Joint least-squares inversion of simultaneous source time-lapse seismic data sets

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ABSTRACT

We present a joint inversion method, based on iterative least-squares migration, for imaging simultaneous source time-lapse seismic data sets. Non-repeatable shot and receiver positions introduce undesirable artifacts into time-lapse seismic images. We conjecture that more artifacts will result from *relative shot-timing non-repeatability* when data sets are acquired with several simultaneously shooting sources. These artifacts can be attenuated by joint inversion of such data sets without need for initial separation. Preconditioning with non-stationary dip filters and with temporal smoothness constraints ensures stability and geologically consistent time-lapse images. Results from a modified Marmousi 2D model show that the proposed method yields reliable time-lapse images.

INTRODUCTION

Time-lapse (4D) seismic is an established technology for monitoring hydrocarbon reservoirs. It is central to most field development and management plans, and many successful applications have been published (Whitcombe et al., 2004; Zou et al., 2006). However, in many time-lapse seismic applications, inaccuracies in replication of acquisition geometries for different surveys (*non-repeatability*) is a recurring problem. Although modern acquisition techniques can improve repeatability of shot-receiver positions, field conditions usually prevent perfect repeatability.

Recently, several authors have suggested acquisition with multiple simultaneously shooting seismic sources. Although, not a new technology (Womack et al., 1990; Beasley et al., 1998), modern acquisition and imaging techniques now make simultaneous source (or blended) acquisition both appealing and practical. Some advantages of this acquisition method include:

- Improved shot-sampling: *reduces shot-interpolation requirements in conventional narrow-azimuth data.*
- Lower acquisition cost: *enables acquisition of several azimuths in 3D wide-azimuth data sets at lower cost.*
- Longer offsets: *enables better imaging and improved AVO information.*
- Shorter acquisition time-window: *makes acquisition practical where operational, climatic, political or other uncontrollable factors could have prevented it.*

Different processing schemes have been proposed for simultaneous source data sets. Most of these schemes rely on separation of the data sets into different shot components before standard processing (Hampson et al., 2008; Spitz et al., 2008). Processing schemes that require no separation have also been suggested (Berkhout et al., 2008; Tang and Biondi, 2009). However, there has been little discussion on the implications of this acquisition technique for time-lapse seismic.

We introduce the term *relative shot-timing non-repeatability* to describe a source of artifacts for simultaneous source time-lapse data sets. Because current simultaneous source acquisition designs generally rely on randomized shot-timings, it will be difficult to accurately reproduce the relative shot-receiver positions and at the same time maintain the relative shot-timing for different surveys. Shot-receiver non-repeatability, together with the predicted relative shot-timing non-repeatability, will lead to strong degradation of time-lapse seismic images. Because of the complexity introduced by non-repeatability of both shot-receiver positions and relative shot-timing, *conventional* cross-equalization methods for time-lapse seismic data sets will fail. Therefore, we explore least-squares inversion methods of such data sets.

Iterative linear least-squares migration/inversion can improve structural and amplitude information in seismic images (Nemeth et al., 1999; Kühl and Sacchi, 2003; Plessix and Mulder, 2004; Clapp, 2005). An extension of model-space least-squares inversion (Valenciano, 2008) to time-lapse imaging has been shown to improve time-lapse seismic images (Ayeni and Biondi, 2008). In this paper, we propose a data-space joint inversion method for imaging simultaneous source time-lapse seismic data sets. The proposed method combines the cost-saving advantages of both simultaneous source acquisition and phase encoded migration (Romero et al., 2000). We demonstrate that preconditioning with non-stationary dip filters and temporal smoothness further improves the time-lapse seismic images.

We assume a known, slowly changing background baseline velocity. Therefore, we propose baseline data acquisition with separate or few simultaneous sources and monitor data acquisition with several simultaneous sources. We assume careful processing of the baseline data such that the baseline image can be used for dip estimation or interpretation. Furthermore, we assume that the shot-receiver positions and relative shot-timing are known for all surveys. Integration of background velocity and geomechanical changes into the joint inversion formulation is ongoing and will be discussed elsewhere.

In this paper, we first discuss Born modeling of phase-encoded data as an approximation of simultaneous source acquisition. Then, using a phase-encoded modeling/migration formulation, we discuss joint linear least-squares inversion of multiple simultaneous source seismic data sets. We also summarize a spatio-temporal preconditioning scheme based on spatial non-stationary dip-filters and temporal leaky integration. Finally, using two synthetic examples (a simple horizontal-layer model and a modified version of the 2D Marmousi model (Versteeg, 1994)), we show that solving the preconditioned joint inversion problem yields optimal time-lapse seismic images.

