Geophysical data integration and its application to seismic tomography

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ABSTRACT

For oil exploration and reservoir monitoring purposes, we probe the earth's subsurface with a variety of geophysical methods, generating data with different natures, scales and frequency content. This diversity represents a large problem when trying to integrate all the gathered information. The concept of a shared earth by all these geophysical surveys suggests the presence of structural similarities in different data sets. For that purpose, it is necessary to work with geophysical properties that are scale-independent and not physical properties in individual layers. In this paper, I overview two methods for extracting structural information from data and using it as a constraint to the seismic tomography problem to compare different techniques and their effectiveness.

INTRODUCTION

In earth sciences, subsurface structures are studied by collecting large and various types of geophysical data. Different geophysical attributes of the subsurface are measured by a variety of geophysical measuring techniques, including but not limited to seismic, magnetic, and well-logs. For many years, different types of data have been used for specific stages of oil exploration and production. However, in recent years, many authors have considered using diverse data in geophysical inversion can reduce uncertainty (de Nardis et al., 2005; Bosch et al., 2005; Colombo and De Stefano, 2007). One of the main challenges of data integration is the difference in physical nature, scale, and frequency contents. All of the collected data, however, while measuring different properties, sample of the same geophysical structures. We can extract mathematical/geophysical properties from a data set that provides structural information. This structural information can be used in geophysical problems as auxiliary data to improve model estimation results by constraining the optimization problem. Geophysical inversion problems can benefit from this method in the form of a regularization misfit term that imposes structural similarity between the main and the auxiliary data fields. It is also applicable to both joint inversion and inversion using the auxiliary data.

STRUCTURAL SIMILARITY MEASURES

Properties of the subsurface can have very different dynamic ranges of values, the frequency band and scale in which they are measured. For example, magnetotelluric data has a much lower frequency than seismic data, while its dynamic range of amplitudes is broader. In this section, I review two attributes that provide some measure of structural similarity.

Cross-gradient function

Gallardo and Meju (2004) introduce the cross-gradient as follows:

$$\mathbf{g} = \nabla \mathbf{u} \times \nabla \mathbf{v}, \tag{1}$$

where $\mathbf{u}$ and $\mathbf{v}$ are two measured fields. By using normalized values of each data field, the cross-gradient function vanishes where the two fields are similar in structure or either one of them is very smooth. Maysami (2008) and Maysami and Clapp (2009) note that cross-gradient functions can be used as a measure of similarity. The cross-gradient function can be a suitable choice for the regularization (model-shaping) term of an optimization problem that imposes the structure of the auxiliary field on the estimated model. The differentiating nature of these functions, however, raises a sensitivity issue where there is a large gap between the frequency bands of different types of data. It is also expected that cross-gradient function is not a very effective choice for low-frequency data, since the smooth behavior causes the gradient to vanish.

Dip residual

In practice, the frequency spectrum of different types of geophysical data can change very widely, and there might be cases with no overlap in frequency band. To address these types of data integration problems, we need to choose properties that are frequency-independent. Local dip is one such property, which can be estimated by solving a regularized optimization (see Fomel (2000) for more details). Given a 2-D field $u(x, z)$, one can estimate the dip values where the data misfit function is given by

$$\begin{cases} \arg\min_{p_u} \quad \mathbf{C}(p_u)\mathbf{u} \quad \text{subject to} \\ \\ \qquad\qquad \epsilon \mathbf{D} \Delta p_u \approx 0 \end{cases}, \tag{2}$$

where $\mathbf{C}(p_u)$ is a convolution operator with a 2-D filter based on the local dip, and $\mathbf{D}$ represents an appropriate roughening operator. The estimated local dip values are frequency independent, making dip a candidate for carrying geological information from one data field to another. Again, the estimated dips can be used in a regularization term of an optimization problem to impose the structure of the auxiliary data on the estimated model. This model misfit function can be given as follows:

$$r_m(x,z) = (\frac{\partial}{\partial x} + p_u \frac{\partial}{\partial z})v(x,z) \approx 0. \tag{3}$$

Note that the structural similarity measures do not include any physical link that might potentially exist between two fields. In other words, these functions only help the model-shaping part of optimization, and the physics of estimation lies in the data-misfit term with the mapping operator.

