MIGRATION AS AN ADJOINT OF UP-WARD CONTINUATION MODELING IN SHOT-PROFILE DOMAIN

Upward-continuation modeling in the shot-profile domain

Starting from the cross-correlation imaging condition in the shot-profile migration

$$r(\mathbf{x},\mathbf{h}) = \sum_{\mathbf{x}_s}\sum_{\omega} p^*(\mathbf{x}-\mathbf{h},\mathbf{x}_s,\omega)u(\mathbf{x}+\mathbf{h},\mathbf{x}_s,\omega), \tag{1}$$

where $p(\mathbf{x},\mathbf{x}_s,\omega)$ is the source wavefield for a single frequency ω at image point $\mathbf{x} = (x, y, z)$ with the source located at $\mathbf{x}_s = (x_s, y_s, 0)$; $u(\mathbf{x},\mathbf{x}_s,\omega)$ is the receiver wavefield and $\mathbf{h} = (h_x, h_y, h_z)$ is the subsurface half-offset, and '*' stands for the complex-conjugate. We can compute the adjoint of equation 1 as:

$$p(\mathbf{x},\mathbf{x}_s,\omega) = \sum_{\mathbf{h}} u(\mathbf{x}+\mathbf{h},\mathbf{x}_s,\omega)r(\mathbf{x},\mathbf{h}) \tag{2a}$$

$$u(\mathbf{x},\mathbf{x}_s,\omega) = \sum_{\mathbf{h}} p(\mathbf{x}-\mathbf{h},\mathbf{x}_s,\omega)r(\mathbf{x},\mathbf{h}). \tag{2b}$$

In a more compact notation, not explicitly writing the dependencies on x, y and $\mathbf{h}$, and for a given depth level, z, equation 2b can be written as:

$$u^z(\omega) = \mathbf{P}^z(\omega)r^z, \tag{3}$$

where $\mathbf{P}^z$ is a convolutional matrix. If $r = r(\mathbf{x}, h_x)$, for the case of 5 x positions and 3 subsurface offsets and subsurface-offset interval equals to the CMP interval, equation 3 can be explicitly written as:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} 0 & p_1 & 0 & p_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_2 & 0 & p_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_1 & 0 & p_3 & 0 & p_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_2 & 0 & p_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_3 & 0 & p_5 & 0 \end{bmatrix} \begin{bmatrix} r_{1,1} \\ r_{1,2} \\ r_{1,3} \\ r_{2,1} \\ r_{2,2} \\ r_{2,3} \\ r_{3,1} \\ r_{3,2} \\ r_{3,3} \\ r_{4,1} \\ r_{4,2} \\ r_{4,3} \\ r_{5,1} \\ r_{5,2} \\ r_{5,3} \end{bmatrix} \tag{4}$$

where the subscripts of u and p represent x positions. The first and second subscripts of r represent x positions and subsurface offset, respectively.

The recursive upward propagation of $u^z(\omega)$ and recording at the surface are given by the following equations

$$\begin{cases} u^{z-1}(\omega) = T_{\uparrow}^{z}(\omega, s) P^z(\omega) r^z \\ d(\omega) = u^1(\omega) \end{cases} \tag{5}$$

where $T_{\uparrow}^{z}$ is the upward continuation operator, which is function of the slowness, s.

bla, bla ... The matrix representation of equation 5 is

$$\begin{aligned} \underline{\mathbf{u}} &= \mathbf{T}_{\uparrow}\mathbf{P}\mathbf{r} \\ \mathbf{d} &= \mathbf{E}\underline{\mathbf{u}} \end{aligned} \tag{6}$$

Weighted least-squares data fitting

The optimum reflctivity, $\mathbf{r}$, minimizes the objective function

$$J(\mathbf{r}, \underline{\mathbf{u}}) = h(\mathbf{r}, \underline{\mathbf{u}}) = \frac{1}{2} \left|\left|\mathbf{W}\left(\mathbf{d} - \mathbf{d_{obs}}\right)\right|\right|^2. \tag{7}$$

$$\tag{8}$$

The forward equation reads:

$$F(\mathbf{r}, \underline{\mathbf{u}}) = \underline{\mathbf{u}} - \mathbf{T}_{\uparrow}\mathbf{P}\mathbf{r} = 0. \tag{9}$$

- Build the augmented functional

$$\begin{aligned} \mathcal{L}(\underline{\lambda}, \underline{\mathbf{u}}, \mathbf{r}) &= h(\mathbf{r}, \underline{\mathbf{u}}) - \langle \underline{\lambda}, F(\mathbf{r}, \underline{\mathbf{u}}) \rangle \\ \mathcal{L}(\underline{\lambda}, \underline{\mathbf{u}}, \mathbf{r}) &= \tfrac{1}{2} \left|\left|\mathbf{W}\left(\mathbf{d} - \mathbf{d_{obs}}\right)\right|\right|^2 - \langle \underline{\lambda}, \underline{\mathbf{u}} - \mathbf{T}_{\uparrow}\mathbf{P}\mathbf{r} \rangle. \end{aligned} \tag{10}$$

- Define the adjoint state equations

$$\frac{\partial \mathcal{L}(\underline{\lambda}, \underline{\mathbf{u}}, \mathbf{r})}{\partial \underline{\mathbf{u}}} = \mathbf{E}^T \mathbf{W}^2 (\mathbf{d} - \mathbf{d_{obs}}) - \lambda = 0, \tag{11}$$

  which gives

$$\lambda = \mathbf{E}^T \mathbf{W}^2 (\mathbf{d} - \mathbf{d_{obs}}). \tag{12}$$

- Compute the gradient with respect to $\mathbf{r}$

$$\frac{\partial \mathcal{L}(\underline{\lambda}, \underline{\mathbf{u}}, \mathbf{r})}{\partial \mathbf{r}} = \frac{\partial J}{\partial \mathbf{r}} = \lambda^* \mathbf{T}_{\uparrow}\mathbf{P} \tag{13}$$

  and

$$\nabla J = \mathbf{P}^* \mathbf{T}_{\downarrow} \lambda, \tag{14}$$

that corresponds to the downward propagation of the residuals and cross-correlation imaging condition of the downward propagated residuals with the source wavefield.