LINEAR PHASE-ENCODED BORN MODELING

Within limits of the Born approximation of the acoustic wave equation, the seismic data $\mathbf{d}$ recorded by a receiver at $\mathbf{x_r}$ due to a shot at $\mathbf{x_s}$ is given by

$$d(\mathbf{x_s}, \mathbf{x_r}, \omega) = \omega^2 \sum_{\mathbf{x}} f_s(\omega) G(\mathbf{x_s}, \mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x_r}, \omega) m(\mathbf{x}), \tag{1}$$

where ω is frequency, $\mathbf{m}(\mathbf{x})$ is the *reflectivity* at image points $\mathbf{x}$, $f_s(\omega)$ is the source waveform, and $G(\mathbf{x_s}, \mathbf{x}, \omega)$ and $G(\mathbf{x}, \mathbf{x_r}, \omega)$ are respectively the Green's functions from $\mathbf{x_s}$ to $\mathbf{x}$ and from $\mathbf{x}$ to $\mathbf{x_r}$.

By considering randomized simultaneous source data as a special case of linear phase-encoded shot gathers, equation 1 is modified to include a concatenation of phase-shifted shots, $s = q$ to $s = p$:

$$d(\mathbf{x_{s_{pq}}}, \mathbf{x_r}, \omega) = \sum_{s=p}^{q} a(\gamma_s) \omega^2 \sum_{\mathbf{x}} f_s(\omega) G(\mathbf{x}_s, \mathbf{x}, \omega) G(\mathbf{x}, \mathbf{x_r}, \omega) m(\mathbf{x}), \tag{2}$$

where

$$a(\gamma_s) = e^{i\gamma_s} = e^{i\omega t_s}, \tag{3}$$

and γ_s, the linear time-delay function, depends on the delay time t_s at shot s.

Relative shot-timing non-repeatability arises due to the uncertainty (Folland and Sitaram, 1997) associated with correct positioning of shots and receivers while maintaining the correct time delays t_s between shots. This is particularly true for the blended acquisition geometry (Berkhout, 2008), where several (20 or more) shots are encoded into a single record.

LINEAR LEAST-SQUARES MIGRATION/INVERSION

We re-write the linear modeling operation in equation 1 as a convolution of an operator $\mathbf{L}$ with the earth reflectivity $\mathbf{m}$:

$$\mathbf{d} = \mathbf{Lm}, \tag{4}$$

and the encoding (or blending) operation in equation 2 as

$$\widetilde{\mathbf{d}} = \mathbf{BLm} = \widetilde{\mathbf{L}}\mathbf{m}, \tag{5}$$

where $\widetilde{\mathbf{d}}$ is data, $\mathbf{B}$ is the encoding (or blending) operator, and $\widetilde{\mathbf{L}}$ is the combined Born modeling and encoding operator .

Given two surveys (baseline and monitor), acquired over an evolving earth model at times $\mathbf{t = 0}$ and $\mathbf{t = 1}$ respectively, we can write

$$\begin{aligned} \widetilde{\mathbf{d}}_0 &= \widetilde{\mathbf{L}}_0 \mathbf{m}_0, \\ \widetilde{\mathbf{d}}_1 &= \widetilde{\mathbf{L}}_1 \mathbf{m}_1, \end{aligned} \tag{6}$$

where $\mathbf{m}_0$ and $\mathbf{m}_1$ are the baseline and monitor reflectivities, and $\widetilde{\mathbf{d}}_0$ and $\widetilde{\mathbf{d}}_1$ are the encoded seismic data sets. Note that the modeling operators $\widetilde{\mathbf{L}}_0$ and $\widetilde{\mathbf{L}}_1$ in equation 6 can define both different acquisition geometries and relative shot-timings.

By applying the adjoint operators to the data sets, we obtain the migrated images:

$$\begin{aligned} \acute{\mathbf{m}}_0 &= \widetilde{\mathbf{L}}_0^* \widetilde{\mathbf{d}}_0, \\ \acute{\mathbf{m}}_1 &= \widetilde{\mathbf{L}}_1^* \widetilde{\mathbf{d}}_1, \end{aligned} \tag{7}$$

where $\acute{\mathbf{m}}_0$ and $\acute{\mathbf{m}}_1$ are the migrated baseline and monitor images respectively, and the symbol * denotes the conjugate transpose of the modeling operators. The *raw* time-lapse seismic image $\boldsymbol{\Delta}\tilde{\mathbf{m}}$ is the difference between the migrated images:

$$\Delta\acute{\mathbf{m}} = \acute{\mathbf{m}}_1 - \acute{\mathbf{m}}_0. \tag{8}$$

Because of differences in relative shot-timings, cross-term artifacts (Romero et al., 2000; Tang and Biondi, 2009) will be different for each migrated data set. Conventional equalization methods (Rickett and Lumley, 2001; Calvert, 2005) will be inadequate to remove these artifacts.

