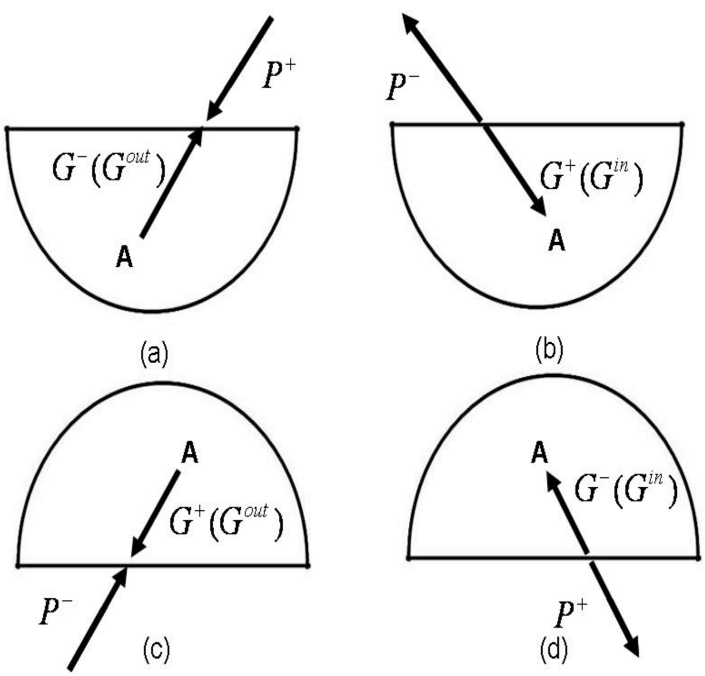


Figure 3: Schematic illustration of the four types of wavefield extrapolations in terms of Kirchhoff integral: (a) downward wavefield extrapolation of the down-going wave; (b) downward wavefield extrapolation of the up-going wave; (c) upward wavefield extrapolation of the up-going wave; and (d) upward wavefield extrapolation of the down-going wave. [**CR**]

Because the basic theory and extrapolation operator based on the Kirchhoff integral are widely known (Schneider, 1978; Berkhout, 1981; Margrave and Daley, 2001), we derive only the formulae corresponding to upward and downward wavefield extrapolation of both the up-going and down-going waves.

In general, the wavefield extrapolation from depth to can be written as follows:

$$P^{+}\left(k_x, k_y, z \pm \Delta z, \omega\right) = P^{+}\left(k_x, k_y, z, \omega\right) e^{-k_z(\pm \Delta z)} \tag{1}$$

$$P^{-}\left(k_x, k_y, z \pm \Delta z, \omega\right) = P^{-}\left(k_x, k_y, z, \omega\right) e^{+k_z(\pm \Delta z)} \tag{2}$$

where $P^{+}\left(k_x, k_y, z, \omega\right)$ and $P^{-}\left(k_x, k_y, z, \omega\right)$ are the Fourier transform over $x$, $y$ and $t$ of the down-gong and up-going waves at position ($x$,$y$,$z$), respectively. The terms $k_x$, $k_y$ and $k_z$ are the three components of the wavenumber vector, and $\omega$ is the angular frequency. The sign $\pm$ before the depth interval $\Delta z$ relates to the upward and downward wavefield extrapolations. Thus, there are four types of wavefield extrapolations in total.

On the other hand, the Kirchhoff integral in the space-frequency domain is

$$\tilde{P}\left(r_A, \omega\right) = \oint_s \left[\tilde{G}\left(r, r_A, \omega\right) \frac{\partial \tilde{P}\left(r, \omega\right)}{\partial n} - \tilde{P}\left(r, \omega\right) \frac{\partial \tilde{G}\left(r, r_A, \omega\right)}{\partial n}\right] ds \tag{3}$$

where $r$ and $r_A$ are the shorthand notations of ($x$,$y$,$z$) and ($x_A$,$y_A$,$z_A$) . The term $n$ is the outward normal of the surface $S$. The wavefield $\tilde{P}\left(r, \omega\right)$ and Green function $\tilde{G}\left(r, r_A, \omega\right)$ satisfy the following Helmholtz equations:

