RETARDED SLANT-MIDPOINT COORDINATES

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Here we will define a coordinate system for reflection seismic data which is organized about the shot/receiver midpoint. Time is moved out linearly with offset and it is retarded so as to remove all shifting terms from the downward continuation equation. Although a data processing application has not yet been completely defined, the following advantages and disadvantages are forseen:

ADVANTAGES

- Wide angle propagation
- Accurate low order schemes
- Unlimited range of velocity $v(z)$
- Processing done in midpoint-offset space
- One pass
- Simultaneous migration and velocity estimation

DISADVANTAGES

- Migration concurrent with stack, not before
- Data access alternates between midpoint and offset
- Lateral velocity problem not addressed
- Multiple reflection problem not addressed

Downward continuation of both geophones and sources may be done in many possible coordinate frames. Linear moveout amounts to organizing the coordinate frame about some particular Snell's parameter p . This in turn achieves the practical advantage that data from rays near this Snell's parameter can be accurately handled with low order approximations. Naturally, rational expansions of the square root operator allow arbitrary propagation angles (up to 90 degrees) at the cost of increased numbers of terms. But why consider p values other than $p=0$? Essentially, the reason is that wide angle rays will generally be present even where the earth has no dip. Choosing pv to be in the midrange of angles, say 25 degrees, then a simple second order differential equation might very well be able to contend with the typical zero to fifty degree range of angles in a velocity analysis.

The definition of the coordinate frame is motivated by the diagram in Figure 1 and is given by

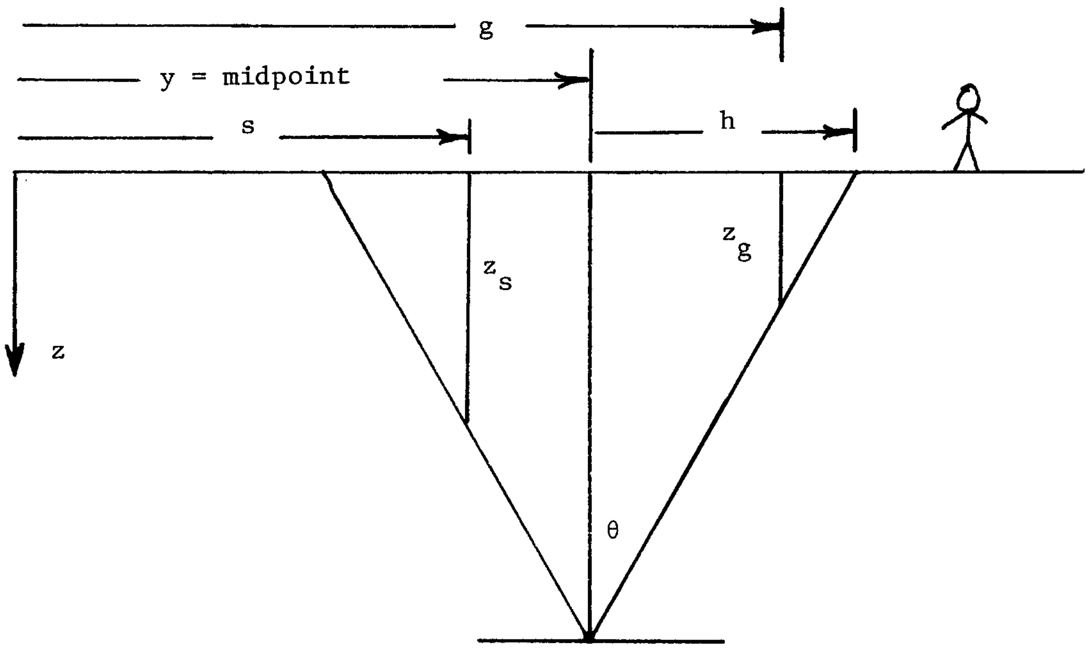
$$t = t' - \int_0^{z'_g} \frac{dz}{v \cos} - \int_0^{z'_s} \frac{dz}{v \cos} + 2ph \tag{1a}$$