The quadratic cost functions for equation 6 are

$$\begin{aligned} S(\mathbf{m_0}) &= \|\widetilde{\mathbf{L}}_0 \mathbf{m}_0 - \widetilde{\mathbf{d}}_0\|^2, \\ S(\mathbf{m_1}) &= \|\widetilde{\mathbf{L}}_1 \mathbf{m}_1 - \widetilde{\mathbf{d}}_1\|^2, \end{aligned} \tag{9}$$

which when minimized gives the inverted baseline $\hat{\mathbf{m}}_0$ and monitor $\hat{\mathbf{m}}_1$ images:

$$\begin{aligned} \hat{\mathbf{m}}_0 &= (\widetilde{\mathbf{L}}_0' \widetilde{\mathbf{L}}_0)^{-1} \widetilde{\mathbf{L}}_0^* \mathbf{d}_0, \\ \hat{\mathbf{m}}_1 &= (\widetilde{\mathbf{L}}_1' \widetilde{\mathbf{L}}_1)^{-1} \widetilde{\mathbf{L}}_1^* \mathbf{d}_1, \end{aligned} \tag{10}$$

This is the so-called least-squares migration method.

Joint-inversion

Instead of solving the two equations in equation 6 independently, we combine then to form a joint system of equations

$$\begin{bmatrix} \widetilde{\mathbf{L}}_0 & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{L}}_1 \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1 \end{bmatrix} = \begin{bmatrix} \widetilde{\mathbf{d}}_0 \\ \widetilde{\mathbf{d}}_1 \end{bmatrix}, \tag{11}$$

for which a solution is obtained by minimizing the objective function

$$S(\mathbf{m_0}, \mathbf{m_1}) = \left\| \begin{bmatrix} \widetilde{\mathbf{L}}_0 & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{L}}_1 \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1 \end{bmatrix} - \begin{bmatrix} \widetilde{\mathbf{d}}_0 \\ \widetilde{\mathbf{d}}_1 \end{bmatrix} \right\|^2. \tag{12}$$

Neglecting stability issues, the computational cost of minimizing equations 12 is approximately twice the cost of minimizing each objective function in 9. Because several shots are encoded and directly imaged, the computational cost of this approach is considerably reduced relative to non-encoded data sets. Equivalent formulations for conventional time-lapse seismic imaging have been shown by previous authors (Ajo-Franklin et al., 2005; Ayeni and Biondi, 2008).

Regularization and Preconditioning

Seismic inversion is an ill-posed problem. Therefore, regularization operators are required to stabilize the inversion and to prevent divergence to unrealistic solutions. A regularized least squares solution $\hat{\mathbf{m}}$ is obtained by minimizing a modified objective function:

$$S(\mathbf{m}) = \|\widetilde{\mathbf{L}}\mathbf{m} - \widetilde{\mathbf{d}}\|^2 + \epsilon^2\|\mathbf{R}\mathbf{m}\|^2, \tag{13}$$

where ϵ is a damping factor that determines the strength of the regularization operator $\mathbf{R}$. In this paper, we consider a fixed damping factor computed as a function of the data as follows:

$$\epsilon = \frac{max|\widetilde{\mathbf{d}}|}{100}. \tag{14}$$

Relevant examples of regularization criteria for geophysical inverse problems include model smoothness (Tikhonov and Arsenin, 1977), temporal smoothness (Ajo-Franklin et al., 2005), and smooth, horizontal angle gathers (Clapp, 2005).

Minimizing equation 13 is equivalent to solving the problem

$$\begin{bmatrix} \widetilde{\mathbf{L}} \\ \epsilon\mathbf{R} \end{bmatrix} \mathbf{m} = \begin{bmatrix} \widetilde{\mathbf{d}} \\ \mathbf{0} \end{bmatrix}. \tag{15}$$

Fast iterative convergence can be obtained by *preconditioning the regularization* (Claerbout and Fomel, 2008). This is equivalent to making the variable substitution $\mathbf{m} = \mathbf{R}^{-1}\mathbf{p} = \mathbf{A}\mathbf{p}$, so that equation 15 becomes

$$\begin{bmatrix} \widetilde{\mathbf{L}}\mathbf{A} \\ \epsilon\mathbf{I} \end{bmatrix} \mathbf{p} = \begin{bmatrix} \widetilde{\mathbf{d}} \\ \mathbf{0} \end{bmatrix}, \tag{16}$$

where $\mathbf{A}$ is the preconditioner, and $\mathbf{p}$ is the preconditioned variable. By selecting an invertible regularization operator $\mathbf{R} = \mathbf{A}^{-1}$, we can solve the preconditioned problem (equation 16) at fewer iterations than the regularized problem (equation 15).