$$\nabla^2 \tilde{P}\left(r, \omega\right) + \frac{\omega^2}{c^2} \tilde{P}\left(r, \omega\right) = 0 \tag{4}$$

$$\nabla^2 \tilde{G}\left(r, r_A, \omega\right) + \frac{\omega^2}{c^2} \tilde{G}\left(r, r_A, \omega\right) = \delta\left(r - r_A\right)) \tag{5}$$

where $c$ is wave propagation velocity. Obviously, it is not easy to relate 3 to the upward and downward wavefield extrapolation of both the up-going and down-going waves. Suppose $S$ consists of a horizontal $S_0$ surface at $z = z_n$ and a hemisphere $S_1$ which contains point A and satisfies the Sommerfeild radiation condition. Transforming the Kirchhoff integral from the ($x$,$y$,$z$,$\omega$) domain into ($k_x$,$k_y$,$z$,$\omega$) domain, 3 becomes (Berkhout, 1989)

$$P\left(r_A, \omega\right) = \frac{1}{\left(2\pi\right)^2} \int\int \left[G \frac{\partial P}{\partial z} - P \frac{\partial G}{\partial z}\right] dk_x dk_y \tag{6}$$

where $G$ and $P$ represent $G\left(-k_x, -k_y, z, r_a, \omega\right)$ and $P\left(k_x, k_y, z, \omega\right)$. Furthermore, 6 can be mathematically expressed by

$$P\left(r_A,\omega\right)=\frac{1}{\left(2\pi\right)^2}\int\int\left[\left(G^{+}+G^{-}\right)\frac{\partial(P^{+}+P^{-})}{\partial z}-\left(P^{+}+P^{-}\right)\frac{\partial\left(G^{+}+G^{-}\right)}{\partial z}\right]dk_x dk_y \tag{7}$$

where $P^+$ and $G^+$ are the down-going wave and Greens function respectively, and $P^-$ and $G^-$ are the up-going wave and Greens function respectively.

Using the definitions of the down-going and up-going waves,

$$\frac{\partial P^{\pm}}{\partial z}=\mp k_z P^{\pm} \tag{8}$$

$$\frac{\partial G^{\pm}}{\partial z}=\mp k_z G^{\pm} \tag{9}$$

7 can be simplified to

$$P\left(r_A,\omega\right)=\frac{1}{\left(2\pi\right)^2}\int\int\left[\left(G^{-}\frac{\partial P^{+}}{\partial z}-P^{+}\frac{\partial G^{-}}{\partial z}\right)+\left(G^{+}\frac{\partial P^{-}}{\partial z}-P^{-}\frac{\partial G^{+}}{\partial z}\right)\right]dk_x dk_y \tag{10}$$

and furthermore to

$$P\left(r_A,\omega\right)=\frac{1}{2\pi^2}\int\int\left[P^{+}\frac{\partial G^{-}}{\partial z}+P^{-}\frac{\partial G^{+}}{\partial z}\right]dk_x dk_y \tag{11}$$

Thus the four types of wavefield extrapolations in terms of the Kirchhoff integral can be easily defined based on 11, as shown in figure 3.

In practice, generally we transform the 11 from the ($k_x$,$k_y$,$z$,$\omega$) domain back into the($x$,$y$,$z$,$\omega$) or ($x$,$y$,$z$,$t$) domain In this case, the down-going and up-going Greens functions will change accordingly, as shown in figure 3. The terms $G^{out}$ and $G^{in}$ are the causal and anticausal Greens functions , or the out-going and in-going waves respectively.

After the above derivation, we have all types of wavefield extrapolations in terms of the Kirchhoff integral. In water-bottom multiple prediction, we use only types (a) and (c), that is, the downward wavefield extrapolation of the down-going wave and the upward wavefield extrapolation of the up-going wave.

