Seismic tomography with co-located soft data

Mohammad Maysami and Robert G. Clapp

ABSTRACT

There is a wide range of uncertainties present in seismic data. Limited subsurface illumination is also common, specially in areas with salt structures. These shortcomings are only a few of many different reasons that makes seismic tomography an under-determined problem with a large null space. We can use additional information to reduce the uncertainty and constrain this large null space. The additional information, also known as co-located soft (secondary) data, can be the result of integrating a non-seismic data from the same subsurface area. A measure of structural similarity between the two given data fields can create a link between the different types of data. We use cross-gradient functions to incorporate this structural information, given by secondary data, into the inverse problem as a constraint.

INTRODUCTION

Seismic data contain a wide range of uncertainties which directly affects the quality of seismic images. Previous studies have tried to extract more information from raw seismic data to reduce the uncertainty in the seismic-imaging problem (Yilmaz, 2001; Aki and Richards, 2002). Since velocity analysis plays a fundamental role in seismic imaging, uncertainties in velocities lead to significant inaccuracies in seismic images. Without an accurate velocity estimate, seismic reflectors are misplaced, the image is unfocused, and seismic images can easily mislead earth scientists (Claerbout, 1999; Clapp, 2001). Defining a reliable velocity model for seismic imaging is a difficult task, especially when sharp lateral and vertical velocity variations are present. Moreover, velocity estimation becomes even more challenging when seismic data are noisy (Clapp, 2001).

In areas with significant lateral velocity variations, reflection tomography methods, where traveltimes are mapped to slowness, are often more effective than conventional velocity-estimation methods based on measurements of stacking velocities (Biondi, 1990; Clapp, 2001). However, reflection tomography may also fail to converge to a geologically reasonable velocity estimation when the wavefield propagation is complex.

Unfortunately, the reflection tomography problem is ill-posed and under-determined. Furthermore, it may not converge to a realistic velocity model without *a priori* information, e.g., regularization constraints and other types of geophysical properties, in addition to seismic data (Clapp, 2001). Better velocity estimation can be achieved by integrating co-located soft data, such as non-seismic geological data, in the reflection tomography problem.

Lack of an analytical relationship between different measured geological properties limits our ability to use co-located soft data. Besides the conventional probabilistic relations, similarity-measurement tools can be used to enforce the structural information contained in soft data into seismic velocity estimates. Based on these tools, differences in two images are classified as structural differences and non-structural differences. Since gradient fields are a good choice for geometrical (structural) comparisons, the cross-gradient function is one useful similarity-measurement tool. This is true because the variations of geophysical properties can be described by a magnitude and a direction (Gallardo and Meju, 2004, 2007).

Here we use the cross-gradient function to integrate a given set of soft data—the resistivity field measured by magnetotelluric (MT) sounding in our case—into the reflection tomography problem. This integration requires consideration of differences in frequency in seismic and resistivity data. In the following sections we study the behavior of cross-gradient functions in different cases and then give an overview of how an understanding of these differences can be used to improve velocity estimates given by seismic tomography.

THE CROSS-GRADIENT FUNCTION: A STRUCTURAL SIMILARITY MEASURE

Integration of soft data into the seismic tomography problem can reduce model uncertainty and result in a better velocity estimation, especially in areas with complex structure. Different geophysical methods probs the same structures in the Earth's subsurface. Among the techniques for integrating different types of geological data, structural similarity-measurement tools may be a good choice for our tomography problem. The cross-gradient function is one tool that measures the structural similarity between any two fields. Following Gallardo and Meju (2004), we can define the cross-gradient function for the tomography problem as