For the current problem, we require two regularization operators (spatial and temporal). The spatial regularization operator is a system of non-stationary dip filters applied on the helix (Claerbout and Fomel, 2008). These symmetric filters, built from *puck* filters (Claerbout and Fomel, 2008; Hale, 2007), are factored into causal filters using the Wilson-Burg factorization (Fomel et al., 2003). The preconditioner, implemented as a helical polynomial division, uses dips estimated from plane-wave destruction (Fomel, 2002) to determine the appropriate filters for each model point. The temporal preconditioner is a two-way leaky integration operator.

The preconditioned joint inverse problem is

$$\begin{bmatrix} \text{Ł}\mathring{\mathbf{A}} \\ \epsilon\mathbf{I} \end{bmatrix} \mathbf{p} = \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix}, \tag{17}$$

where

$$\text{Ł} = \begin{bmatrix} \widetilde{\mathbf{L}}_0 & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{L}}_1 \end{bmatrix}, \tag{18}$$

$$\mathring{\mathbf{A}} = \mathbf{A}\mathbf{T}, \tag{19}$$

with

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_1 \end{bmatrix}, \tag{20}$$

and

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \Lambda \\ \Lambda & \mathbf{I} \end{bmatrix}. \tag{21}$$

The operators $\mathbf{A}_0$ and $\mathbf{A}_1$ are preconditioners for the baseline and monitor images, respectively, while $\mathbf{I}$ is identity and Λ is a diagonal operator containing the leak rates λ. Equation 17 is directly extendable to an arbitrary number of surveys. The proposed method, joint preconditioned least squares inversion (J-PLSI) refers to the definition in equation 17.

Relaxed covariance-based preconditioning

We specialized the spatial and temporal preconditioners such that the dip-discrimination (or range) of the filters decreases as a function of iteration, while the temporal integration leak rate increases as a function of iteration. This preconditioning approach (which should be applicable to other inversion problems) ensures that close to the solution, the data fitting goal is given more importance relative to the regularization goal.

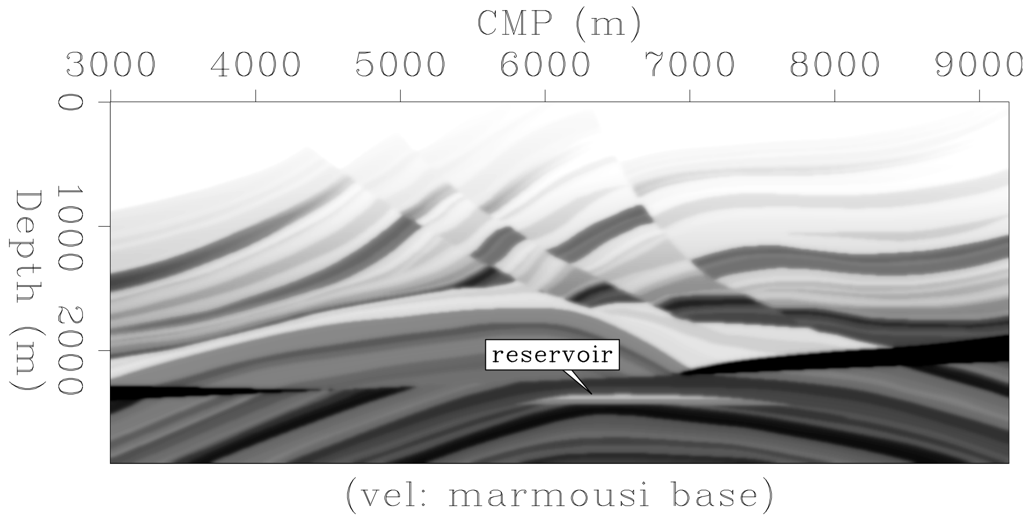
In addition, because non-stationary deconvolution by polynomial division can become unstable at sharp boundaries, the filter range at any image point is a function of dip contrast-dependent covariance. Details of this preconditioning approach is outside the scope of this paper and will be discussed elsewhere.

NUMERICAL EXAMPLE

We demonstrate J-PLSI using a modified section of 2D Marmousi model (Figure 1). The objective is to image the seismic amplitude changes at the reservoir using simultaneous data sets. We assume seismic amplitude changes only within the reservoir and neglect geomechanical changes.