## SYNTHETIC EXAMPLE

To evaluate the usefulness of multiple prediction and adaptive subtraction, we use the geologic model shown in figure 4 to produce the synthetic seismic data. In this model,

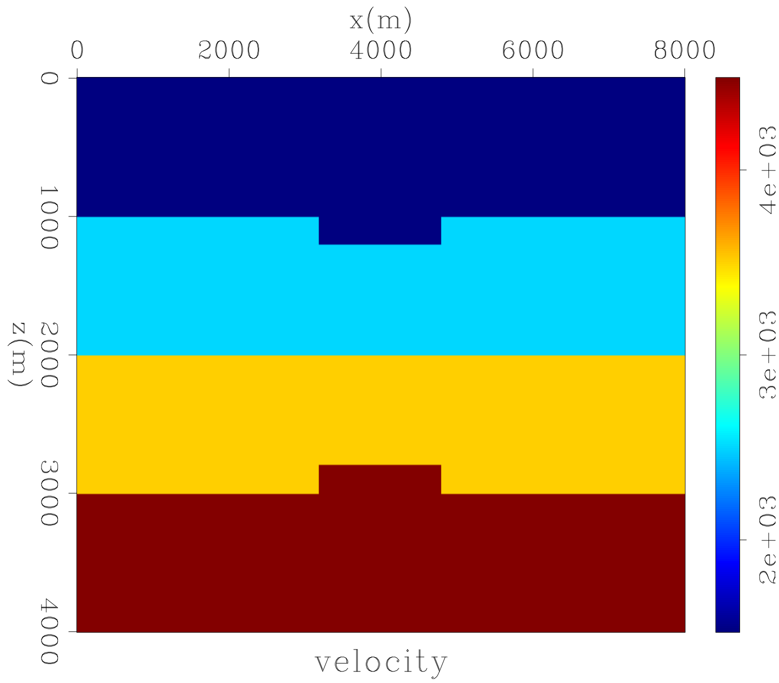


Figure 4: The structure and interval velocity of the geological model. [**CR**]

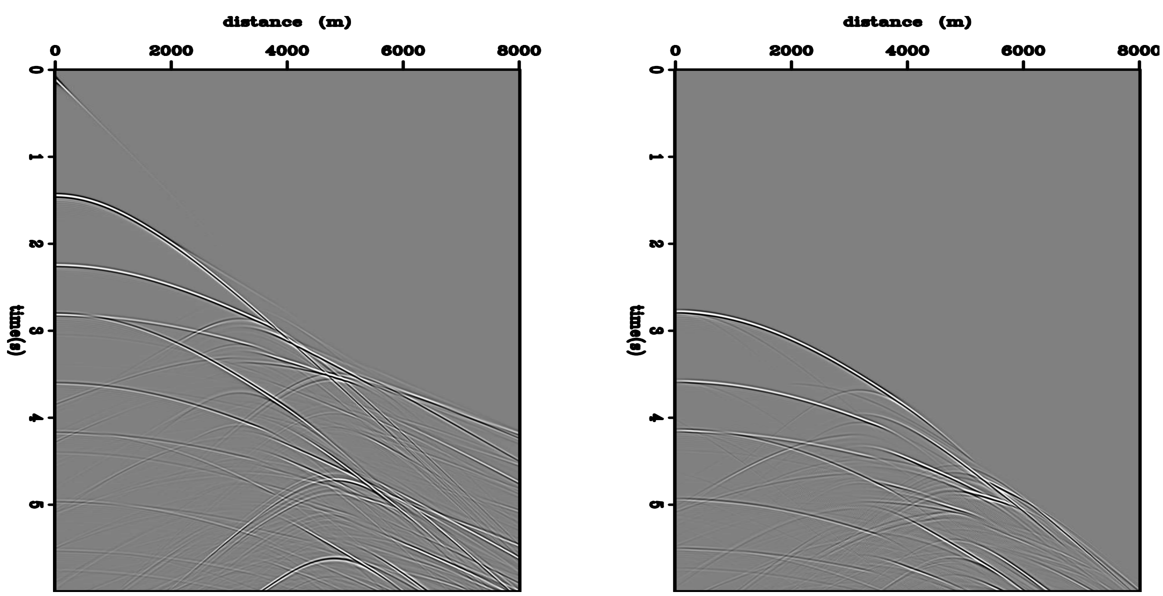


Figure 5: The shot gather at the leftmost location(left) and predicted multiples (right). [**CR**]