$$\mathbf{g} = \nabla \mathbf{r} \times \nabla \mathbf{s}, \tag{1}$$

where $\mathbf{r}$ and $\mathbf{s}$ can represent any two model parameters. In our case, they represent resistivity and slowness, respectively. Zero values of the cross-gradient function correspond to points where spatial changes in both geophysical properties, i.e., $\nabla \mathbf{r}$ and $\nabla \mathbf{s}$, align. However, the function is also zero where the magnitude of spatial variations of either field is negligible, e.g., where either property is smooth. Note that the cross-gradient function is a non-linear function of $\mathbf{r}$ and $\mathbf{s}$ if both are unknowns. In a 2-D problem, $\mathbf{g}$ simplifies to a scalar function at each point, given by

$$\mathbf{g} = \frac{\partial \mathbf{s}}{\partial x}\ \frac{\partial \mathbf{r}}{\partial z} - \frac{\partial \mathbf{s}}{\partial z}\ \frac{\partial \mathbf{r}}{\partial x}\ , \tag{2}$$

where the model parameters are given in the $x - z$ plane. To compute the cross-gradient function, we can further simplify it by using first-order forward-differences approximations of the first derivative operators.

Figures 1(a) and 1(b) show the smooth Marmousi synthetic 2-D velocity model (Versteeg and Grau, 1991) and its cross-gradient with itself, respectively. The cross-gradient of a field with itself is called the auto-gradient hereafter. Note that the auto-gradient of a field should be zero everywhere; however, since the figures are prepared with a first-order linear approximation of the cross-gradient function, it is not zero, especially in areas with sharp edges.

Although we expect different types of geophysical methods to result in similar structural maps, in practice each method maps the subsurface through different filters and frequency contents. Typical frequencies in magnetotelluric data are much lower than those of seismic data (Kaufman and Keller, 1981). This difference in the frequency content of two fields may affect how the cross-gradient represents the structural similarity of two fields. To investigate the effect of different spatial frequency content, we prepared Figure 1, in which the cross-gradient of the Marmousi velocity model and a smooth version of it is computed. In Figure 1(d), we have increased the smoothing factor. This increase is equivalent to a lower cut-off spatial frequency for a lowpass filter. Note that because of the relatively sharp edges in the original velocity model, the cross-gradients in Figures 1(c) and 1(d) seem to include some structure as well as higher amplitudes as compared with Figure 1(b). However, this synthetic example is an extreme case of complexity and sharp edges. As shown by the results for the Pillow velocity model in Figure 2, in simpler cases of subsurface structure, the cross-gradient with a smooth version of the velocity model leads to an acceptable similarity indicator. The amplitude may be improved by using a higher-order linear approximation of the cross-gradient computation. These figures in general may imply that the cross-gradient function can be used as a constraint for joint data inversion problems or to integrate *a priori* information from other fields into the seismic tomography problem.

REFLECTION TOMOGRAPHY

By definition, tomography is an inverse problem, in which a field is reconstructed from its known linear path integrals, i.e., projections (Clayton, 1984; Iyer and Hirahara, 1993). Tomography can be represented by a matrix operator $\mathbf{T}$, which integrates slowness along the raypath. The tomography problem can then be stated as

$$\mathbf{t} = \mathbf{T}\ \mathbf{s}, \tag{3}$$

where $\mathbf{t}$ and $\mathbf{s}$ are traveltime and slowness vector, respectively (Clapp, 2001). The tomography operator is a function of the model parameters, since the raypaths depend on the velocity field. Consequently, the tomography problem is non-linear. A common technique to overcome this non-linearity is to iteratively linearize the operator around