The data consist of two sets of 29 encoded shot records over the 8x8 m grid model. Both data sets used were fully encoded because we use a known background baseline velocity model. The random encoding function, with a maximum delay of 1 s, is different for each data set. Shot positions vary randomly between surveys with a maximum displacement of 32 m, whereas the receiver array is the same for both surveys and fixed for all shots. Dips were computed from a single source migrated baseline image (not shown).

The downgoing baseline and monitor source wavefields at time 0.52 s are shown Figure 3. The migrated single and simultaneous source monitor images are shown in Figures 4(a) and 4(b), respectively. The inverted simultaneous source monitor image obtained from J-PLSI is shown in Figure 4(c). Figure 5 shows the migrated and inverted time-lapse images.



(a)

Figure 1: 2D Marmousi velocity model.

DISCUSSION AND CONCLUSIONS

We presented a joint least-squares wave-equation inversion method (J-PLSI) for imaging simultaneous source (or blended) time-lapse seismic data sets. J-PLSI directly inverts the simultaneous source data sets without need for prior separation, combining the cost savings advantages of simultaneous source acquisition and phase-encoded migration.

Direct migration of simultaneous source data generates strong cross-term artifacts (Figure 4(b)) relative to conventional single source data (Figure 4(a)). J-PLSI attenuates these artifacts giving images with better resolution and more balanced amplitudes than migration (Figure 4). This translates to relatively high-quality, high-resolution time-lapse images (Figure 5(c)). Because J-PLSI provides a way of obtaining good-quality inverted time-lapse images at a fraction of least-squares migration of single source data sets, it can be applied to conventional single source time-lapse data sets, specifically encoded for computational cost savings.

Because simultaneous source acquisition reduces the overall data acquisition and the data can be efficiently processed, we recommend shorter survey intervals. By acquiring time-lapse seismic data sets in this manner and processing them using the J-PLSI method, we can get closer to the goal of *continuous* seismic reservoir monitoring.

ACKNOWLEDGEMENTS

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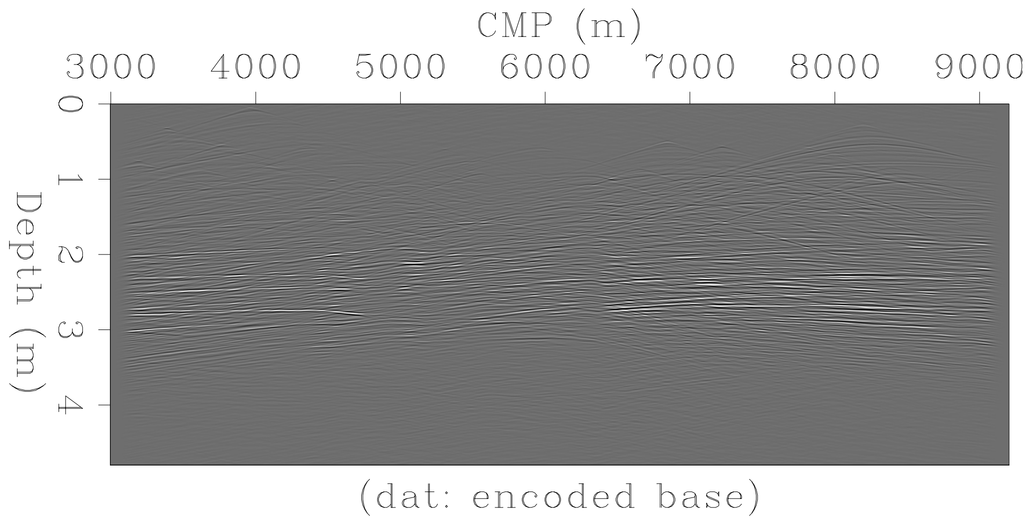
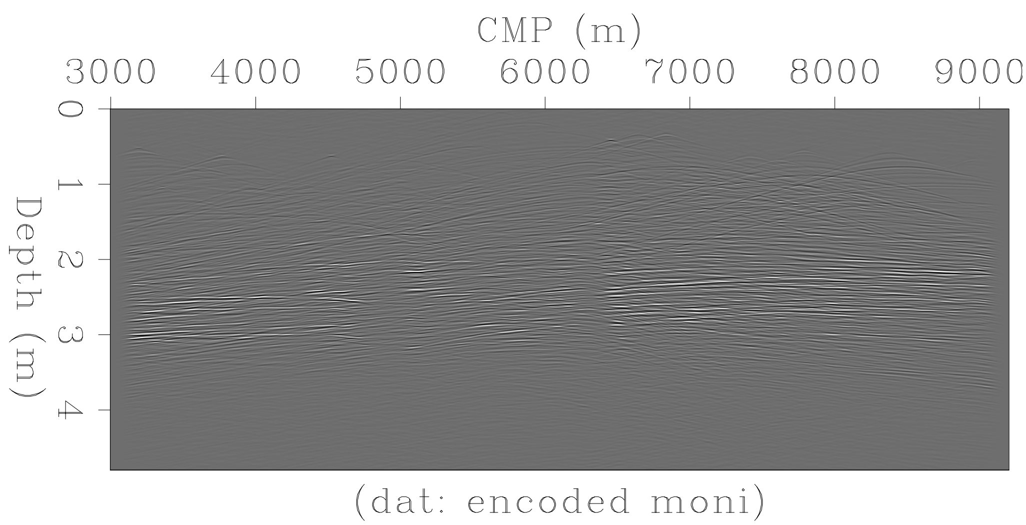
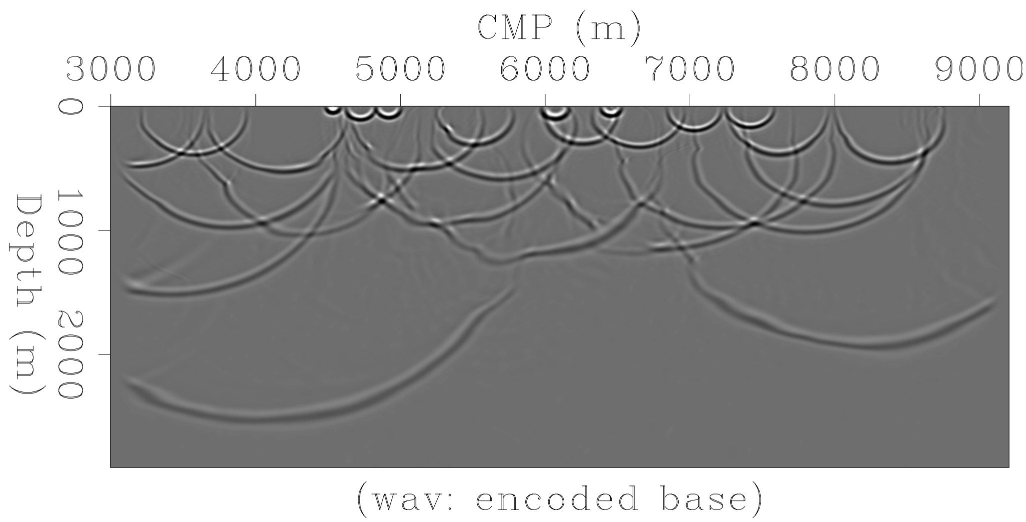


