Migration velocity analysis with cross-gradient constraint

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ABSTRACT

Velocity analysis plays a fundamental to seismic imaging. A variety of techniques using pre-stack seismic data exist for migration-velocity analysis, including reflection tomographic inversion methods. However, when the wavefield propagation is complex, reflection tomography may fail to converge to a geologically reasonable velocity estimation. Non-seismic geological properties can be integrated in the reflection-seismic tomography problem to achieve better velocity estimation. Here, I propose to use cross-gradients function as a similarity measure to constrain the tomography problem and enforce a general geometrical structure for the seismic velocity estimates.

INTRODUCTION

Precise estimation of subsurface velocities is a requirement for high quality seismic imaging. Without an accurate velocity, seismic reflectors are misplaced, the image is unfocused, and seismic images can easily mislead earth scientists (Claerbout, 1999; Clapp, 2001). Defining a reliable velocity model for seismic imaging is a difficult task, especially when sharp lateral and vertical velocity variations are present. Velocity estimation becomes even more challenging when seismic data are noisy. Therefore it is harder to extract velocity information (Clapp, 2001).

In areas with complex structures, and significant lateral velocity variations, velocity analysis is a challenging task. In these areas, reflection-tomography methods are often more effective than conventional velocity-estimation methods based on measurements of stacking velocities (Biondi, 1990; Clapp, 2001). Unfortunately, the reflection-tomography problem is ill-posed and under-determined. Furthermore, it may not converge to a realistic velocity model without *a priori* information, e.g., regularization constraints and other types of geophysical properties in addition to seismic data (Clapp, 2001).

The main challenge in integrating different geophysical data sets is the absence of an analytical relationship between properties exploited by different geophysical surveys. Most often, probabilistic relations among these geophysical properties are used to address this shortcoming. A different approach would be to use gradients field as an objective measure of geometrical similarity. This is true since the variations of geophysical properties can be described by a magnitude and a direction (Gallardo and Meju, 2004, 2007).

Here, I use the cross-gradients function introduced by Gallardo and Meju (2004, 2007) to integrate the resistivity field measured by electromagnetic surveys into the reflection-seismic tomography problem. The integration of this additional piece of information may lead to velocity estimates that are geologically reasonable.

My paper is organized as follows. First, I present a short overview of reflection tomography, followed by introducing the cross-gradients function as a structural similarity measure. Next, I show how we can use this measure as an extra constraint for the reflection-tomography problem, with the goal of obtaining at a more accurate velocity estimation. Last, I discuss the future work based on these ideas.

REFLECTION SEISMIC-TOMOGRAPHY

By definition, tomography is an inverse problem, where a field is reconstructed from its known linear path integrals, i.e., projections (Clayton, 1984; Iyer and Hirahara, 1993). We can think of tomography as a matrix operator $\mathbf{T}$, which integrates slowness along the raypath. The tomography problem can then be stated as

$$\mathbf{t} = \mathbf{T}\ \mathbf{s}, \tag{1}$$

where $\mathbf{t}$ and $\mathbf{s}$ are travel time and slowness vector, respectively (Clapp, 2001).

The raypaths are dependent on the velocity field. Consequently, the tomography operator is a function of the model parameters. This dependency causes the tomography problem to be nonlinear, which makes it difficult to solve. A common technique to overcome this non-linearity is to iteratively linearize the operator around *a prior* estimation of the slowness field $\mathbf{s}_0$ (Biondi, 1990; Etgen, 1990; Clapp, 2001). The linearization of the tomography problem by using a Taylor expansion is then given by

$$\mathbf{t} \approx \mathbf{T}\mathbf{s}_0 + \frac{\partial \mathbf{T}}{\partial \mathbf{s}}\Big|_{\mathbf{s}=\mathbf{s}_0} \Delta\mathbf{s}. \tag{2}$$

Here, $\Delta\mathbf{s} = \mathbf{s} - \mathbf{s}_0$ represents the update in the slowness field with respect to the *a priori* slowness estimation, $\mathbf{s}_0$. Equation 2 can be simplified as

$$\Delta\mathbf{t} = \mathbf{t} - \mathbf{T}\mathbf{s}_0 \approx \mathbf{T}_L \Delta\mathbf{s}, \tag{3}$$

where $\mathbf{T}_L = \frac{\partial \mathbf{T}}{\partial \mathbf{s}}\Big|_{\mathbf{s}=\mathbf{s}_0}$ is a linear approximation of $\mathbf{T}$. A second, but not least, difficulty arises because the location of reflection points are unknown and a function of the velocity field (van Trier, 1990; Stork, 1992).

