Schoenberg's angle on fractures and anisotropy: A study in orthotropy

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ABSTRACT

For vertical-fracture sets at arbitrary orientation angles to each other – but not perfectly randomly oriented, I present a detailed model in which the resulting anisotropic fractured medium generally has orthorhombic symmetry overall. Analysis methods of Schoenberg are emphasized, together with their connections to other similarly motivated and conceptually related methods by Sayers and Kachanov, among others. Examples show how parallel vertical fracture sets having HTI symmetry turn into orthotropic fractured media if some subsets of the vertical fractures are misaligned with the others, and then the fractured system can have VTI symmetry if all the fractures are aligned either randomly, or half parallel and half perpendicular to a given vertical plane. Another orthotropic case of vertical fractures in an otherwise VTI earth system treated previously by Schoenberg and Helbig is compared to, and contrasted with, other examples treated here.

INTRODUCTION

The present work covers various issues related to fractures and anisotropy, especially in relation to some of the published work of Michael Schoenberg (Schoenberg, 1980; Schoenberg and Muir, 1989; Schoenberg and Sayers, 1995; Schoenberg and Helbig, 1997). Details of methods presented here will also make use of an approach outlined by Sayers and Kachanov (1991), and used previously by Berryman (2006, 2008, 2009) in a recent series of published papers. [However, it is important to recognize that in earlier work it has also been shown (Berryman, 2006) that the general results of Bakulin et al. (2000) (for example) for the Thomsen (1986, 2002) weak anisotropy seismic parameters and contained in their Figure 6, are both qualitatively and even (reasonably) quantitatively consistent with each other, as well as being consistent with results from the method of Sayers and Kachanov (1991) treated explicitly here.] Thus, a high degree of consistency has been established among fracture-influence results that are based in part on the linear-slip model of fractures by Schoenberg (1980) and in part on penny-shaped (or approximately penny-shaped) cracks. [Also see Grechka et al. (2006).] This fact is important to the theme of the paper, because it shows that the details are often less important than the grand scheme of how fractures affect both the elastic system response and the wave propagation results.

FRACTURE ANALYSIS

For waves propagating in the $[x_1\text{-}x_3]$-plane with wavenumbers $k_1 = k\sin\theta$ and $k_3 = k\cos\theta$ where $k^2 = k_1^2 + k_3^2$, Tsvankin (1997) shows that we have the following equations [patterned here after the notation of Berryman (1979)]:

$$\rho\omega_\pm^2 = \frac{1}{2}\left[(c_{11}+c_{55})k_1^2 + (c_{33}+c_{55})k_3^2 \pm R\right], \tag{1}$$

where

$$R \equiv \sqrt{\left[(c_{11}-c_{55})k_1^2 - (c_{33}-c_{55})k_3^2\right]^2 + 4(c_{13}+c_{55})^2k_1^2k_3^2} \tag{2}$$

and where ρ (with no subscript) is the inertial density. Equation (1) determines the two wave speeds

$$V_\pm^2 = \frac{\omega_\pm^2}{k^2}. \tag{3}$$

The quantities $\omega_\pm$ have dimensions of angular frequency, but they are introduced mostly to simplify the form of the equations. The pertinent phase speeds V_+ for quasi-P-waves and V_- for quasi-SV-waves are given respectively by values corresponding to the $+$ and $-$ subscripts in the velocity equation (3). Group velocities [Brillouin (1946); Rüger (2002); Tsvankin (2005)] can be computed relatively easily as well when using the methods outlined earlier by Berryman (1979).

Sayers and Kachanov (1991) consider a model with two sets of possibly nonorthogonal fractures, also possibly having two different fracture density values ρ_a and ρ_b. Total fracture density is therefore $\rho_f = \rho_a + \rho_b$. These authors found that the pertinent fracture influence parameters were multiplied in this situation, when the angle between the fracture sets is ϕ, either solely by ρ_f itself or by one of the two factors:

$$\begin{aligned} A &= \rho_f + \left[\rho_f^2 - 4\rho_a\rho_b\sin^2\phi\right]^{1/2}, \\ B &= \rho_f - \left[\rho_f^2 - 4\rho_a\rho_b\sin^2\phi\right]^{1/2}. \end{aligned} \tag{4}$$

Table 1 shows the Sayers and Kachanov (1991) results for corrections to the isotropic background values of compliance (in Voigt 6×6 matrix notation — the original paper had results expressed in terms of tensor notation). Those background values are specfically for one model considered having Poisson's ratio $\nu = 0.4375$ (dimensionless), effective bulk modulus $K = 16.87$, shear modulus $\mu = 2.20$, and Young's modulus $E = 6.325$, with all moduli measured in units of GPa. For the assumed inertial density $\rho = 2200.0$ kg/m^3, the resulting isotropic background compressional wave speed is $V_p = 3.0$ km/s and shear wave speed is $V_s = 1.0$ km/s. For our computations, we also need the isotropic background compliance values, which are $S_{11} = S_{22} = S_{33} = 6.325$, $S_{12} = S_{13} = S_{23} = -2.767$, and $S_{44} = S_{55} = S_{66} = 0.4545$. The fracture influence factors η_1 and η_2, found for this specific model by Berryman and Grechka (2006), are displayed in Table 2. Some higher order fracture-influence factors were also obtained in the earlier work, but I will not be considering such factors in this short paper.