Figures 1 and 2 show examples where the reflectivity of the Marmousi synthetic model is used as auxiliary data to impose the geophysical structures on a random noise field and a smooth velocity field. The optimization problem used to generate the estimated model shown by Figures 1 and 2 is given by

$$\begin{cases} \arg\min_m \quad \left\| \mathbf{d} - \mathbf{I}\ \mathbf{m} \right\|_2^2 \quad \text{subject to} \\ \\ \qquad \epsilon\ \mathbf{A}(\mathbf{u})\ \mathbf{m} \approx 0 \end{cases}, \tag{4}$$

where $\mathbf{I}$ is identity matrix; $\mathbf{A}$ is either the cross-gradient or dip residual operator; $\mathbf{u}$ and $\mathbf{d}$ represent the auxiliary reflectivity field and the data (noise or smooth velocity field), respectively; and $\mathbf{m}$ is the estimated model which incorporates some of the structural information provided by $\mathbf{u}$. Note that a large ϵ is needed to emphasize on model-shaping term and impose structure. Figures 1(a) and 2(b) show the data $\mathbf{d}$ and the auxiliary field $\mathbf{u}$. Figure 1(d) shows a better reconstruction of the Marmousi structure with dip residual technique than the cross-gradient function (Figure 1(c)). This is clearly visible by comparing the continuity of reconstructed amplitude along the geological dips in Figure 1(d)).

Figure 2 shows a similar problem where we start with a smooth velocity of the Marmousi model instead of noise. Similarly, Figure 2(d) suggests that the dip residual technique leads to a less noisy partial reconstruction of the Marmousi structure; but not all the details are included. However, some of the horizontal parts of structures are reconstructed by the cross-gradient function (Figure 2(c)), but not with the dip residual (Figure 2(d)). Note that frequency ranges in smooth velocity and reflectivity are low and high, respectively.

APPLICATION TO SEISMIC TOMOGRAPHY

Following the earlier work by Maysami (2008) and Maysami and Clapp (2009), I apply the methods discussed above to regularize the seismic velocity tomography problem. In this optimization problem, the data misfit term includes the physics and kinematics of the tomography problem, while the regularization (model-shaping) term imposes the structure present in the auxiliary field, which is chosen to be either the reflectivity