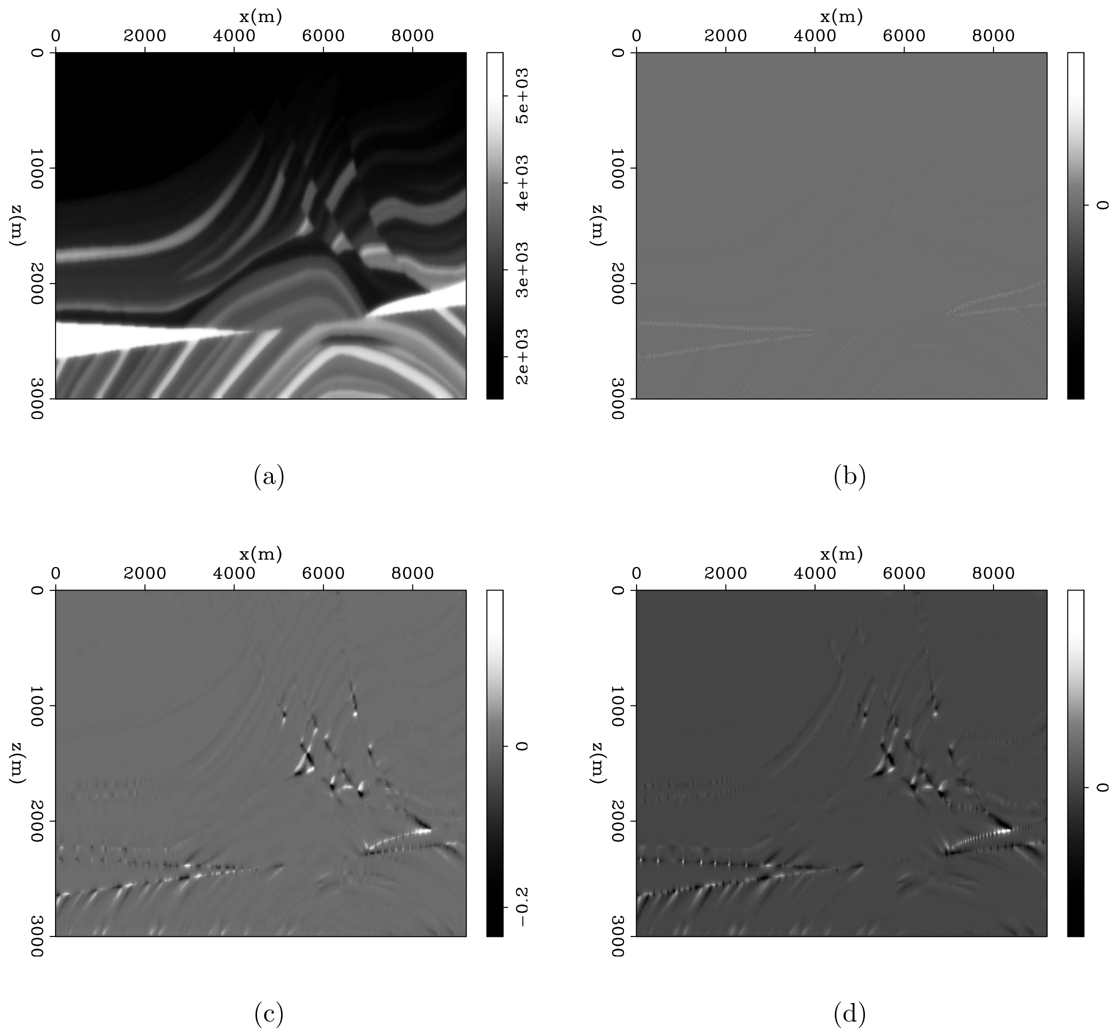


Figure 1: Frequency sensitivity of the cross-gradient function: **(a)** The Marmousi velocity model; **(b)** its auto-gradient. Cross-gradient values of the Marmousi velocity model and its **(c)** smooth and **(d)** very smooth copies. [**ER**]