$$g = y + h - \int_0^{z'_g} \frac{\sin}{\cos}\, dz \tag{1b}$$

$$s = y - h + \int_0^{z'_s} \frac{\sin}{\cos}\, dz \tag{1c}$$

$$z_g = z'_g \tag{1d}$$

$$z_s = z'_s \tag{1e}$$



Although the coordinate transformation depends on the angle θ , it should be understood that in the final analysis these dependencies are eliminated by Snell's law $pv(z) = \sin\theta(z)$. The definition of retarded time is by no means obvious. The two integrals account for the travel time from the surface to the shot and receiver depths along the slanted paths and the $2ph$ term is the linear moveout. That it is correct will be seen later when the ∂_{tt} and ∂_{yt} shifting terms cancel from the downward continuation equations. The equations (1) are a coordinate transformation from data coordinates (y,h,t',z') to wave equation coordinates (g,s,t,z) . Really the equations (1a-1e) are implicit *definitions* of the coordinates (y,h,t',z') which we wish to use to reference our data. The Jacobian of the transformation (1) is given by the partial derivative matrix

$$\begin{bmatrix} dt \\ dg \\ ds \\ d_{zg} \\ d_{zs} \end{bmatrix} = \begin{bmatrix} t_{t'} & t_y & t_h & t_{z'_g} & t_{z'_s} \\ g_{t'} & g_y & & & \\ s_{t'} & s_y & & & \\ & & & & \\ & & & & \text{etc.} \end{bmatrix} \begin{bmatrix} dt' \\ dy \\ dh \\ d_{z'g} \\ d_{z's} \end{bmatrix} \tag{2}$$

Performing the partial differentiations in (2) we get

$$\begin{bmatrix} dt \\ dg \\ ds \\ d_{zg} \\ d_{zs} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{2\sin}{v} & \frac{-1}{v\cos} & \frac{-1}{v\cos} \\ 0 & 1 & 1 & -\frac{\sin}{\cos} & 0 \\ 0 & 1 & -1 & 0 & \frac{\sin}{\cos} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dt' \\ dy \\ dh \\ d_{z'g} \\ d_{z's} \end{bmatrix} \quad (3)$$

A nice property about this Jacobian is that it is a function of $v(z)$ and p through Snell's Law but it is not a function of the independent variables (g,s,y, or h) . This will mean later that when taking second derivatives we will not be generating a vast number of low order terms. Also it may be noted that we are not carefully distinguishing $v(z_s)$ from $v(z_g)$ because at a later time we will set z_s equal z_g . It can readily be verified that the inverse Jacobian is given by the matrix of

$$\begin{bmatrix} dt' \\ dy \\ dh \\ d_{z'g} \\ d_{z's} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\sin}{v} & \frac{\sin}{v} & \frac{\cos}{v} & \frac{\cos}{v} \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{\sin}{2\cos} & \frac{-\sin}{2\cos} \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{\sin}{2\cos} & \frac{\sin}{2\cos} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dt \\ dg \\ ds \\ d_{zg} \\ d_{zs} \end{bmatrix} \quad (4)$$

We have the usual statement that different mathematical functions describe the wavefield in different coordinate systems

$$P(t,g,s,z_g,z_s) = Q(t',y,h,z'_g,z'_s) \quad (5)$$

The chain rule for partial differentiation says that it is the transpose of the matrix (4) which helps us convert the wave equation from physical coordinates to data coordinates, namely

$$\begin{bmatrix} \partial_t \\ \partial_g \\ \partial_s \\ \partial_{z_g} \\ \partial_{z_s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{\sin}{v} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{\sin}{v} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ \frac{\cos}{v} & \frac{\sin}{2\cos} & \frac{\sin}{2\cos} & 1 & 0 \\ \frac{\cos}{v} & \frac{-\sin}{2\cos} & \frac{\sin}{2\cos} & 0 & 1 \end{bmatrix} \begin{bmatrix} \partial_{t'} \\ \partial_y \\ \partial_h \\ \partial_{z'_g} \\ \partial_{z'_s} \end{bmatrix} \tag{6}$$

The wave equation for geophones is

$$(\partial^2_{z_g} + \partial^2_g - \frac{1}{v^2} \partial^2_t) P = 0$$

Making substitutions from (6) we get

$$\left[\left(\frac{\cos}{v} \partial_{t'} + \frac{\sin}{2\cos} \partial_y + \frac{\sin}{2\cos} \partial_h + \partial_{z'_g}\right)^2 + \left(-\frac{\sin}{v} \partial_{t'} + \frac{1}{2} \partial_y + \frac{1}{2} \partial_h\right)^2 - \frac{1}{v^2} \partial^2_{t'}\right] Q = 0 \tag{7a}$$

