Inversion of up and down going signal for ocean bottom data

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ABSTRACT

We formulate an inversion problem using the up- and down-going signals of ocean bottom data to imaging primaries and multiples. The method involves separating pressure (P) and vertical particle velocity (V_z) data into up- and down-going components. Afterward, the up- and down-going data can be used for inversion with appropiate modeling operators. To a first-order accuracy, we use mirror imaging to define the up- and down-going modeling operator. A complete modeling scheme can be defined by the composition of the over-under modeling operator and the up-down decomposition operator. This scheme effectively models all primaries, water reveberations and multiples.

INTRODUCTION

Traditionally, multiples in seismic surveying are considered as noise to be removed because most migration algorithims do not account for multiples. Recently, there have been efforts to use multiples as signals. For example, Berkhout and Verschuur (2003) and Guitton (2002) image the multiples with shot-profile migration while Shan (2003) transforms multiples into pseudo-primaries by cross-correlation in the souce-receiver domain.

One motivation to image with multiples is that multiples can provide subsurface information not found in primaries. The angular and spatial ranges covered by multiples are different than that of primaries (Figure 1). Ronen et al. (2005); Guimaraes et al. (1998) propose a mirror imaging technique that takes advantage of this property for ocean bottom and vertical cable acquisition geometry, respectively. By using receiver ghosts as signals, mirror imaging provides a much wider aperture in the image space, given the same set of data. While mirror imaging correctly images multiples by using down-going receiver ghost at the ocean bottom, the primary signal is imaged separately. On the other hand, Brown (2004); Brown and Guitton (2005) proposed joint imaging between primaries and multiples by using least-square inversion. One advantage of joint inversion is that both the primary and multiple signals are used.

For the case of ocean bottom data, signals can be separated into up- and down-going parts. Traditioanlly, up- and down-going signals are used for de-ghosting (Canales and Bell, 1996). Muijs et al. (2007) use this property to formulate prestack depth migration of primary and surface-related multiple using downward continuation. In

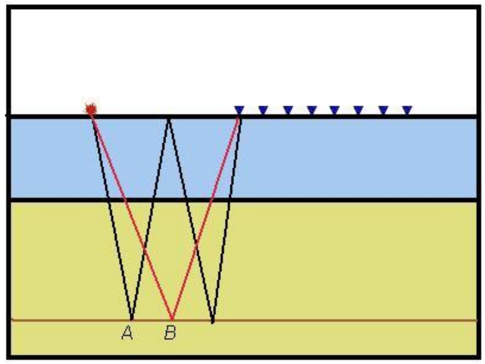

Figure 1: The angular and spatial ranges covered by multiples is different than that of primary. Near the edge of the receiver array, A can only be illuminated by multiple as primary can only illuminate up to a spatial extend of B.[**NR**]

this study, we carry on the work of Brown (2004) and discuss the theory of joint imaging of multiples and primaries using up- and down-going signals of ocean bottom data.

We will first discuss the geometry of ocean bottom data acquisition. Next, we will consider the techniques available to separate data into up-going and down-going wavefields. We will discuss the theory of joint imaging of multiples and primaries using up- and down-going ocean bottom data. Such an inversion scheme requires a good modeling operator for up- and down-going signals. To a first order accuracy and flat water bottom, we use mirror imaging to define the two modeling operators. A more accurate modeling scheme requires an over-under modeling and an up-down decomposition operator. This scheme effectively images all primaries, water reveberations, and multiples. It also reduces crosstalk leakage between up- and down-going signals and hence reduces incorrectly placed reflectors. The testing of this thoery would be the focus of my research for the next quarter.

SEISMIC ACQUISITION OF OCEAN BOTTOM DATA

Although conventional streamers acquisition is well developed in terms of technology and processing techniques, it has significant limitations. For example, in obstructed oil fields, working with streamers could be difficult. In addition, streamers are more prone to drift and to be affected by weather conditon, which may compromise repeatability in time-lapse reservoir monitoring (4D). These limitations bring out a growing demand for ocean bottom seismometers (OBS). OBS data acquisition is an alternate approach in which seismometers are placed at the ocean bottom and shots are fired at the ocean surface.