ΔS_{11}	=	$(\eta_1+\eta_2)A$
ΔS_{22}	=	$(\eta_1+\eta_2)B$
ΔS_{33}	=	0
ΔS_{12}	=	$\eta_1\rho_f$
ΔS_{13}	=	$\eta_1 A/2$
ΔS_{23}	=	$\eta_1 B/2$
ΔS_{44}	=	$\eta_2 B$
ΔS_{55}	=	$\eta_2 A$
ΔS_{66}	=	$2\eta_2\rho_f$

Table 1: Compliance matrix correction values for the vertical fracture model considered in Eq. 4, which are also true for the specific limit of Eq. 5, as a special case of the general result.

In the examples that follow, I will consider only the case of equal fracture densities $\rho_a = \rho_b = \rho_f/2$. For this somewhat simpler situation, I also have

$$\begin{aligned} A &= \rho_f(1+\cos\phi), \\ B &= \rho_f(1-\cos\phi). \end{aligned} \tag{5}$$

Fracture parameter	GPa^{-1}
η_1	-0.0192
η_2	0.3944

Table 2: Fracture-influence parameters (see Table 1 for usage) in a model reservoir having isotropic background with Poisson's ratio $\nu = 0.4375$, $V_p = 3.0$ km/s, and $V_s = 1.0$ km/s.

There may also be some uncertainty about exactly which of these factors is which in this degenerate case, because of the sign ambiguity in taking the square root of $\cos^2\phi$. But I will not concern myself with this detail here.

VANISHING OF THE ANELLIPTICITY PARAMETERS

One observation made immediately upon computing sample results for the model specified here is that the quasi-SV-wave propagating in the $[x_1$-$x_3]$-plane apparently has constant (or very nearly constant to numerical accuracy) wave speed at all angles in this plane (see Table 3). This result is startling when first seen, but it has been remarked upon previously in the literature by Gassmann (1964) and Schoenberg and Sayers (1995). One useful interpretation of this fact is obtained by noting that for quasi-SV-waves to have constant velocity in the plane, it is necessary for the pertinent effective anellipticity parameter to vanish for this plane of propagation. If it does not vanish identically, then it must at least vanish to first order in the fracture dependent correction factors (shown here in Table 1) in order to explain the numerical results for the relatively small fracture densities considered here. The condition required for exact vanishing of the pertinent anellipticity parameter in terms of stiffness coefficients turns out to be:

$$c_{11}c_{33} - c_{13}^2 = c_{55}\left(c_{11} + c_{33} + 2c_{13}\right), \tag{6}$$

as was already known to Gassmann (1964). But going farther in the analysis, it takes a fair amount of algebra to show that — to first order in the correction factors — the result (6) amounts to a condition relating compliance correction factors:

$$\Delta S_{55} = \Delta S_{11} + \Delta S_{33} - 2\Delta S_{13}. \tag{7}$$

This condition holds when we can ignore the higher order terms $O(\Delta^2)$, as well as all still higher orders (think of the proportionality $\Delta \propto \rho_f$). Then, we find that (7) is in fact satisfied identically when the expressions in Table 1 are substituted. It is important to notice as well that exact satisfaction of the condition in (7) is true for the general form of the definitions in Table 1, and not just for the restricted definitions used in the examples we computed, where the simplified definitions of (5) were employed to reduce our bookkeeping load. So the result in (7) is more general than just the specific examples I have computed. But the result is certainly not expected to be true for arbitrary compliance matrices. Nevertheless, it does seem to be true for a wide range of compliance matrices having vertical fractures in an otherwise isotropic earth.

ϕ	V_{sv} (km/s)
0^o	0.8602
30^o	0.8678
45^o	0.8771
60^o	0.8896
90^o	0.9222

Table 3: Constant V_{sv} wave speeds in the $[x_1$-$x_3]$-plane found for various values of the angle ϕ between fracture planes, and for the fixed value of fracture density $\rho_f = 0.20$.