Clapp (2001) attempts to resolve some of the non-linearity issues with the introduction of a new tomography operator in the tau domain and use of steering filters. In addition to geologic models other types of geophysical data can also be extremely important. In the following section, I show how the cross-gradients function can be used to add constraints to the seismic tomography problem.

THE CROSS-GRADIENTS FUNCTION AS A CONSTRAINT FOR THE TOMOGRAPHY PROBLEM

As mentioned in the previous section, integrating different types of geophysical data can lead to improvements in reflection-seismic tomography results due to reduction of model uncertainty. For this purpose, I propose the cross-gradients function, which can also be considered as a metric to measure the structural similarity between two fields. Following Gallardo and Meju (2004), we can define the cross-gradients function for the tomography problem as

$$\mathbf{g} = \nabla \mathbf{r} \times \nabla \mathbf{s}, \tag{4}$$

where $\mathbf{r}$ and $\mathbf{s}$ can represent any two model parameters, e.g., resistivity and slowness in our case, respectively. Zero values of the cross-gradients function correspond to points where spatial changes in both geophysical properties, i.e., $\nabla \mathbf{r}$ and $\nabla \mathbf{s}$, align. However, the function is also zero where the magnitude of spatial variations of either field is negligible, e.g., where either property is smooth. Note that the cross-gradients function is a non-linear function of $\mathbf{s}$ and $\mathbf{r}$ if both are unknowns.

Figure 1 shows a synthetic 2-D resistivity profile with two anomalies in a constant resistivity background. Ideally, we expect different types of geophysical measurements to produce a geometrically similar image of the subsurface. The cross-property relations between pairs of geophysical properties, e.g., seismic velocity and electrical resistivity of rocks (for more details refer to Hacikoylu et al., 2006; Carcione et al., 2007) also support this similarity. Figure 2 shows the corresponding 2-D velocity profile of the modeled subsurface region in Figure 1, which includes both fast and slow anomalies in comparison to the background velocity. The velocity profile is computed using the *Archie/time-average* cross-property relation (Carcione et al., 2007) with arbitrary parameter values. Note that, the structural similarity of Figures 1 and 2 suggest that the cross-gradients function should vanish almost everywhere.

In a 2-D problem, $\mathbf{g}$ simplifies to a scalar function at each point, given by

$$\mathbf{g} = \frac{\partial \mathbf{s}}{\partial x}\,\frac{\partial \mathbf{r}}{\partial z} - \frac{\partial \mathbf{s}}{\partial z}\,\frac{\partial \mathbf{r}}{\partial x}\,, \tag{5}$$

where the model parameters are given in $x - z$ plane. In order to compute the cross-gradients function, we can further simplify it by using first-order forward differences approximation of the first derivative operators. Figure 3 shows the estimated cross-gradients function. Note that it is approximately zero everywhere as expected. Negligible non-zero values are caused by errors in forward-difference estimation. This implies that geometrical changes, e.g., layer boundaries and other subsurface structures, should be sensed by measurement of both geophysical properties, i.e., seismic slowness and electrical resistivity. Therefore, the cross-gradients function can be used as a constraint for joint data inversion problems or to integrate *a priori* information from other fields into the seismic tomography problem.