OBS data acquisition can be done with either ocean bottom cables (OBC) or with nodes (OBN). Because the OBS method uses geophones and hydrophones, it can measure both compressional and shear waves. This capability permits separating up- and down-going waves at the seabed and therefore provides good opportunities for imaging with multiples.

Multi-component streamers (Pharez et al., 2008) also permit separation of up-going and down-going waves, but because the data are recorded near the sea surface, imaging with multiples is much more limited than with OBS in deep water.

To understand the events represented by the up-going an down-going signals, consider Figure 2. For ocean bottom data acquisition, down-going events include direct arrival, receiver ghosts, and higher-order pegleg multiples. On the other hand, up-going events include primaries, and pegleg multiples. Since the kinematics of these events are quite distinct for up-going and down-going signals, an inverse problem can be formulated to fit the up and down signals jointly. Before formulating the inverse problem, we will dicuss methods for separating receiver signals into their up- and down-going parts.

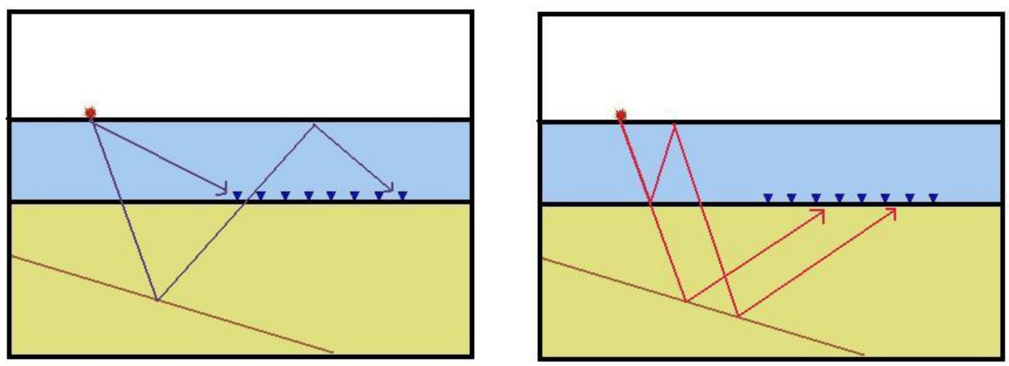

Figure 2: Left: The down-going signal consists of events such as direct arrival, receiver ghosts, and higher-order pegleg multiples. Right: The up-going signal consists of events such as primaries and other pegleg multiples. [**NR**]

SEPARATION OF UP- AND DOWN-GOING WAVEFIELD

There are two well developed techniques to separate recorded signal into up- and down-going waves. For multi-component data, such separation can be done with the pressure and particle velocity recordings. Another way is to have two sets of hydrophones with one set on top of the other (an over-under arrangement). Although the over-under arrangement is used near the ocean's surface, it is an easy way to obtain up- and down-going wavefields for synthetic data examples.

Separation using pressure and particle velocity recordings

The basic idea of up-down separation using pressure and vertical particle velocity is quite simple. Hydrophones measure compressional waves (P) regardless of their direction. Ocean bottom seismometers measure vertical particle velocity (V_z) that depends on the direction of the waves measured. Figure 3 illustrates the measurement of a positive pulse coming from above and from below.

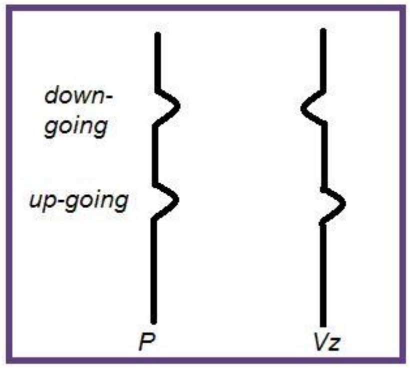

Figure 3: This Figure illustrates pressure **P** and verticle particle velocity V_z measurement of a positive pulse coming from above and from below. Down-going events have opposite polarity while up-going events have the same polarity. [**NR**]