This result surely seems quite interesting all by itself, but the curiously symmetric nature of the full fracture model can be highlighted further by noting that the following two expressions analogous to (7):

$$\Delta S_{44} = \Delta S_{22} + \Delta S_{33} - 2\Delta S_{23}, \tag{8}$$

and

$$\Delta S_{66} = \Delta S_{11} + \Delta S_{22} - 2\Delta S_{12}, \tag{9}$$

which are also both satisfied identically by the same set of expressions found in Table 1. These facts indicate that the model also has vanishing anellipticity factors (at least to the precision at which we are working) in the other orthogonal planes of propagation [x_2-x_3] and [x_1-x_2], as well.

As is well-known [see Gassmann (1964)], vanishing of the anellipticity factors means that there will be no triplications of wave arrivals for these models from propagation in any of these planes we have considered. Triplications arise because of complications from taking the derivatives required to compute group velocity from the phase velocities quoted here [also see Berryman (1979) for examples]. Group velocity determines the wave speed of signals and/or pulses of seismic energy [see Brillouin (1946) for a discussion].

VERTICAL FRACTURES IN VTI EARTH

Another model in a very similar context that has been discussed frequently by Schoenberg and Helbig (1997), Bakulin et al. (2000), and others is the model of a VTI earth system (where the background elastic medium is transversely isotropic with vertical axis of system, as it would be in a layered earth model having isotropic layers [Backus (1962)]) with superposed vertical fractures. A result that is often quoted in this context concerns a condition that is necessarily satisfied by the elastic stiffness matrix elements for such a system:

$$c_{13}(c_{22}+c_{12}) = c_{23}(c_{11}+c_{12}). \tag{10}$$

Using the same ideas applied here already, we can reduce this equation to a simple statement about the system compliances. The resulting statement is the formula:

$$S_{13} = S_{23}, \tag{11}$$

by which we mean to say that the only requirement imposed on the compliances after the introduction of the vertical fractures to the VTI earth background is that the new overall system compliance must satisfy the conditions in (11) after the changes due to the fractures are included in the values of these two compliance components. No other special constraints appear.

To see that (11) is the correct condition, note that

$$\begin{aligned} S_{13} &= (c_{12}c_{23} - c_{13}c_{22})/\det(C) \\ S_{23} &= (c_{12}c_{13} - c_{23}c_{11})/\det(C), \end{aligned} \tag{12}$$

where $\det(C)$ is the determinant of the upper left 3×3 sub-matrix of the orthotropic stiffness matrix C. Equating these two expressions from (12) and rearranging the final result gives a formula that is precisely the same condition (10) given by Schoenberg and Helbig (1997). What this condition implies for the physical system — since the background earth medium is assumed to be VTI with vertical axis of symmetry and also since $S_{13} = S_{23}$ for the background medium itself (before the fractures are added to it) — is that our final result must be the condition:

$$\Delta S_{13} = \Delta S_{23}. \tag{13}$$

This very simple equality means the changes (i.e., increases) in these off-diagonal compliances — when due to the addition of the vertical fractures to this model — have just one constraint, and that single constraint is that changes in these two off-diagonal compliances S_{13} and S_{23} must occur in unison. This result is also seen as a limiting case found in Table 1, when $A = B$, which occurs only when $\phi = 0^o$. Since $\phi = 0^o$ means that all the vertical fractures are parallel, and therefore being all aligned fractures, I therefore then have exactly the case studied explicitly by Schoenberg and Helbig (1997).

DISCUSSION OF VARIOUS FRACTURE MODELS

Among others, there are two main methods that are typically used in the seismic exploration literature for modeling the effects of fractures on seismic wave propagation: One is the method introduced originally by Schoenberg (1980) and the other was introduced at about the same time by Kachanov (1980). These two methods have both been used extensively in the exporation community, especially since the work of Sayers and Kachanov (1991) and Schoenberg and Sayers (1995). Connections, including many similarities and a few differences, are discussed in an overview by Schoenberg and Sayers (1995). I have some personal preference for the analytical version of Sayers and Kachanov (1991), because it is so explicit and also permits deep connections to be made to effective medium theory, especially through the work of Eshelby (1957). In particular, some recent and related work on fractures in outcrops using an effective medium theory approach by Berryman and Aydin (2009) has made use of the approach put forward by Sayers and Kachanov (1991) for modeling higher fracture density media. This study might have been more difficult to carry through using the original Schoenberg (1980) approach. But, once these higher fracture density results were known, it became straightforward to incorporate them into the layer-averaging approach emphasized by Backus (1962) and Schoenberg and Muir (1989). Thus, the effective medium theory for layering was found to be most useful for providing a means of determining fracture-fracture interactions when the fracture sets are close, but not actually intersecting.

Effects of fluids on the fracture behavior have not been emphasized here, but this issue is obviously a very important one for our applications, and it has been treated in other recent work by Daley et al. (2006) and Berryman (2006). Effects that liquids have on elastic moduli may be incorporated fairly easily using results of Gassmann (1951) and Skempton (1954), as shown by Berryman (2006).