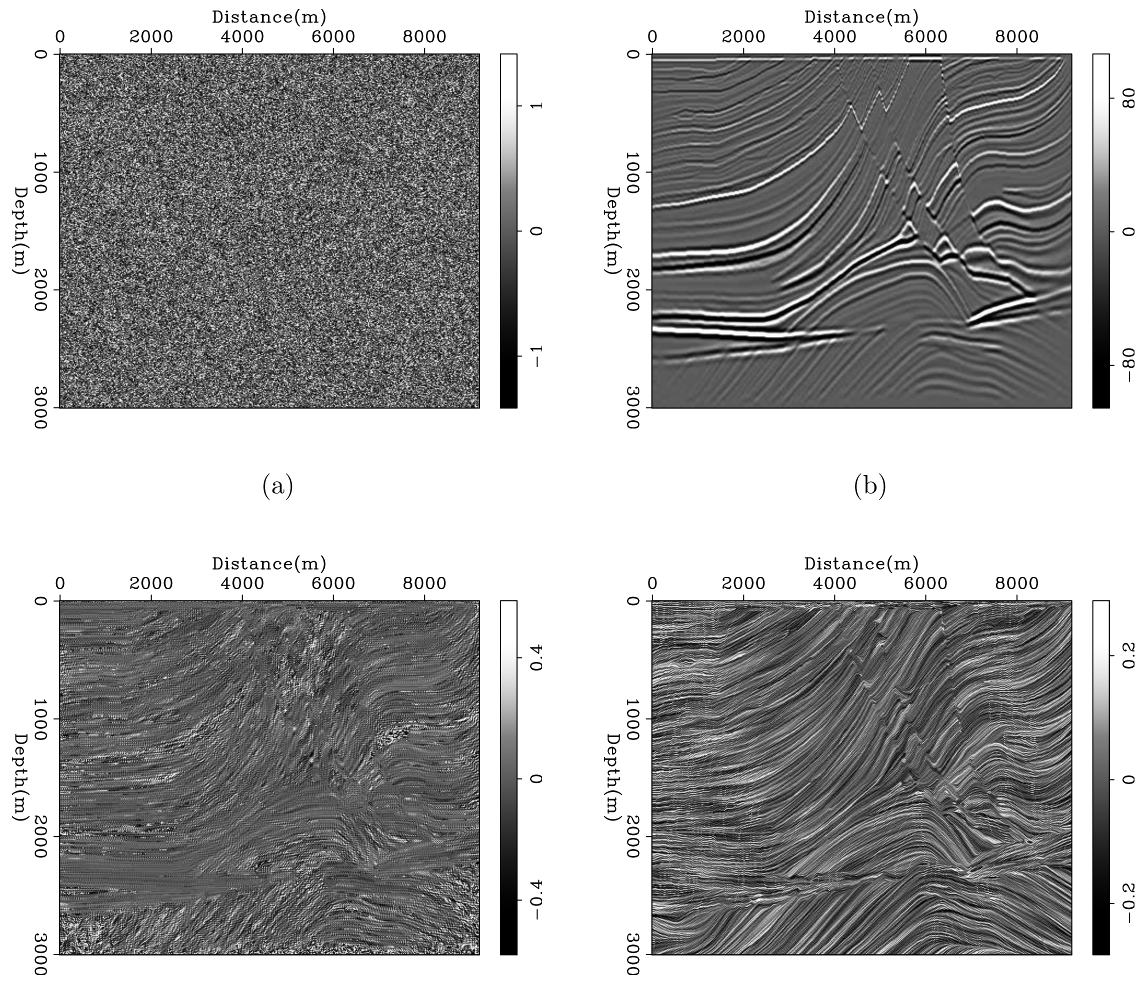


Figure 1: Data integration problem 1: The starting model is random noise **(a)** and the auxiliary data is the reflectivity of the Marmousi model **(b)**. Panel **(c)** shows the estimated model with the cross-gradient function and panel **(d)** shows the estimated model with the dip residual. Note the reconstruction of the structure in the estimated models and how each method provides different quality of reconstruction. [**ER**]