Since the polarity of the P and V_z signal is the same for up-going waves and opposite for down-going waves, one can decompose the P and V_z measurements into up-going (U) and down-going (D) pressure components:

$$\begin{aligned} P(z_r) &= [U(z_r) + D(z_r)], \\ V_z(z_r) &= [U(z_r) - D(z_r)]/I, \end{aligned} \tag{1}$$

where z_r is the receiver depth and I is an impedance factor that scales vertical velocity value to pressure value. The impedance can be offset, frequency, wavenumber, or density dependent depending on the method used. One way to perform PZ summation is in the Fourier $(\omega - k_x - k_y)$domain as

$$\begin{aligned} U(z_r) &= \frac{1}{2}\left[P(z_r) - \frac{\rho\omega}{k_z}V_z(z_r)\right], \\ D(z_r) &= \frac{1}{2}\left[P(z_r) + \frac{\rho\omega}{k_z}V_z(z_r)\right], \end{aligned} \tag{2}$$

where ω is frequency in time. $k_z = \sqrt{\frac{w^2}{v^2} - k_x^2 - k_y^2}$ is the vertical wavenumber calculated from horizontal wave numbers k_x and k_y. For a complete derivation of equation 2, please refer to Amundsen (1993).

Separation using over-under recordings

The derivation for decomposing over-under pressure waves into up-going and down-going signals is best done in the Fourier domain. Denote $S_1(\omega, k_x)$ and $S_2(\omega, k_x)$ to be the Fourier transformed measurement of compressional waves at depth z_1 (over) and z_2 (under). Theoretically, $S_1(\omega, k_x)$ can be viewed as a sum of the up-going $U_1(\omega, k_x)$ and down-going $D_1(\omega, k_x)$ components. Likewise for $S_2(\omega, k_x)$:

$$\begin{aligned} S_1(\omega, k_x) &= U_1(\omega, k_x) + D_1(\omega, k_x), \\ S_2(\omega, k_x) &= U_2(\omega, k_x) + D_2(\omega, k_x). \end{aligned} \tag{3}$$

Down-going waves visit the over array (D_1) before visiting the under array (D_2). Therefore, D_1, when shifted forward in time, would match the signal D_2. Similarily, up-going waves visit the under array first. Therefore, U_2, when shifted forward in time would match the signal U_1. This relationship is equivalent to a phase-shift in the Fourier domain:

$$\begin{aligned} e^{ik_z \Delta z} D_1 &= D_2, \\ U_1 &= e^{ik_z \Delta z} U_2, \end{aligned} \tag{4}$$

where $\Delta z = z_2 - z_1$ and k_z is the usual dispersion relation. Finally, substituting equation 4 into 3 yields the formula for the up-going and down-going waves at the receivers:

$$\begin{aligned} U_2 &= \frac{S_2 - e^{ik_z \Delta z} S_1}{1 - e^{2ik_z \Delta z}}, \\ D_2 &= \frac{e^{ik_z \Delta z} S_1 - e^{2ik_z \Delta z} S_2}{1 - e^{2ik_z \Delta z}}. \end{aligned}$$

Data acquisition using over-under arrangement is often used to elimate receiver ghosts and water reverberation. For a thorough review of this method, please see Sonneland et al. (1986). Although over-under arrays are rarely placed at the sea floor in real seismic surveying, this technique allows easy generation of up- and down-going data at the sea bottom in synthetic examples using the simpler acoustic wave equation.

THE INVERSE PROBLEM FOR IMAGING MULTIPLES USING UP- AND DOWN-GOING DATA

The inverse problem for imaging multiples using up- and down-going data can be fomulated as follow. We first break down the recorded data as the superposition of up- $\mathbf{d}_\uparrow$ and down-going $\mathbf{d}_\downarrow$ signals at the receivers. This can be done by using PZ data to give up-going and down-going data as discussed in the PZ summation section above.

$$\begin{bmatrix} \mathbf{d}_\uparrow \\ \mathbf{d}_\downarrow \end{bmatrix} = \mathbf{S}_{pz} \begin{bmatrix} \mathbf{d}_p \\ \mathbf{d}_z \end{bmatrix}$$

The above construction assumes that the vertical particle velocity d_z contains mostly pressure ($\mathbf{P}$) waves. A pre-processing step can be included into $\mathbf{S}_{pz}$ to separate the $\mathbf{P}-$ and converted $\mathbf{S}-$wave arrivals (Helbig and Mesdag, 1982; Dankbaar, 1985). Next, we denote the modelling operator for up-going signals at the receivers as $\mathbf{L}_\uparrow$. Similarily, denote the modeling operator for down-going signals at the receivers as $\mathbf{L}_\downarrow$. The two modeling operators provide the up- and down-going modeled data, denoted as $\mathbf{d}_\uparrow^{mod}$ and $\mathbf{d}_\downarrow^{mod}$;

$$\begin{aligned} \mathbf{d}_\uparrow^{mod} &= \mathbf{L}_\uparrow \mathbf{m}, \\ \mathbf{d}_\downarrow^{mod} &= \mathbf{L}_\downarrow \mathbf{m}. \end{aligned} \tag{5}$$

