Seismic reflector characterization by a multiscale detection-estimation method

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ABSTRACT

Seismic reflector interfaces in the subsurface are typically idealized as zero-order discontinuities. According to this model, the Earth's subsurface is represented by a spiky reflection coefficient sequence for which deconvolution methods have been derived. However, multiscale analysis on sedimentary basins reveals the existence of accumulation of varying-order singularities in the subsurface. Seismic traces with varying-order singularities violate most of the assumptions underlying conventional deconvolution methods. We avoid this problem by proposing a nonlinear parametric method based on a two-step approach that divides the initial problem of delineating and characterizing transitions over the whole seismic signal into two easier subproblems. The waveform characterization implies a representation for the geological record that provide information on the basic subsurface stratigraphy and lithology.

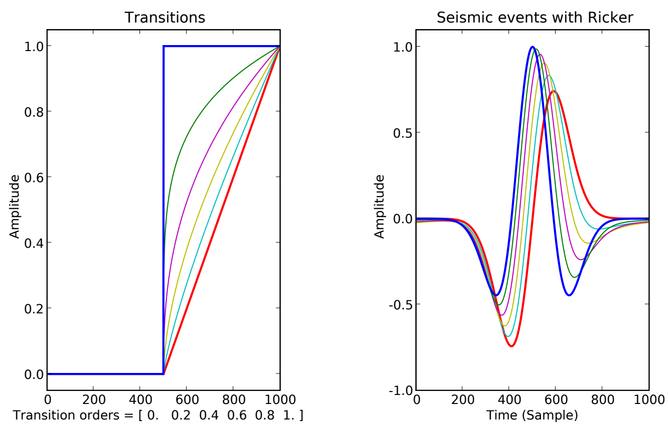
INTRODUCTION

The Earth's subsurface consists of layers of different materials separated by interfaces, also called transitions. Transitions are characteristic of regions where acoustic properties of the Earth vary rapidly compared to the length-scale of the seismic source wavelet. Extracting information about the accurate locations and nature of transitions from seismic data recently has received increasing interest, as it provides geologists with quantitative and complementary information useful in various geophysical applications, ranging from improving geological interpretation to detecting lithology changes.

The aim of seismic deconvolution is to find the locations of seismic reflectors based on certain assumptions on the reflectivity. Robinson (1957) proposed the first method to solve the blind deconvolution problem with an unknown source function. Other approaches are based on the sparsity assumption, where the reflectivity is considered to be given by a sparse spike train (see e.g. Dossal and Mallat (2005) and the references therein).

Multiscale studies of seismic and well data has established the diversity of transitions in the subsurface (Herrmann, 1998; Herrmann et al., 2001). These results reveal the existence of transitions with varying singularity orders in the subsurface. This generalized type of fractional-order singularity is given by fractionally differentiating/integrating a zero-order discontinuity (see Figure 1). Consequently, valuable information is lost when assuming a reflectivity sequence consists of only zero- or first-order singularities. Despite recent developments (Saggaf and Robinson, 2000), the existence of different fractional-order transitions renders conventional deconvolution techniques ineffective. Other methods to extract the singularity orders from seismic data based on wavelet coefficient decay (Liner et al., 2004) yield ambiguous estimates since seismic data is bandlimited. These observations led to a new type of parametrization, where the observed reflectivity is a superposition of parametrized waveforms. Maysami and Herrmann (2007) proposed a new detection-estimation method (see also Maysami, 2008) to estimate parameters that provide information on the transition sharpness that is related to the lithology (Herrmann et al., 2001; Liner et al., 2004).

The report is organized as follows. First, we present the parametric reflector model. Next we review the individual steps of our new method, i.e., detection, partitioning, and estimation. We also apply this method to a synthetic seismic trace and show the output of each step.



(a) (b)

Figure 1: Generalized transition model with singularity orders varying from zero- to first-order. **(a)** Zero- (step), first- (ramp), and fractional-order transitions. **(b)** Corresponding seismic waveforms, yielded by the convolution of the reflectivity with a Ricker wavelet. [**NR**]

THE EARTH'S MODEL

We represent a vertical 1-D profile of the Earth, either a time series or depth profile, as a superposition of parametrized waveforms. Without loss of generality, we use the time series notation represented by

$$s(t) = \sum_{i \in I} \; c_i \; \psi_{\sigma_i}^{\alpha_i}(t - \tau_i) \; e^{j\pi\phi_i}, \tag{1}$$

where t represents time, and c_i and ϕ_i $(0 \leq \phi < 2)$ are the amplitude and the phase for the i^{th} transition, respectively. Furthermore, $\psi_{\sigma_i}^{\alpha_i}(t - \tau_i)$ is a translated (τ_i) and scaled (σ_i) source wavelet ψ, which is fractionally differentiated $(\alpha < 0)$ or integrated $(\alpha > 0)$. The parameters of interest in this case, hereafter referred to as attributes, are the amplitude c_i, the location τ_i, the scale σ_i, the singularity order α_i, and the phase ϕ_i. The location (offset) of the waveforms corresponds to subsurface transition depths (stratigraphy), whereas their fractional-order constitutes a measure of transition sharpness (lithology).

