

INTERPOLATION WITH PREDICTION-ERROR FILTERS AND  
TRAINING DATA

A DISSERTATION  
SUBMITTED TO THE DEPARTMENT OF GEOPHYSICS  
AND THE COMMITTEE ON GRADUATE STUDIES  
OF STANFORD UNIVERSITY  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY

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June 2008

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I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

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# Abstract

With finite capital and logistical means, interpolation is a necessary part of seismic processing, especially for such data-dependent methods as surface-related multiple elimination. One such method is to first estimate a prediction-error filter (PEF) on a training data set, and then use that PEF to estimate missing data, where each step is a least-squares problem. This approach is useful because it can interpolate multiple simultaneous, aliased slopes, but requires regularly-sampled data. I adapt this approach to interpolate irregularly-sampled data, marine data with a large near-offset gap, and 3D prestack marine data with many dimensions.

I estimate a PEF from irregularly-sampled data in order to interpolate these data onto a regular grid. I do this by regridding the data onto multiple different grids and estimate a PEF simultaneously on all of the regridded data. I use this approach to interpolate both irregularly-sampled 3D synthetic data and 2D prestack land data using nonstationary PEFs, both with reasonable results.

Marine data typically contains a near-offset gap of several traces, which can be larger when surface obstacles are present, such as offshore platforms. Most methods that depend on lower-frequency information from the data fail for these large gaps. I estimate nonstationary PEFs from pseudoprimary data, which is generated by cross-correlating data within each shot, so that the correlation of multiples with primaries creates data at the near offsets that were not originally recorded. I use this approach in  $t$ - $x$ - $y$  and  $f$ - $x$ - $y$ , on both the Sigsbee2B 2D prestack synthetic dataset, and a 2D prestack field data set. While this method is currently unfeasible in the crossline direction in 3D marine data, the larger crossline aperture of a wide-azimuth dataset

would improve results considerably

Finally, I estimate nonstationary PEFs in many dimensions using the approximation that slope is constant as a function of frequency, and interpolate data in two, three, four, and five dimensions simultaneously by using nonstationary PEFs on frequency slices. This method correctly interpolates multiple simultaneous aliased slopes in the data. I interpolate both prestack 3D synthetic as well as field data to the densities required for future processing.

# Preface

The markings [ER], [CR], and [NR] included in the figure captions in this thesis denote the extent to which each figure is reproducible by anyone desiring to do so. The electronic version of this thesis provides the original figures and Fortran90 programs written to produce all of the results. Most programs are included locally in the chapter directories. I assume you have a UNIX-based workstation with Fortran90 and C compilers, an X-Windows environment, and the Stanford Exploration Project (SEP) software available from our website (make rules, programming libraries, and L<sup>A</sup>T<sub>E</sub>X packages).

Reproducibility is a way of organizing computational research that allows both the author and the reader to verify and regenerate results. Reproducibility also facilitates the transfer of knowledge within SEP and between SEP, its sponsors, and the geophysical community at large.

**ER** denotes Easily Reproducible results of processing described in the paper. I claim that you can reproduce such a figure from the programs, parameters, and makefiles included in the electronic document. The data must either be included in the electronic distribution, be easily available to all researchers (e.g., SEG-EAGE data sets), or be available in the SEP data library. Before the publication of the electronic document, someone other than me tested my claim by destroying and rebuilding all ER figures.

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Our testing is currently limited to LINUX 2.6 (using the Intel Fortran90 compiler), but the code should be portable to other architectures. Reader's suggestions are welcome. For more information on reproducing SEP's electronic documents, please visit

<http://sepwww.stanford.edu/research/redoc/>.

# Acknowledgments

A Ph.D. is a much longer endeavor than I had originally thought, and I have many people to thank for getting me through it. This list is far from exhaustive, but I would like to single out people who played particularly strong roles during my extended stay at SEP.

I would like to start by thanking my senior students and mentors Morgan Brown, Bob Clapp, and Antoine Guitton. They led me toward a research project and provided many hours of conversation about it and many other topics of mutual interest in geophysics and computing. I have since had the pleasure of having Bob on my defense committee and Antoine as a supervisor during an internship, and learned even more from them as a result. I thank my cohort, Gabriel Alvarez, Brad Artman, Andrey Karpushin, and Nick Vlad, who I shared many experiences with, and who gave me much needed support. I also thank the many students who arrived before and after me, in particular Guojian Shan, Jesse Lomask, Daniel Rosales, Doug Gratwick, Alejandro Valenciano, Jeff Shragge, Yaxun Tang, Madhav Vyas, Pierre Jouselin, Ben Witten, Claudio Cardoso, Gboyega Ayeni, and Roland Gunther for many useful discussions about our research and the world at large.

I owe a large debt to Jon Claerbout and Biondo Biondi for establishing and maintaining the intellectual framework of SEP and for keeping us focused on interesting topics that have applicability in the real world. They have given me the viewpoint that research is the most fun when working on new ways to approach a difficult field data set. Diane Lau deserves much credit for keeping a group as large as SEP running smoothly, and I owe the sponsors of SEP much for their financial support.



Anyone reading this thesis should thank Ken Lerner for taking what was a semi-intelligible mess and helping me rewrite and refine it into what you are reading today. He was extremely supportive and helpful during his brief stay at SEP. Norm Sleep also deserves much credit for his thorough review of this thesis. It is much stronger because of his help.

Doug Schmitt and Mauricio Sacchi at the University of Alberta sparked my interest in geophysics, and I enjoyed their courses greatly. Bill MacDonald was my supervisor during an extended internship in Calgary. He sent me out on a seismic survey for a week, which got me thinking about interpolation as a research topic.

I would like to thank my many friends at Stanford and elsewhere who helped me in my attempt to remain as well-rounded as possible. They are too numerous to mention, and are in geophysics, earth sciences, and further abroad.

Finally, I thank my family: my father and my brother for their focus on science and education, and my mother for her continual support.

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