If an accurate estimate of the electrical resistivity profile is provided, we can use the cross-gradients function as a constraint for the reflection-seismic tomography

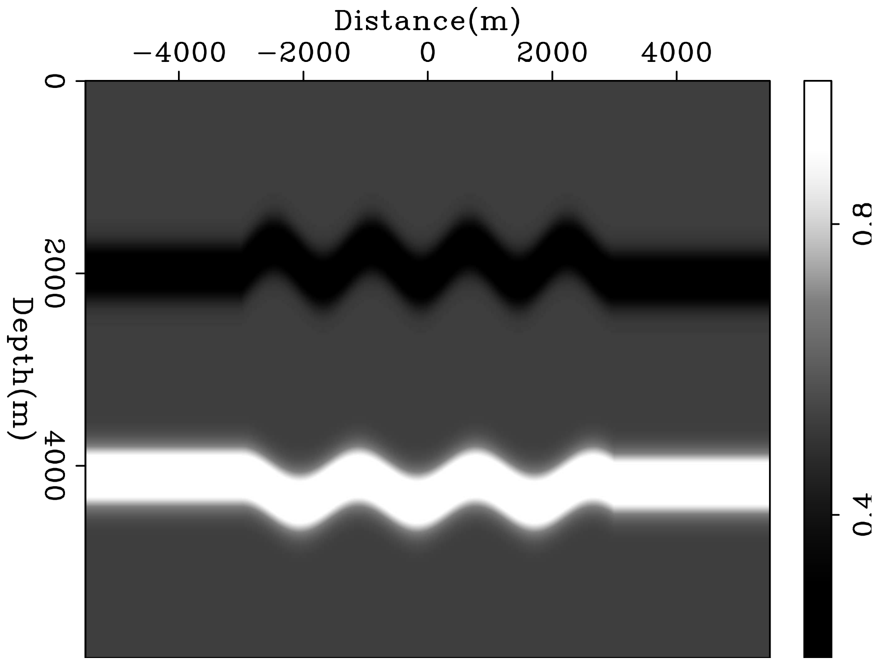

Figure 1: Synthetic resistivity ($Ohm.m$). [**ER**]

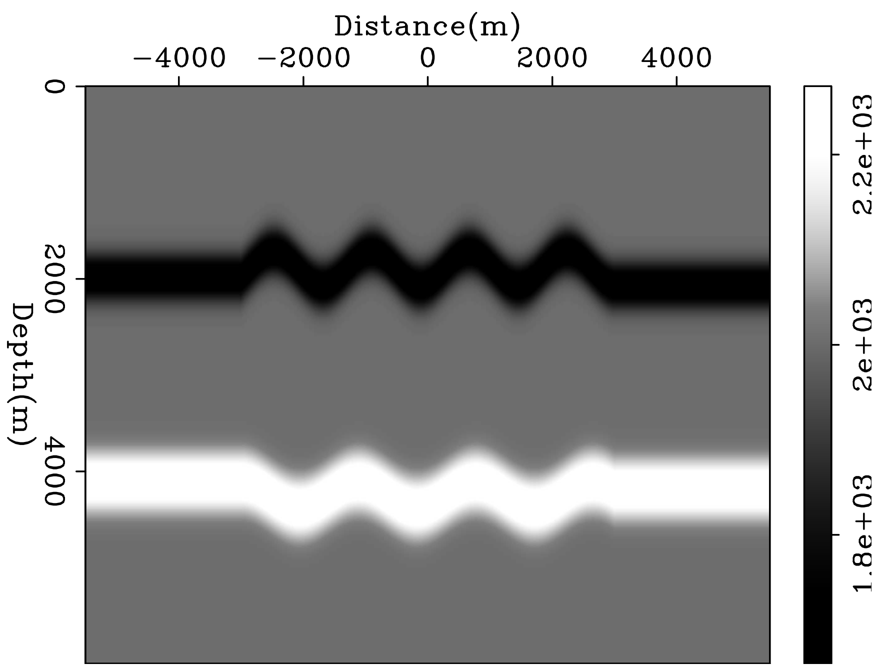

Figure 2: Synthetic velocity (m/s) associated with Figure 1. [**ER**]