(a)

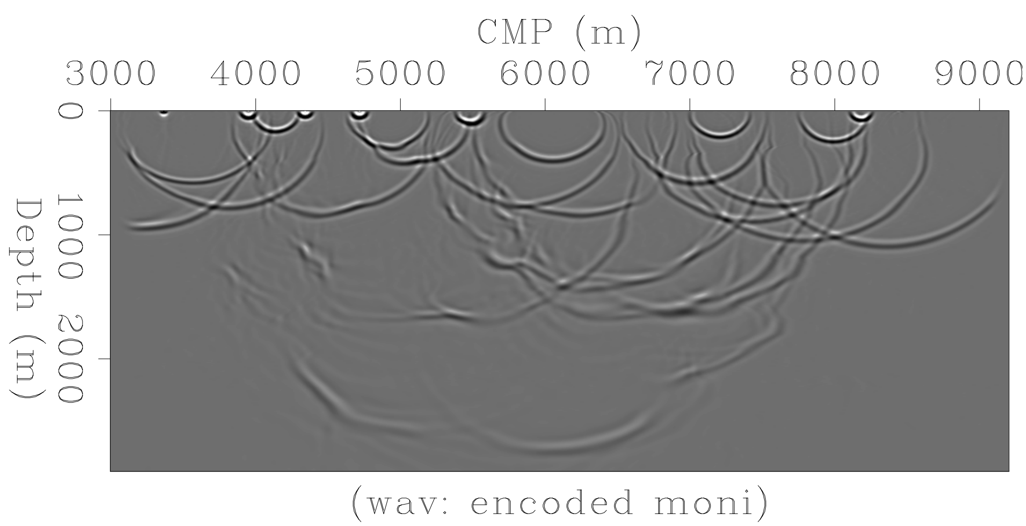


(b)

Figure 2: Encoded (a) baseline and (b) monitor data sets.



(a)



(b)

Figure 3: Downgoing source wavefields at 0.52 s for (a) the baseline and (b) monitor data sets. Note the difference in shot-timing.