The inverse problem is defined as minimizing the L_2 norm of the two data residuals $\mathbf{r}_\uparrow$ and $\mathbf{r}_\downarrow$, with respect to a single model $\mathbf{m}$. The data residuals are defined as the difference between the recorded data and the modelled data,

$$\begin{aligned} \mathbf{r}_\uparrow &= \mathbf{d}_\uparrow - \mathbf{d}_\uparrow^{mod} = \mathbf{d}_\uparrow - \mathbf{L}_\uparrow \mathbf{m}, \\ \mathbf{r}_\downarrow &= \mathbf{d}_\downarrow - \mathbf{d}_\downarrow^{mod} = \mathbf{d}_\downarrow - \mathbf{L}_\downarrow \mathbf{m}, \end{aligned} \tag{6}$$

$$min \left(\|\mathbf{L}_\uparrow \mathbf{m} - \mathbf{d}_\uparrow\|_2^2 + \|\mathbf{L}_\downarrow \mathbf{m} - \mathbf{d}_\downarrow\|_2^2 \right), \tag{7}$$

where $\|.\|_2$ represents the L_2 norm. In matrix form, the fitting goal can be written as

$$0 \approx \begin{bmatrix} \mathbf{L}_\uparrow \\ \mathbf{L}_\downarrow \end{bmatrix} \mathbf{m} - \begin{bmatrix} \mathbf{d}_\uparrow \\ \mathbf{d}_\downarrow \end{bmatrix}.$$

With the conjugate gradient method, the model update $\Delta\mathbf{m}$ at each iteration has contributions from both the up-going and down-going parts of the inversion;

$$\Delta\mathbf{m} = \mathbf{L}_\uparrow' \mathbf{r}_\uparrow + \mathbf{L}_\downarrow' \mathbf{r}_\downarrow. \tag{8}$$

where $\mathbf{r}_\uparrow = \mathbf{L}_\uparrow \mathbf{m} - \mathbf{d}_\uparrow$ and $\mathbf{r}_\downarrow = \mathbf{L}_\downarrow \mathbf{m} - \mathbf{d}_\downarrow$ is the up- and down-going part of the residual, respectively.

The justification for this inverse problem is to reduce the wrong placement of image point with the use of both $\mathbf{d}_\uparrow$ and $\mathbf{d}_\downarrow$ signals. Traditional migration scheme only uses $d_\uparrow$ to determine the image since all primary signal can be found in $\mathbf{d}_\uparrow$. However, migration of primaries can give incorrect image point as well. In Figure 4, a primary event with a given travel time can indicate a correct image point at A and an incorrect image point at B. If we include the previously ignored information $\mathbf{d}_\downarrow$ into a joint inversion, some wrongly placed image point can be refuted. Joint imaging allow us to use both primaries and multiples to estimate the image. This can be a distinct advantage because multiples and primaries illuminate different parts of the sub-surface. For ocean bottom data with sparse receiver spacing, multiples illuminate more than primaries.

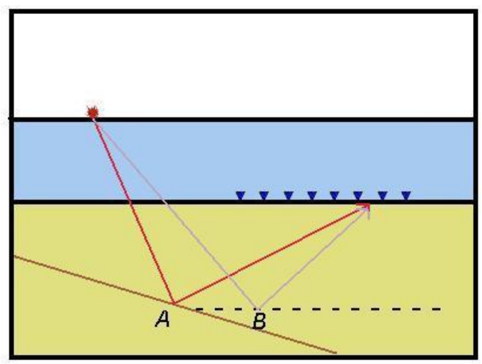

Figure 4: An up-going signal with a given travel time can indicate a correctly placed reflector at A and an incorrectly placed reflector at B. Reflector A will be supported by down-going data while reflector B will be refuted. [**NR**]

The quality of the inverse problem would depend on the implemetation of the modeling operator. The next section will discuss how to approximate the modeling operator.