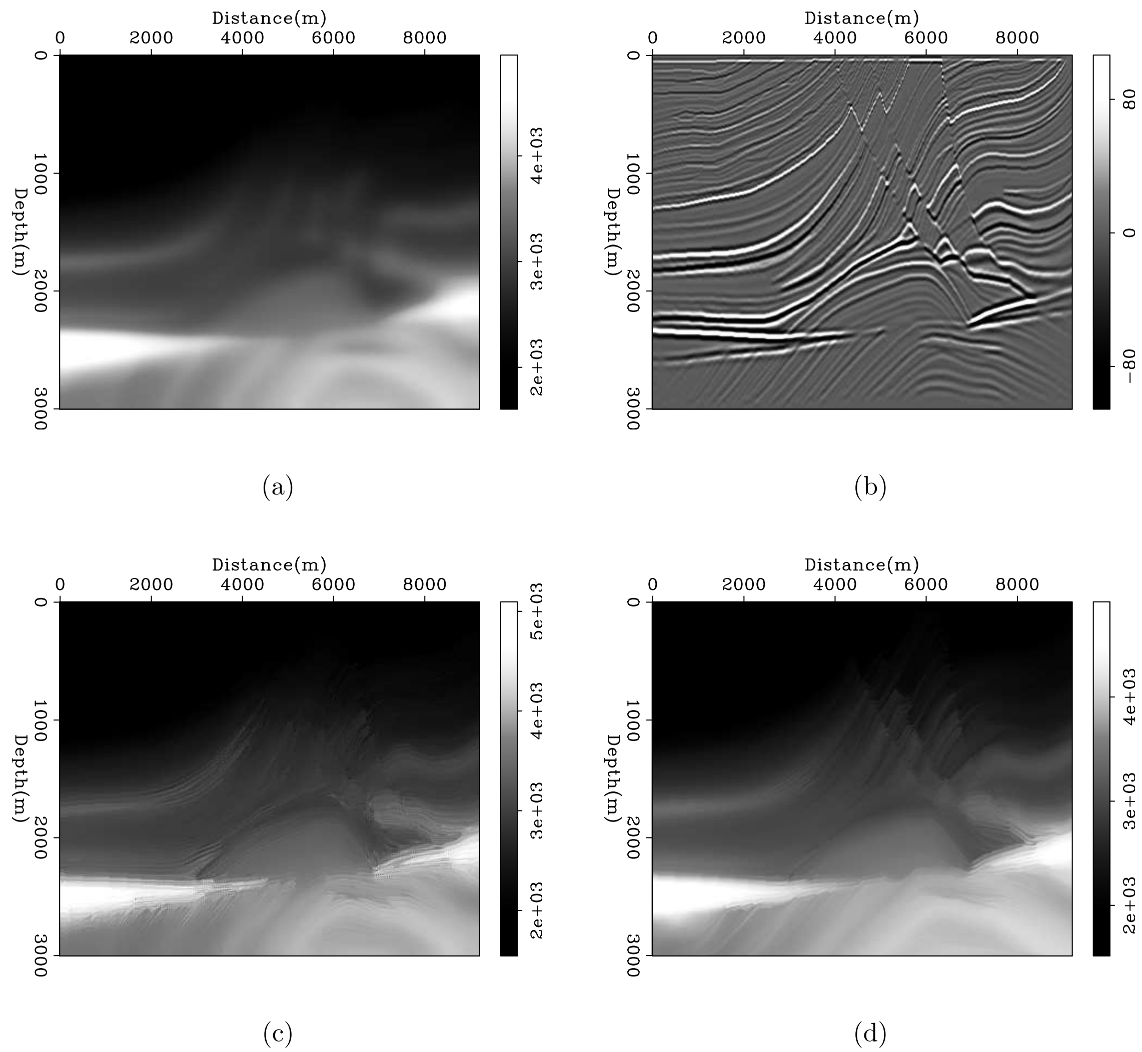


Figure 2: Data integration problem 2: The starting model is now changed to a smooth version of velocity **(a)** and the auxiliary data is the reflectivity of the Marmousi model **(b)** . Next panels show the estimated model with **(c)** the cross-gradient function and **(d)** dip residual. Note that we only expect to reconstruct the structure of the model, and the formulation of our optimization problems does not include the physics behind the wave propagation. [**ER**]

or the resistivity field (see Figure 3). This problem can be stated as

$$\begin{cases} \arg\min_{\Delta \mathbf{s}} \quad \left\| \ \Delta \mathbf{t} - \mathbf{T}_L \Delta \mathbf{s} \right\|_2^2 \quad \text{subject to} \\ \\ \qquad \epsilon \left\| \ \mathbf{A}(\mathbf{r})(\mathbf{s}_0 + \Delta \mathbf{s}) \ \right\|_2^2 \quad \approx \ 0 \end{cases} , \tag{5}$$

where $\mathbf{s}$ and $\mathbf{r}$ are the slowness and auxiliary fields, respectively; $\Delta\mathbf{t}$ and $\mathbf{T}_L$ represents traveltime updates and the linearized tomography mapping operator; and $\mathbf{A}$ is the model-shaping operator, which is picked as either the cross-gradient operator or the dip residual operator. In the latter case, the regularization operator is given by $\mathbf{A}(\mathbf{r}) = \frac{\partial}{\partial x} + p_r \ \frac{\partial}{\partial z}$.

Although the reflectivity field is in practice a function of velocity, we assume that it is an accurate representation. One may consider a more general case, where the regularization operator is also a function of model, which needs to be linearized around the current estimation.

Figure 4 shows the results of solving the tomography problem (Equation 5) for an optimal update in velocity. I start with a smooth velocity as the initial guess. I use either reflectivity or resistivity as auxiliary data and I choose either the cross-gradient function or the dip residual for the regularization operator. This leads to four different updates based on the choice of auxiliary data and data integration method. Note the different frequency contents of the two types of auxiliary data in Figure 3. Updated velocities with cross-gradient functions (Figure 4(a) and 4(b)) seem to have more contribution of structure (model-shaping term) than the physics (data misfit term) of the problem, as the structure of model is clearly visible in the results. The dip residual method, however, shows a better portion of physics in the results and less of the actual structure. This suggests that we may be able to benefit from combining these methods in a specific fashion to obtain a better velocity estimation.