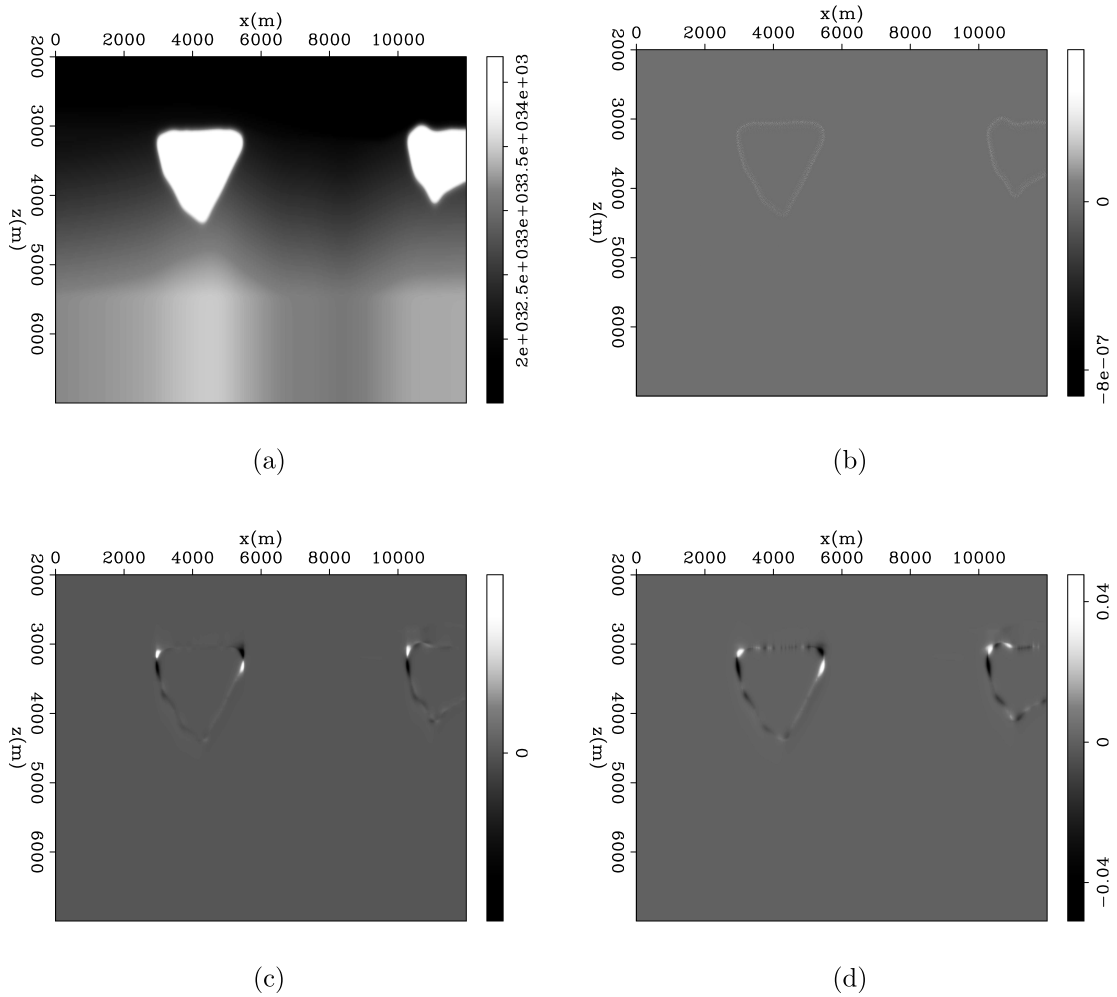


Figure 2: Frequency sensitivity of the cross-gradient function: **(a)** The Pillow velocity model; **(b)** its auto-gradient. Cross-gradient values of the Pillow velocity model and its **(c)** smooth and **(d)** very smooth copies. [**ER**]

an *a priori* estimation of the slowness field $\mathbf{s}_0$ (Biondi, 1990; Etgen, 1990; Clapp, 2001). The linearization of the tomography problem by using a Taylor expansion is given by

$$\mathbf{t} \approx \mathbf{T}\mathbf{s}_0 + \frac{\partial \mathbf{T}}{\partial \mathbf{s}}\Big|_{\mathbf{s}=\mathbf{s}_0} \Delta\mathbf{s}. \tag{4}$$

Here, $\Delta\mathbf{s} = \mathbf{s} - \mathbf{s}_0$ represents the update in the slowness field with respect to the *a priori* slowness estimation, $\mathbf{s}_0$. Equation 4 can be simplified as

$$\Delta\mathbf{t} = \mathbf{t} - \mathbf{T}\mathbf{s}_0 \approx \mathbf{T}_L \Delta\mathbf{s}, \tag{5}$$

where $\mathbf{T}_L = \frac{\partial \mathbf{T}}{\partial \mathbf{s}}\Big|_{\mathbf{s}=\mathbf{s}_0}$ is a linear approximation of $\mathbf{T}$. A second, but not lesser, difficulty arises because the locations of reflection points are unknown and are a function of the velocity field (van Trier, 1990; Stork, 1992).