THE CHARACTERIZATION PROBLEM

Given the above signal presentation, Maysami and Herrmann (2007) proposed a new method to estimate attributes of major transitions in seismic data (see also Maysami, 2008). They divided the initial problem of delineating and characterizing transitions over the seismic signal into two easier steps: detection and estimation. The first stage locates the major components of the seismic signal and segments it into individual waveforms. The second step estimates attributes of single windowed waveforms by using a nonlinear parametric inversion. Since our problem does not fit into the classical deconvolution framework, we use a multiscale wavelet technique to locate the main events. After segmentation, we input the individual waveforms to a nonlinear inversion procedure to estimate the attributes. This procedure uses rough estimates for the location, scale and phase from the detection stage as initial guesses.

Detection This part aims to approximate stratigraphic information of seismic signal regardless of nature of the transitions. The variety of different orders of transitions in the subsurface calls for a seismic-event-detection technique that does not make any assumptions regarding the transition type. Edge detection based on the multiscale continuous (complex) wavelet transform modulus maxima (Mallat, 1997) offers an approach that is robust for different waveforms reflecting different transition types. First, the method calculates the forward wavelet transform of a seismic trace, s, as a convolution product

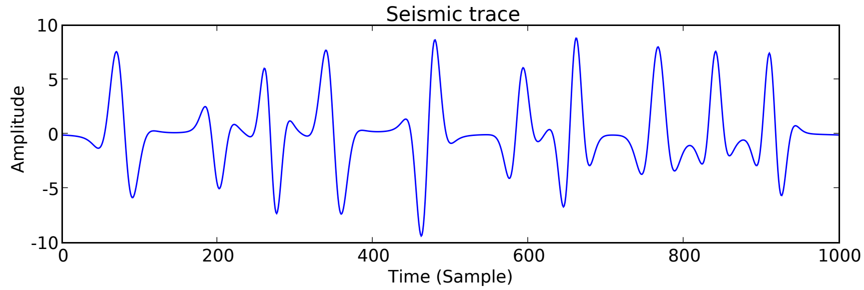
$$\mathcal{W}s(t,\sigma) = \big(s * \bar{\psi}_\sigma\big)(t), \tag{2}$$

where $\bar{\psi}_\sigma(t) = \frac{1}{\sqrt{\sigma}}\psi^*(\frac{-t}{\sigma})$, and $\sigma \geq 0$ is the scale of wavelet ψ. Here, the symbol * denotes the complex conjugate. The range of scales σ for the wavelet is adapted to the seismic source function. After forming the modulus maxima lines (MML) from the wavelet coefficients (Mallat, 1997), the maximum points along these lines are calculated, yielding rough estimates for the scale (i.e., bandwidth) and position of the reflection events (see Figure 2). The result of this stage is a set of locations, scales, and phase rotations $\{\tau^{(n)}, \sigma^{(n)}, \phi^{(n)}\}$ with $n = 1, N$, and N is the number of detected maxima, which corresponds to location and scale. We use these approximated values as initial guesses in the nonlinear inversion during the estimation stage.

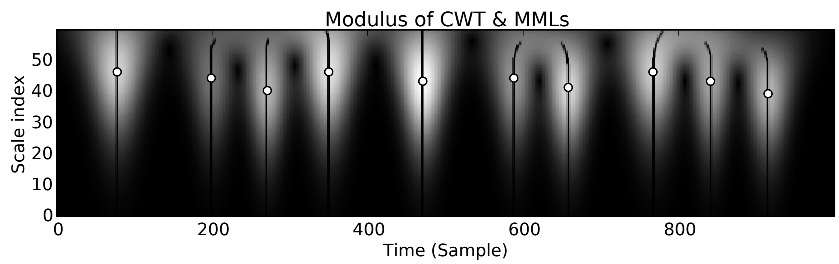
Partitioning Given estimates of the location and scale of the detected events, the trace is segmented into separate events. We extract the n^{th} detected waveform by multiplying the seismic trace by a window function centered at $\tau^{(n)}$ and with a support proportional to $\sigma^{(n)}$ (see Figure 3). The isolated waveforms are given by

$$s^{(n)}(t) = \mathbf{W}[\tau^{(n)}, \sigma^{(n)}]s(t) \quad \text{with} \quad n = 1, N \ , \tag{3}$$

where $\mathbf{W}[\cdot]$ is the windowing operator. This procedure outputs N vectors containing 'isolated' events. Even though this segmentation procedure is somewhat arbitrary, e.g., it depends on a width parameter, we found this method to perform reasonably well for most cases.