CONCLUSIONS

As mentioned above, geophysical data integration can greatly affect the quality of estimation of attributes by reducing the uncertainty in the model. In this paper, I reviewed the structural similarity and some measurement techniques. The efficiency of these techniques was compared with a simple example, where the physics were not taken into account to emphasize the effect of data-integration techniques. The results suggest that the dip-residual method provides better continuity in estimated structures than does the cross-gradient function. Model estimation results using cross-gradient function shows higher sensitivity, which can be explained by its differentiating nature.

I also compared these methods on a velocity estimation problem, where the physics are also included in the problem statement. I used two different types of auxiliary data

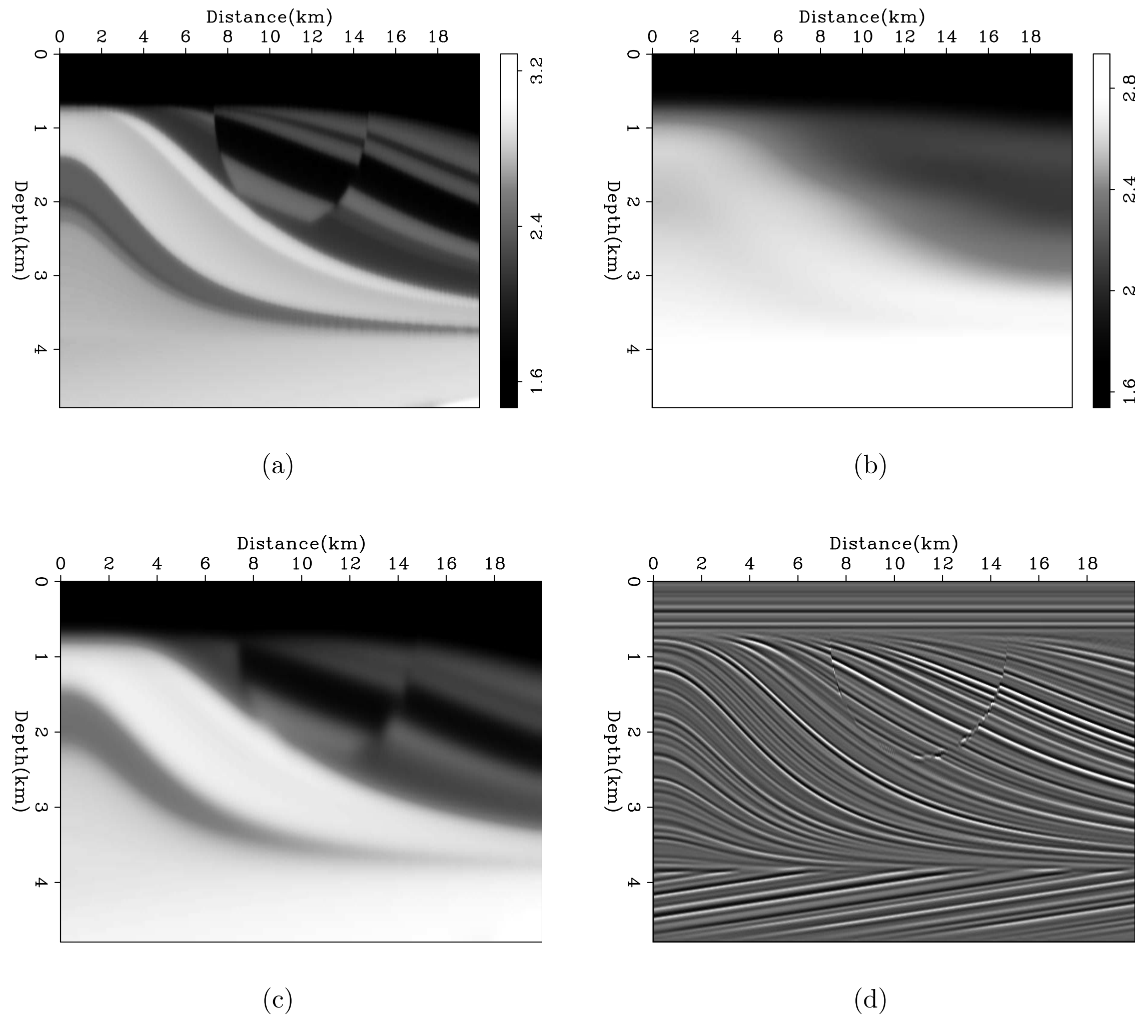
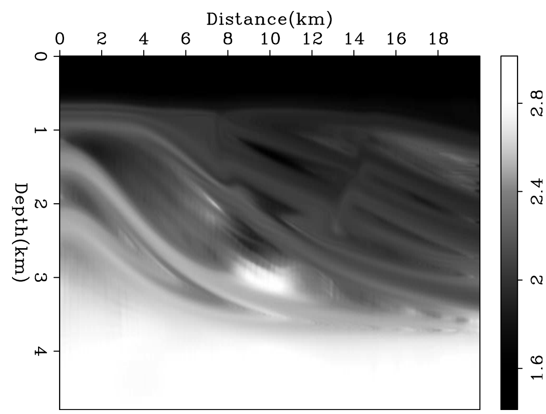
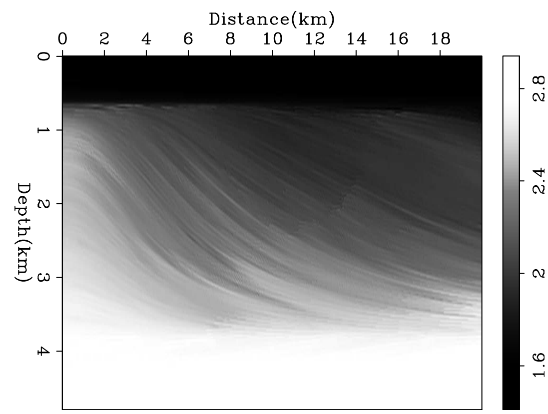