MODELLING OPERATORS

To implement for a modeling operator that maps the image only into up-going signal $d_\uparrow$ or only into down-going signal $d_\downarrow$ at the receiver, we can use wave equation extrapolation, choosing either an one-way or a two-way wave equation. We can define the modeling operator with different levels of accuracy. As a first-order of accuracy, we can use the mirror imaging operator to model the down-going wave as described in the next section.

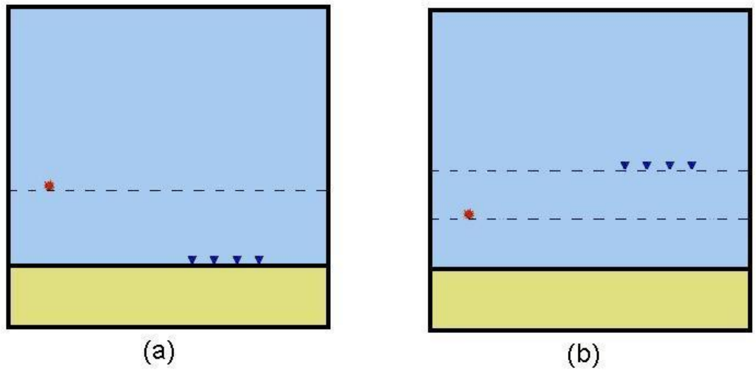

Figure 5: (a) shows the set-up of a first-order modelling operator for the up-going signal. The air-water interface is removed and filled with a half space of water. (b) shows the set-up of a first-order modeling operator for down-going signal. The air-water interface is removed and the receivers is elevated to twice the water depth. [**NR**]

Mirror Imaging modeling opeators

The first-order implementation of $\mathbf{L}_{\uparrow}$, denoted $\mathbf{L}_1^{\uparrow}$ performs wave equation modeling in a model space without the sea surface as shown in Figure 5 (a). Our up-going modeling operator is now approximated as

$$\mathbf{L}_{\uparrow} \approx \mathbf{L}_1^{\uparrow}. \tag{9}$$

When the sea surface is removed, waves always return to the receiver going upward. There are obvious limitations to $\mathbf{L}_1^{\uparrow}$. It can only model primaries and internal multiples. Higher order water reverberations are excluded. Figure 6 shows some events that are captured and excluded by $\mathbf{L}_1^{\uparrow}$.

On the other hand, we can use the mirror imaging modeling operator to get a first-order estimate of down-going signals, denoted $\mathbf{L}_1^{\downarrow}$. This operator performs wave equation modeling by raising the receivers to twice the water depth level as shown in Figure 5 (b). Our down-going modeling operator is now approximated as

$$\mathbf{L}_{\downarrow} \approx \mathbf{L}_1^{\downarrow}. \tag{10}$$

The limitation of $\mathbf{L}_1^{\downarrow}$ is that it can only image direct arrival and pegleg multiples having only one bounce from the sea surface. Higher-order water reverberations and pegleg multiples with two or more bounces from the sea surface are excluded. Figure 7 shows some events that are captured by $\mathbf{L}_1^{\downarrow}$. To do mirror imaging, we had to assume that the sea bottom is flat.

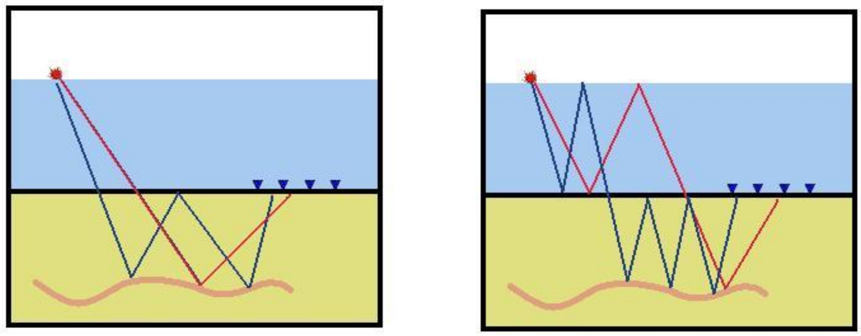

Figure 6: Left: The first-order up-going operator captures all the primaries and internal multiples. Right: It does not capture higher-order water reverberations and any pegleg multiples with a bounce at the sea surface. [**NR**]