Clapp (2001) attempts to resolve some of the difficulties caused by the non-linearity of the seismic tomography problem by introducing a new tomography operator in the tau domain and by using steering filters. In addition to geological models, other types of geophysical data can also be extremely important for yielding improved velocity estimates. In the following section, we show how the cross-gradient function can be used to add constraints to the seismic tomography problem in order to decrease the uncertainties in the estimated velocity model.

APPLICATION OF THE CROSS-GRADIENT FUNCTION IN SEISMIC TOMOGRAPHY

Figure 3 shows a velocity map and corresponding resistivity map of a synthetic 2-D model. That includes a water velocity of about 1.5 $\frac{km}{s}$ at the top and a semi-circular fault in the middle of the ocean bottom. There are also laterally smooth velocity anomalies in the model. The resistivity profile and velocity profile are connected using the *Archie/time-average* cross-property relation (Carcione et al., 2007) with arbitrary parameter values.

We use the resistivity map as soft data to constrain the tomography problem with the cross-gradient function. In this case, we can write the cross-gradient function given in equation 2 as a linear operator $\mathbf{G}$ on the slowness field, $\mathbf{s}_0 + \Delta\mathbf{s}$. We can then extend the linearized tomography problem by employing $\mathbf{G}$ as an additional constraint. The objective function, $\mathcal{P}(\Delta\mathbf{s})$, of this extended problem becomes

$$\mathcal{P}(\Delta\mathbf{s}) = ||\Delta\mathbf{t} - \mathbf{T}_L\Delta\mathbf{s}||^2 + \epsilon_1^2 \; ||\mathbf{G}(\mathbf{s}_0 + \Delta\mathbf{s})||^2, \tag{6}$$

where ϵ_1 is a problem-specific weight factor to regularize the tomography problem (Clapp, 2001).

Figure 4 shows the initial velocity and the estimated velocities found by solving the tomography problem both with steering filters and the cross-gradient constraint. The results show that steering filters yield a good result for low frequency features such as