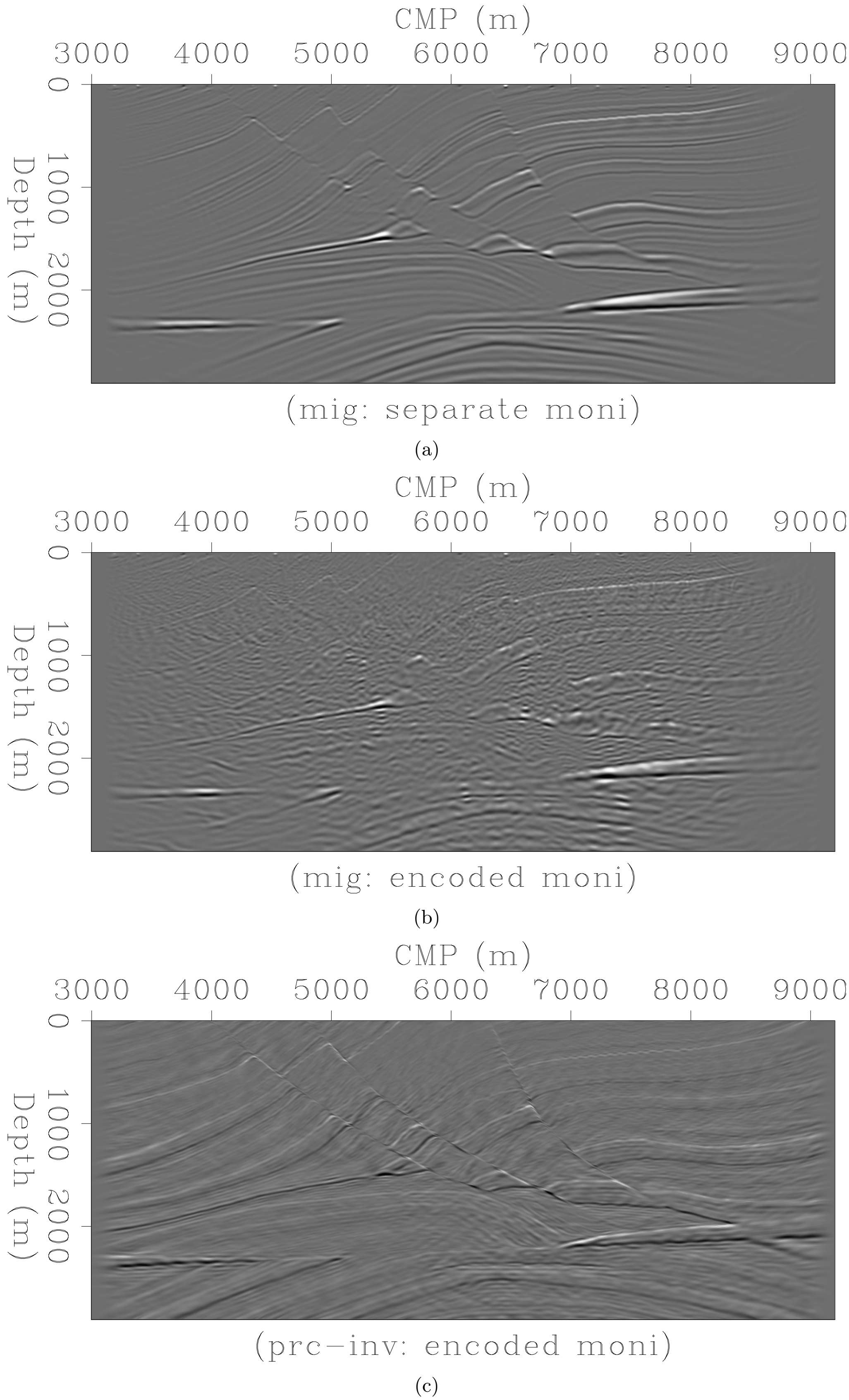
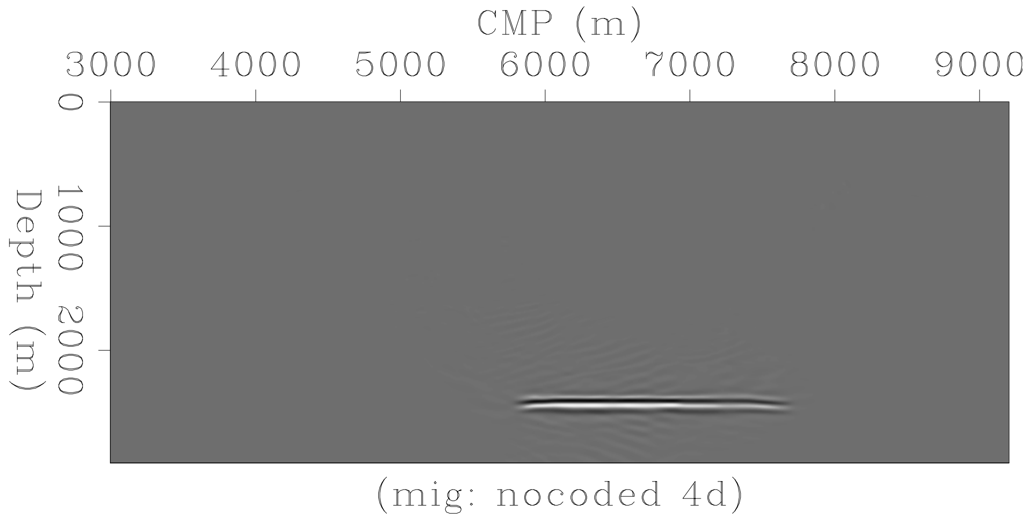
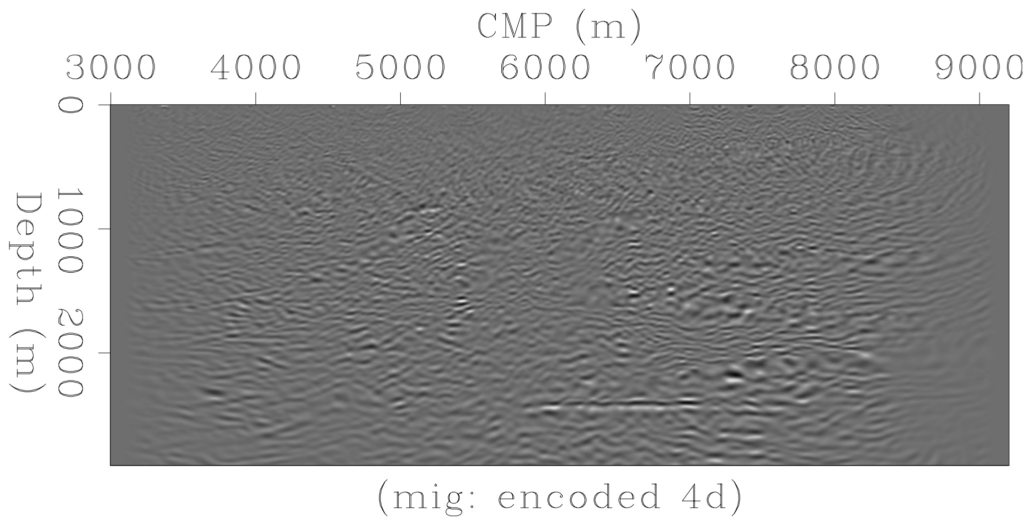