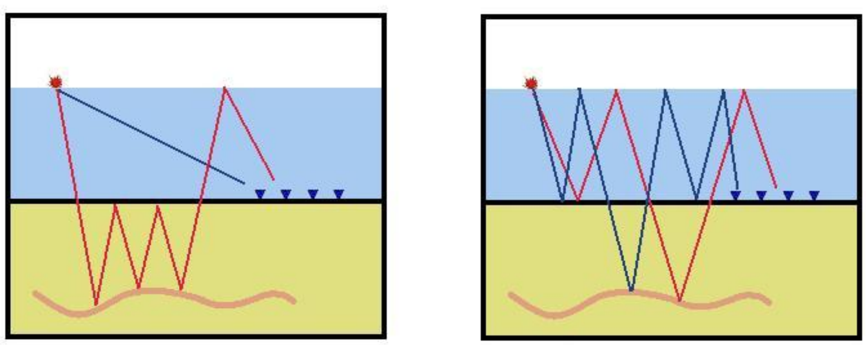

Figure 7: Left: The first-order down-going operator captures the direct arrival and pegleg multiples with only one reflection from the sea surface. Right: It does not capture higher-order water reverberation and any pegleg multiples with two or more bounces at the sea surface. [**NR**]

Complete modeling

To model beyond primaries and first order receiver pegleg multiples, one can use the separation operator that maps over-under data to up-going and down-going data,

$$\begin{bmatrix} \mathbf{L}_{\uparrow} \\ \mathbf{L}_{\downarrow} \end{bmatrix} = \mathbf{S}_{ou}\mathbf{A},$$

where $\mathbf{A}$ represents the wave equation forward modelling operator that generates over and under signals. $\mathbf{S}_{ou}$ is a separation operator that extracts the up- and down-going signals from over-under data. In matrix form, the inversion scheme has the following fitting goal:

$$0 \approx \begin{bmatrix} \mathbf{L}_{\uparrow} \\ \mathbf{L}_{\downarrow} \end{bmatrix} \mathbf{m} - \begin{bmatrix} \mathbf{d}_{\uparrow} \\ \mathbf{d}_{\downarrow} \end{bmatrix} = \mathbf{S}_{ou}\mathbf{A}\mathbf{m} - \mathbf{S}_{pz} \begin{bmatrix} \mathbf{d}_{p} \\ \mathbf{d}_{z} \end{bmatrix},$$

where $\mathbf{S}_{pz}$ is a separation operator that extracts the up- and down-going signals from PZ data. Note that $\mathbf{d}_{\uparrow}$ and $\mathbf{d}_{\downarrow}$ can be viewed as processed data from the orignal recorded $\mathbf{d}_p$ and $\mathbf{d}_Z$. The advantage of this joint modeling is that we are now imaging all multiples event that return to the ocean bottom receivers going upward or downward.

For our complete modeling operator, $\mathbf{L}_{\uparrow}$ and $\mathbf{L}_{\downarrow}$, there is an alternate way to interpret the inverson problem we have set-up from above. Consider an equivalent fitting goal below,

$$0 \approx \mathbf{S}_{pz}^{-1}\mathbf{S}_{ou}\mathbf{A}\mathbf{m} - \begin{bmatrix} \mathbf{d}_{p} \\ \mathbf{d}_{z} \end{bmatrix}.$$

The above fitting goal converts our model into over-under data. Afterward, over-under data are separated into up-going and down-going data. Finally, the up-going and down-going data are converted into PZ data using $\mathbf{S}_{pz}^{-1}$. Therefore, this inversion scheme can be interpreted as fitting both P and Z data using only acoustic equation.

SUMMARY

We have discussed the theory of joint imaging of multiples and primaries using up- and down-going ocean bottom data. To a first order accuracy, we can use mirror imaging to define $\mathbf{L}_\uparrow$ and $\mathbf{L}_\downarrow$. A complete modeling scheme first models over-under data and then decompose them into up- and down-going data. This scheme effective uses water reveberations and multiples as signal instead of noise. It also reduces crosstalk leakage between up- and down-goign signals and hence reduces incorrectly placed reflectors. The testing of this thoery would be the focus of my research for the next quarter.

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