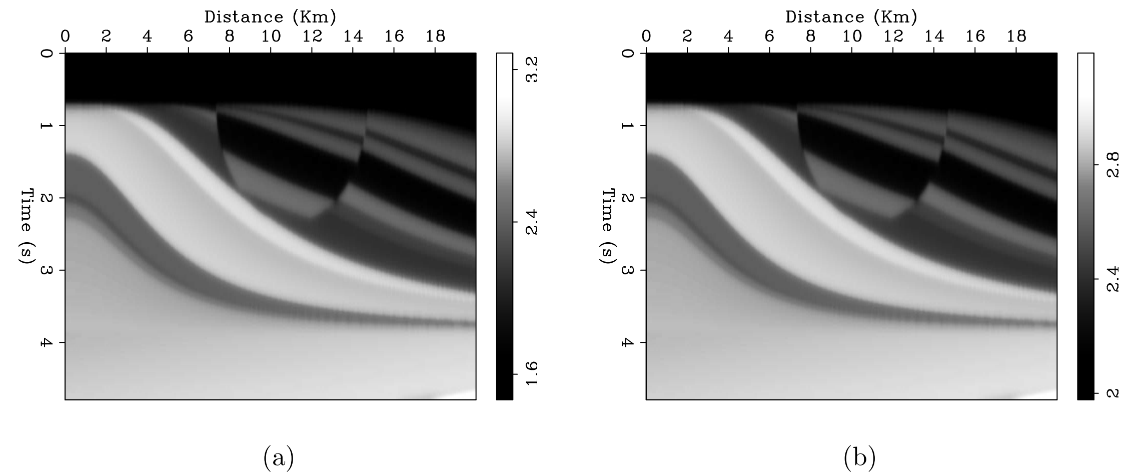


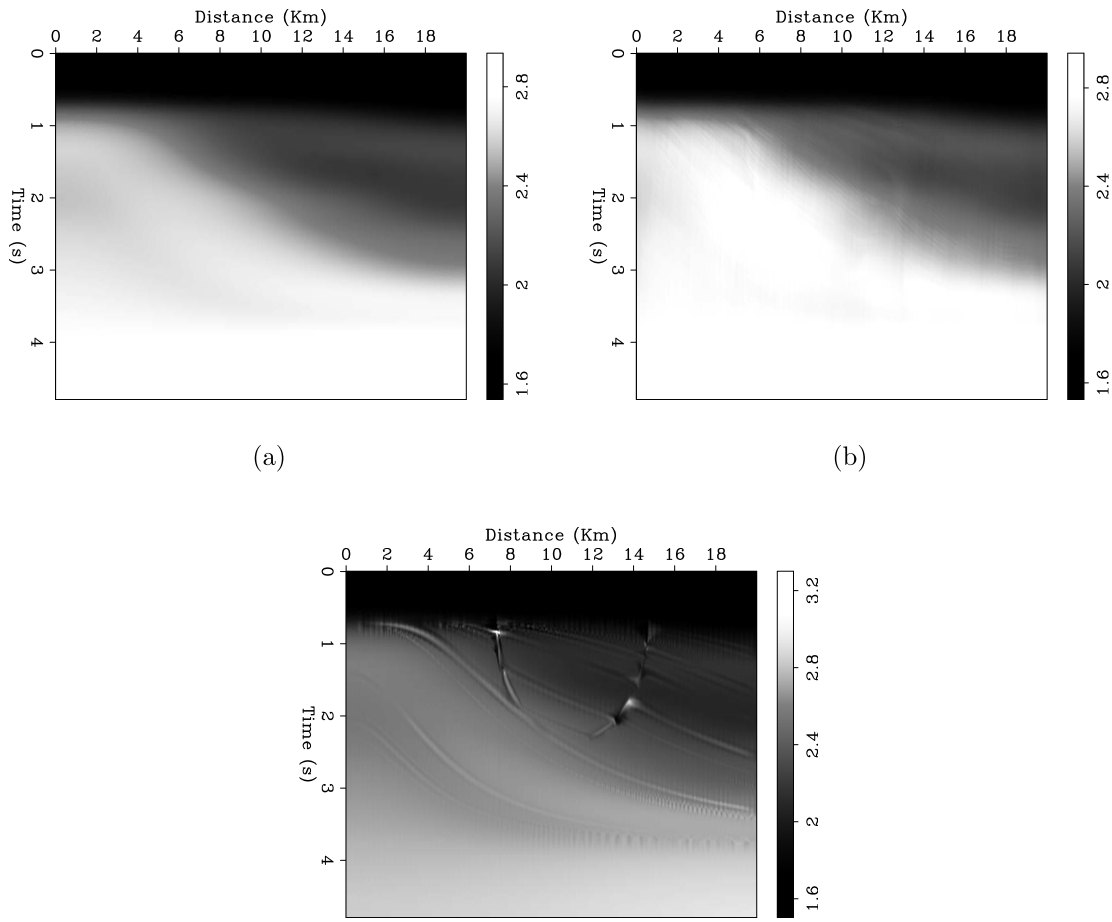
Figure 3: Synthetic sinusoidal model with **(a)** two velocity anomalies and corresponding **(b)** resistivity model. [**ER**]

smooth lateral velocity anomalies; however, it ignores high-frequency structures of the velocity model. On the other hand, the cross-gradient functions are able to provide better estimates for high-frequency features of the velocity model, such as sharp salt boundaries and faults. Steering filters assume *a priori* knowledge of the model parameters, while the cross-gradient function uses the co-located soft data field to build this information. The combination of these two method may be an optimal tool for addressing the velocity estimation problem in more general subsurface structures.