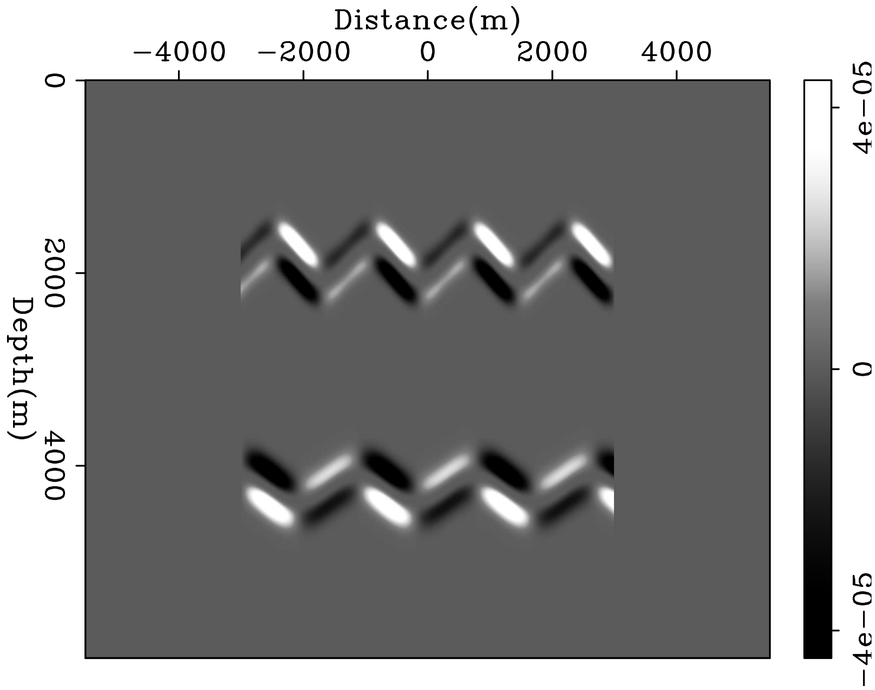

Figure 3: Cross-gradients function of the velocity and resistivity shown in Figures 1 and 2. [**ER**]

problem to improve the accuracy of the velocity estimations. In this case, we can write the cross-gradients function given in equation 5 as a linear operator $\mathbf{G}$ on the slowness field, $\mathbf{s}_0 + \Delta\mathbf{s}$. We can then extend the linearized tomography problem by employing $\mathbf{G}$ as an additional constraint. The objective function, $\mathcal{P}(\Delta\mathbf{s})$, of this extended problem becomes

$$\mathcal{P}(\Delta\mathbf{s}) = ||\Delta\mathbf{t} - \mathbf{T}_L\Delta\mathbf{s}||^2 + \epsilon_1^2 \; ||\mathbf{A}\Delta\mathbf{s}||^2 + \epsilon_2^2 \; ||\mathbf{G}(\mathbf{s}_0 + \Delta\mathbf{s})||^2, \tag{6}$$

where ϵ_1 and ϵ_2 are problem-specific weights, and $\mathbf{A}$ represents any regularization operator other than cross-gradients function such as smoothing operator.

The important advantage of using the cross-gradients function over using steering filters may not be very clear in this synthetic example. Steering filters are most effective for continuous anomalies with smooth boundaries. However, in the case of sharp boundaries, e.g., Gaussian anomalies or salt boundaries, the cross-gradients function is better able to handle the seismic tomography problem. As mentioned in previous section, we can also use the cross-gradients function as a constraint for joint inversion, where steering filters are not effective. This is true because steering filters assume *a priori* knowledge of the model parameters while the cross-gradients function use the collocated data field to build this information.

FUTURE WORK

The tomography problem stated in Equation 6 is based on the assumption that we have a reasonably accurate estimate of the collocated resistivity field. Given this assumption, I expect the similarity constraint to improve the estimation of slowness profile. I will first incorporate the similarity constraint into the reflection-seismic tomography problem for the synthetic model shown above. Then, I will extend the application of the idea to the tomography problem for 2-D sections of a field data set.

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