(a)



(b)

Figure 2: A typical example for detection stage of a synthetic seismic trace **(a)**. Wavelet coefficients for the signal are plotted in **(b)** with dark colors corresponding to small magnitudes. The vertical and horizontal axes show scale and location, respectively. Modulus maxima lines are shown as dark lines where white circles identify the scale and the location for the corresponding events. [**NR**]

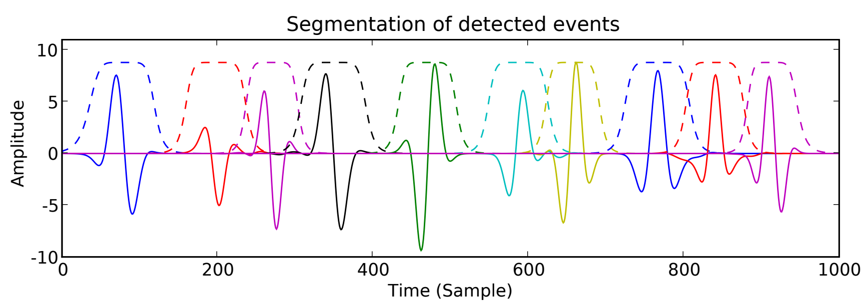


Figure 3: Segmentation of detected events. Each individual event (solid waveform) is extracted by using a window function (dashed line). [**NR**]

Estimation To complete the characterization, we apply a parametric nonlinear inversion procedure to the segmented events. In order to set up this procedure, we first need to refine our mathematical model for the parametrized waveforms in equation 1. We derive our model from a Gaussian bell-shaped waveform. Each element of the parametric family, also known as a manifold, is given by a fractional derivative/integrate of the shifted, scaled, and phase-rotated Gaussian. In the time domain, these waveforms are defined by a nonlinear function, $f_\theta : \mathbb{R}^5 \mapsto \mathbb{R}$, given by

$$f_\theta(t) = D^\alpha \left(\frac{1}{\sqrt{2\pi\sigma^2}} \; e^{(t-\tau)^2/2\sigma^2} \right) \; e^{j\pi\phi}, \tag{4}$$

with $\theta = [\tau, \sigma, \alpha, \phi]$ the set of parameters and D^α the α-order integration operator. Inspired by the work of Wakin et al. (2005), we can parametrize the windowed signals with the set of $\theta = [\sigma, \tau, \alpha]$ as

$$s^{(n)}(t) = M[\theta](t), \tag{5}$$

where $M[\theta] = \{f_\theta : \theta \in \Theta\}$ and Θ is feasible space for θ.

To estimate the different attributes for individual windowed waveforms, we need to find the optimal θ that minimizes the estimation error $e^{(n)}(\theta)$ given by

$$e^{(n)}(\theta) = \left\| f_\theta - s^{(n)} \right\|_2^2 . \tag{6}$$

The minimization problem for isolated event $s^{(n)}$ is then given by

$$\tilde{\theta}^{(n)} = \arg\min_{\theta\in\Theta} \; \left\| s^{(n)} - M[\theta] \right\|_2^2 \quad \text{with} \quad n = 1, N \; . \tag{7}$$

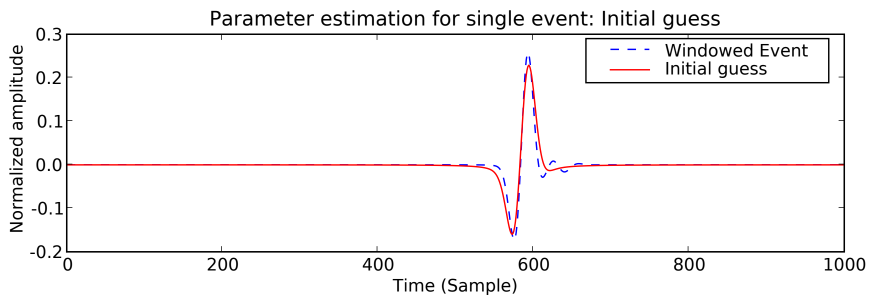
To solve this optimization problem, we employ the BFGS quasi-Newton method (Nocedal and Wright, 1999; Kelley, 1999). In quasi-Newton methods, one need not compute the second derivatives of the objective function for the Hessian matrix. Instead, the Hessian is updated by analyzing successive gradients. Alternatively, a trust region method with the Levenberg-Marquardt parameter (Kelley, 1999) can be used to solve the minimization problem. As with the BFGS method, trust region methods also require the manifold to be smooth. Partial derivatives of our model can be derived analytically since smoothness of Gaussian signals provides differentiability. This suggests we solve the above minimization problem in the frequency domain, where the fractional derivatives are known analytically. The elements of the Gaussian waveform family in the Fourier domain are given by (Blu and Unser, 2003)