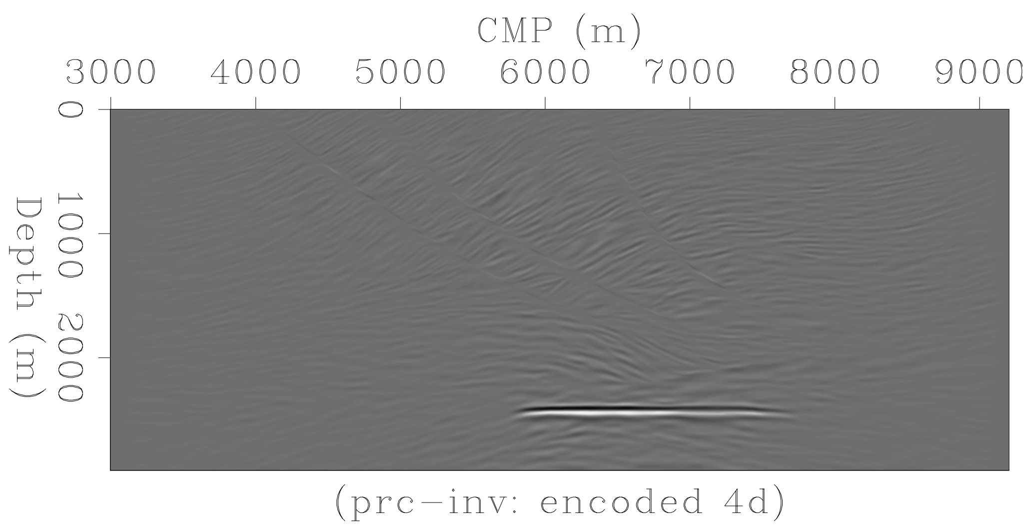
Figure 4: (a) Single source migrated monitor image. Simultaneous source (b) migrated and (c) inverted monitor images.



(a)



(b)



(c)

Figure 5: (a) Single source migrated time-lapse image. Simultaneous source (b) migrated and (c) inverted time-lapse images. Note the cross-term artifacts in (b).

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