CONCLUSIONS AND FUTURE WORK

We have reviewed the issues involved in solving a typical seismic tomography problem and how we can address some of them by introduction of additional information. We also discussed our motivations for using the cross-gradient function to incorporate this additional information. The preliminary sensitivity analysis on two synthetic velocity models shows that the cross-gradient functions are a potential tool to integrate different types of geophysical data into the tomography problem. Finally, the comparison between estimated velocities by use of steering filters and cross-gradients functions suggest that we may use these two types of constraints to resolve more general cases of velocity models, including sharp boundaries and smooth anomalies.

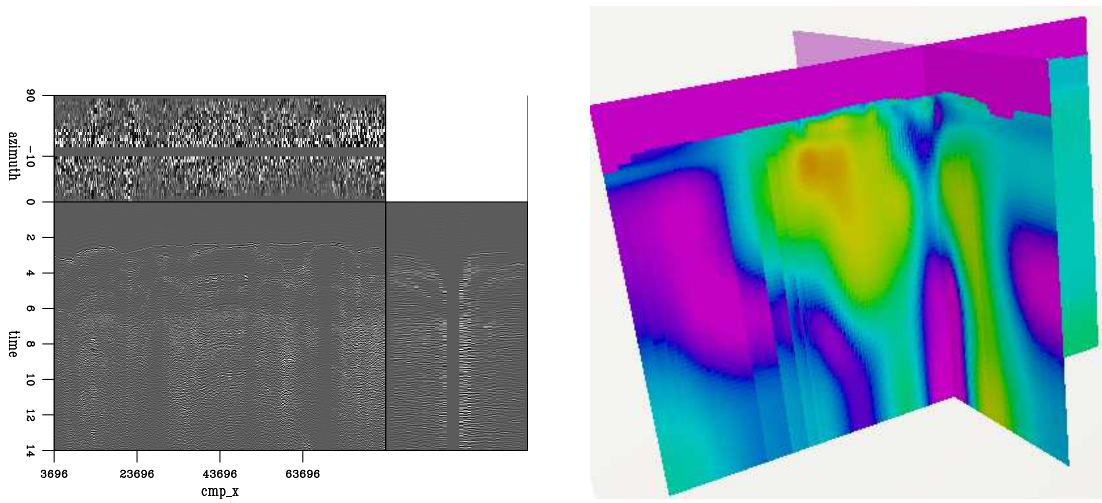
This method may lead to improved subsurface interpretations in regions mapped using more than one geophysical method. Figures 5(a) and 5(b) show a CMP gather of seismic data from a marine field dataset and co-located inverted MT resistivity data, respectively. We hope to improve the velocity estimations given by the seismic data itself by including the co-located smooth resistivity map in the tomography problem. Note that the frequency contents of seismic and resistivity data are different, and the



(a) (b) (c)

Figure 4: Velocity estimates by seismic tomography: Initial velocity estimate **(a)** and estimated velocity **(b)** with steering filers and **(c)** with cross-gradient constraint on soft-data. [**CR**]

resistivity field provides only a low frequency estimation of the subsurface structure. However, we hope to enforce a reasonable geological structure on the output of the seismic tomography problem by using this smooth image as the constraint.



(a) (b)

Figure 5: Field data provided by WesternGeco company: **(a)** a seismic CMP gather from field data and **(b)** The inverted resistivity map from the MT survey. [**NR**]

This method can be extended to seismic tomography constrained by training images, where we can also aim for different realizations of the velocity model by altering the co-located data or training image.

ACKNOWLEDGMENTS

We thank WesternGeco company and Olav Lindtjorn for providing the field dataset.

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