Likewise, the equation for shots is

$$\left[\left(\frac{\cos}{v} \partial_{t'} - \frac{\sin}{2\cos} \partial_y + \frac{\sin}{2\cos} \partial_h + \partial_{z'_s}\right)^2 + \left(+\frac{\sin}{v} \partial_{t'} + \frac{1}{2} \partial_y - \frac{1}{2} \partial_h\right)^2 - \frac{1}{v^2} \partial^2_{t'}\right] Q = 0 \tag{7b}$$

Study of (7a) and (7b) shows that coefficients of $\partial^2_{t'}$, $\partial_{t'y}$, and $\partial_{t'h}$ all vanish identically confirming our earlier assertion that equation (1) is a correct definition of retarding coordinates. Low order square root approximations are obtained by simply neglecting the Fresnel terms ∂_{zz}, ∂_{zy}, and ∂_{zh} . What we have left of (7a) and (7b) is

$$\left[\partial_{t'z_g} + \frac{v}{8 \cos^3 \theta} (\partial_y + \partial_h)^2\right] Q = 0 \tag{8a}$$

$$\left[\partial_{t'z_s} + \frac{v}{8 \cos^3 \theta} (\partial_y - \partial_h)^2\right] Q = 0 \tag{8b}$$

Now let us push downward shots and geophones simultaneously, namely, choose

$$\partial_z Q = (\partial_{z_s} + \partial_{z_g}) Q \tag{9}$$

With this we lose ∂_{yh} and (8) reduces to

$$Q_{zt'} = - \frac{v}{4 \cos^3 \theta} (\partial_{yy} + \partial_{hh}) Q \tag{10}$$

Equation (10) is the basic equation for downward continuation of unstacked data. Define a common midpoint slant stack by S , namely

$$S(y,t',z) = \int Q(y,t',z,h)\, dh \tag{11}$$

Integrating (10) over offset we see that a stack can be migrated with the equation

$$S_{zt'} = - \frac{v}{4 \cos^3 \theta} S_{yy} \tag{12}$$

STOPPING CONDITION

When the shots and geophones have been downward continued to the appropriate depth we have

$$t = 0 \tag{13a}$$

$$g = s \tag{13b}$$

$$z_g = z_s = z \tag{13c}$$

In other words the reflection coefficient at (y,z) is seen at zero travel time on the shot-geophone combination located at (y,z). Combining equations (1) and (13) we have

$$t = 0 = t' - 2\int_0^z \frac{dz}{v \cos \theta} + 2p \int_0^z \tan \theta \, dz$$

or

$$t' = 2\int \left(\frac{1}{v \cos \theta} - \frac{\sin^2 \theta}{v \cos \theta} \right) dz$$

$$t' = 2 \int_0^z \frac{\cos \theta}{v} \, dz \tag{14a}$$

which is the stopping conditions for the downward continuation. To get an equation for velocity estimation we use (13b,c) and (1b,c) to get

$$h = 2 \int_0^z \tan \theta \, dz$$

$$h = 2 \int_0^{t'} \tan \theta \frac{dz}{dt'} \, dt'$$

which using (14a) for dt'/dz gives

$$h = \int_0^{t'} \frac{v \sin \theta}{\cos^2 \theta} \, dt' \tag{14b}$$

Equation (14b) provides the possibility for doing velocity analysis as migration proceeds. This equation defines a line $h(t')$ on a common midpoint gather. On surface data this line is a line connecting the tops of the skewed hyperboloids. (See *How to Measure RMS Velocity with a Pencil and a Straightedge*, SEP 11 pages 41-44). The interval velocity at time t' is exactly and readily obtained by differentiating (14b) with respect to t' getting $h_{t'}$, then solving for $v(t')$ getting

$$v(t') = \left[\frac{h_{t'}}{p(1 + h_{t'})} \right]^{1/2} \tag{15}$$

Interval velocity can be determined after migration by searching the common midpoint gather [(h,t') space] for maxima (which were hyperboloid tops before migration). Connecting two maxima gives an estimate of $h_{t'} = dh/dt'$ which can be inserted into (15) to determine the interval velocity between the two events. In this way velocity estimation can take place during downward continuation so that in principle both the velocity and the migrated section is simultaneously determined.