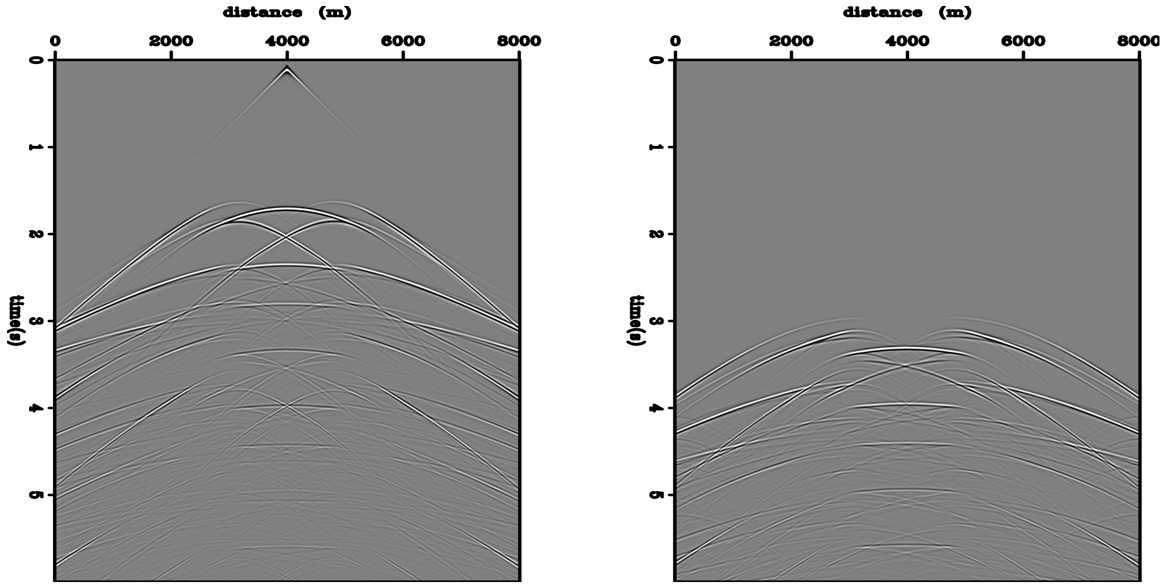


Figure 6: The shot gather at the central location (left) and predicted multiples (right). [**CR**]

there are four layers, including the water layer. The interval velocities from the top to bottom layer are 1500, 2500, 3500 and 4500 m/s respectively, and the depths of the first to third interfaces are 1000, 2000 and 3000 m respectively. As there is a notch (trench) on the first interface, the synthetic seismic data contain not only specularly reflected but also diffracted water-bottom multiples and peg-legs. Furthermore, the model has a convex feature (fault or salt) on the third interface to produce specularly reflected and diffracted interbed multiples. In addition, to test the ability to preserve the primary, we design the interface depths such that that the zero-offset travel time of the first order water-bottom multiples are very close to the zero-offset reflection time of the third interface. Figure 5 and figure 6 show shot gathers and predicted multiples at the leftmost and central locations. Figure 7 shows the zero-offset sections of the original seismic data and predicted multiples. Comparing the original and predicted multiples both in the shot gathers and the zero-offset section, we can see that both the specularly reflected and diffracted water bottom multiples and peg-legs are well predicted. The results also demonstrate the limitation of both the convolution-based and WEM-modeling-based surface-related multiple prediction methods; neither can predict the interbed multiples.