CONCLUSIONS

In this paper I have treated various methods for quantifying the geomechanical effects of fracture sets on reservoirs. Special emphasis has been given to recent work, and also to the influence that the work of Michael Schoenberg has had on this subject. I conclude that the methods currently in use provide a consistent and presumably quite accurate picture of the influence of fractures on wave propagation in many cases, and that the various methods in use, although sometimes presented quite differently, are actually very closely related both conceptually and also in the quantitative details of their predictions.

Of course, elastic orthotropy is not universal in the earth [see Sayers et al. (2009)], so we should not and I do not assume that all the problems in exploration seismology will be solved by such relatively simple models. One final conclusion is that, although good progress has been made, there is also clearly more work to be done relative to the influence of fractures and anisotropy on seismic waves.

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REFERENCES

Backus, G. E., 1962, Long-wave elastic anisotropy produced by horizontal layering: Journal of Geophysical Research, **67**, 4427–4440.

Bakulin, A., V. Grechka, and I. Tsvankin, 2000, Estimation of fracture parameters from reflection seismic data – Part I: HTI model due to a single fracture set: Geophysics, **65**, 1788–1802.

Berryman, J. G., 1979, Long-wave elastic anisotropy in transversely isotropic media: Geophysics, **44**, 896–917.

———, 2006, Seismic waves in rocks with fluids and fractures: Geophysical Journal International, **171**, 954–974.

———, 2008, Exact seismic velocities for transversely isotropic media and extended Thomsen formulas for stronger anisotropies: Geophysics, **73**, D1–D10.

———, 2009, Aligned vertical fractures, HTI reservoir symmetry and Thomsen seismic anisotropy parameters for polar media: Geophysical Prospecting, **57**, no. 2, 193–208.

Berryman, J. G. and A. Aydin, 2009, Quasi-static analysis of elastic behavior for some systems having higher fracture densities. submitted for publication 2009.

Berryman, J. G. and V. Grechka, 2006, Random polycrystals of grains containing cracks: Model of quasistatic elastic behavior for fractured systems: Journal of Applied Physics, **100**, 113527.

Brillouin, L., 1946, Wave Propagation in Periodic Structures: Dover.

Daley, T. M., M. Schoenberg, J. Rutqvist, and K. T. Nihei, 2006, Fractured reservoirs: An analysis of coupled elastodynamic and permeability changes from pore-pressure variation: Geophysics, **71**, no. 5, O33–O41.

Eshelby, J. D., 1957, The determination of the elastic field of an ellipsoidal inclusion, and related problems: Proceedings of the Royal Society of London A, **241**, no. 1226, 376–396.

Gassmann, F., 1951, Über die Elastizität poröser Medien: Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, **96**, 1–23.

———, 1964, Introduction to seismic travel-time methods in anisotropic media: Pure and Applied Geophysics, **58**, 53–112.

Grechka, V., I. Vasconcelos, and M. Kachanov, 2006, Influence of crack shape on the effective elasticity of fractured rocks: Geophysics, **71**, no. 5, D153–D160.

Kachanov, M., 1980, Continuum model of medium with cracks: Journal of the Engineering Mechanics Division of ASCE, **106**, 1039–1051.

Rüger, A., 2002, Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media: Society of Exploration Geophysicists, Tulsa, OK.

Sayers, C. M. and M. Kachanov, 1991, A simple technique for finding effective elastic constants of cracked solids for arbitrary crack orientation statistics: International Journal of Solids and Structures, **27**, 671–680.

Sayers, C. M., A. D. Taleghani, and J. Adachi, 2009, The effect of mineralization on the ratio of normal to tangential compliance of fractures: Geophysical Prospecting, **57**, no. 3, 439–446.

Schoenberg, M., 1980, Elastic wave behavior across linear slip interfaces: Journal of the Acoustical Society of America, **68**, 1516–1521.

Schoenberg, M. and K. Helbig, 1997, Orthorhombic media: Modeling elastic wave behavior in a vertically fractured earth: Geophysics, **62**, no. 6, 1954–1974.

Schoenberg, M. and F. Muir, 1989, A calculus for finely layered anisotropic media: Geophysics, **54**, no. 4, 581–589.

Schoenberg, M. and C. M. Sayers, 1995, Seismic anisotropy of fractured rock: Geophysics, **60**, no. 1, 204–211.

Skempton, A. W., 1954, The pore-pressure coefficients A and B: Geotechnique, **4**, 143–147.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, **51**, 1954–1966.

———, 2002, Understanding Seismic Anisotropic in Exploration and Exploitation: Society of Exploration Geophysicists, Tulsa, OK.

Tsvankin, I., 1997, Anisotropic parameters and P-wave velocity for orthorhombic media: Geophysics, **72**, 1292–1309.

———, 2005, Seismic Signatures and Analysis of Reflection Data in Anisotropic Media: Elsevier, Oxford, UK.