Figure 3: Synthetic 2-D model used as an example for comparison of different data integration methods: **(a)** True velocity, **(b)** initial guess for velocity, **(c)** approximation of resistivity, and **(d)** true reflectivity. [**ER**]



(a)



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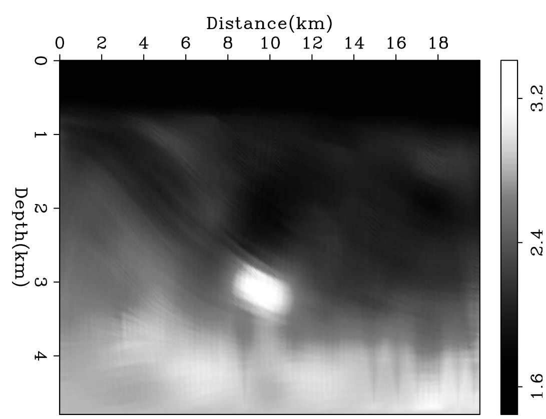
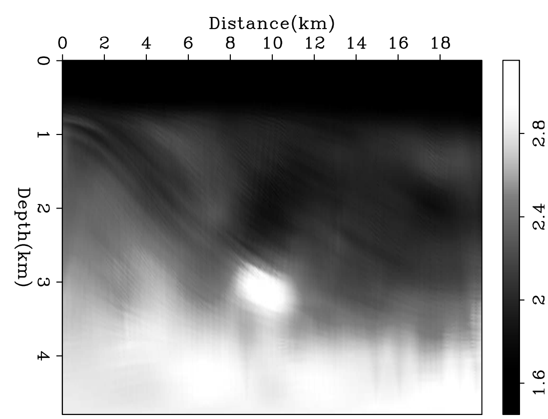


(c)



(d)

Figure 4: Updated velocities for the model shown in Figure 3 obtained by solving the tomography problem using **(a)** resistivity and the cross-gradient function, **(b)** reflectivity and the cross-gradient function **(c)**, resistivity and the dip residual, and **(d)** reflectivity and dip residual. [**CR**]

with different frequency content to compare the efficiency of the techniques reviewed above. In this case, the cross-gradient function shows a stronger structure-imposing effect. However, the cross-gradient functions are not guaranteed to produce improved results when the frequency contents of the main data field and the auxiliary data are very different. The dip- residual method integrates less structural information by missing some of the anomalies. More examples are suggested to validate the comparison results.

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