## CONCLUSIONS

In the case of a structured or rugged sea floor, wavefield extrapolation with rejection and injection is an efficient and effective technique for the prediction of the rugged water-bottom multiple and peg-legs. The Kirchhoff integral is the algorithm best suited to both the downward wavefield extrapolation of the down-going wave and the

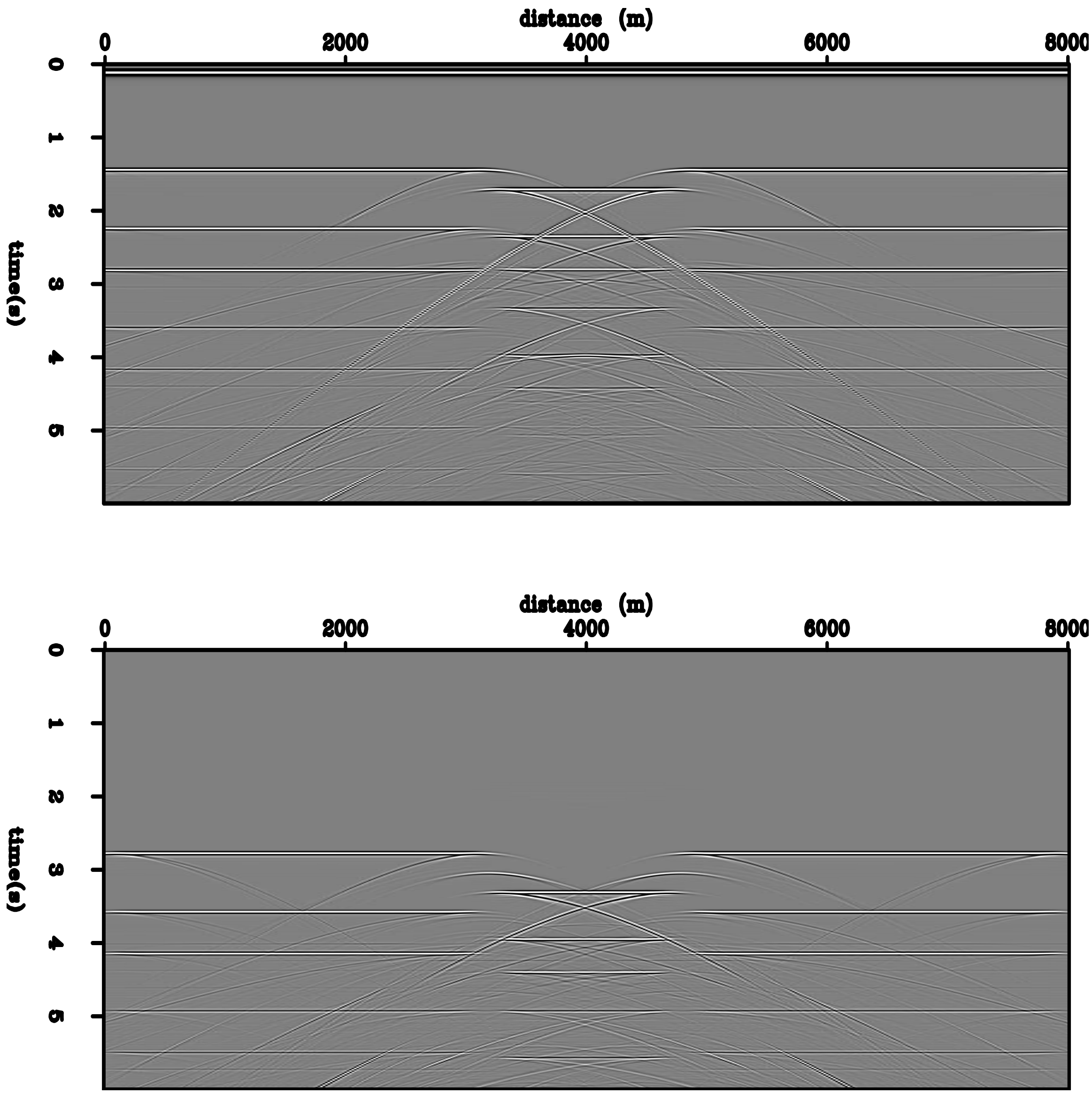


Figure 7: The zero-offset sections of the original seismic data (top) andthe predicted multiples (bottom). [**CR**]

upward wavefield extrapolation of the up-going wave. The synthetic data example shows that both the specularly-reflected and diffracted water-bottom multiples and peg-legs are well predicted by our method.

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