$$\hat{f}_\theta(\omega) = (j\omega)^{-\alpha/2+\phi} \; (-j\omega)^{-\alpha/2-\phi} \; e^{-\frac{(\sigma^2\omega^2)}{2}} \; e^{-j\omega\tau}, \tag{8}$$

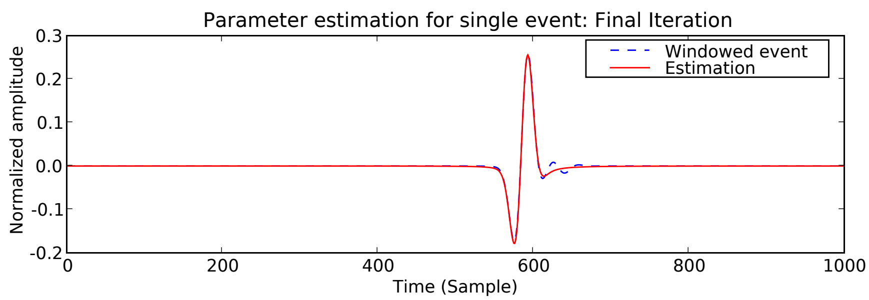
where $\hat{f}_\theta(\omega) = \mathcal{F}(f_\theta(t))$ represents Fourier transform of $f_\theta(t)$. Analytical expressions for partial derivatives of the manifold in the frequency domain are given by

$$\begin{aligned} \frac{\partial}{\partial\tau} \; \hat{f}_\theta(\omega) &= -j\omega \; \hat{f}_\theta(\omega), & \frac{\partial}{\partial\alpha} \; \hat{f}_\theta(\omega) &= -\ln(\omega) \; \hat{f}_\theta(\omega), \\ \frac{\partial}{\partial\sigma} \; \hat{f}_\theta(\omega) &= -\sigma\omega^2 \; \hat{f}_\theta(\omega), & \frac{\partial}{\partial\phi} \; \hat{f}_\theta(\omega) &= j\pi \; \hat{f}_\theta(\omega). \end{aligned} \tag{9}$$

Our experiments using the BFGS solver have shown that this optimization method provides acceptable and robust solutions for the estimation problem (see Figure 4). Figure 5 compares the original trace and the reconstructed trace by superposition of the estimated waveforms.



(a)



(b)

Figure 4: Parameter estimation for an individual event in Figure 3. **(a)** Initial iteration of parameter estimation for the isolated event where dashed line shows windowed event and solid line shows our guess. **(b)** Final iteration of parameter estimation for the isolated event where the estimated waveform matches the actual event. [**NR**]

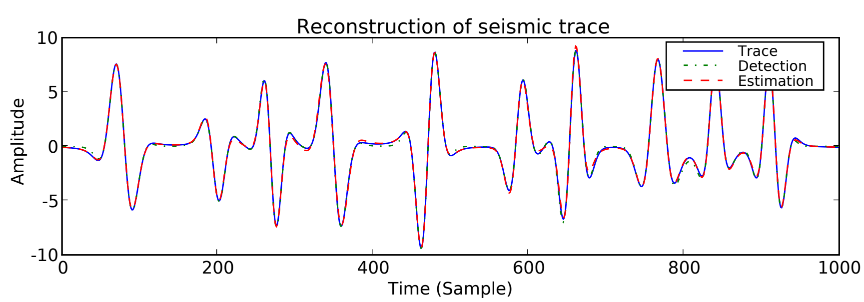


Figure 5: Estimated seismic signal formed by superposition of all characterized events and compared with the original seismic trace. [**NR**]

DISCUSSION AND CONCLUSIONS

In this report, we review a new characterization method allowing for the estimation of fractional-order discontinuities. These scale attributes may lead to an improvement of geological interpretation from seismic traces. The examples we present indicate that the proposed characterization method generates accurate results. In addition, our method is well-suited for estimating scale exponent attributes from bandlimited data. As opposed to wavelet-coefficient-decay-based methods, such as SPICE, our method does not generate possibly ambiguous estimates since we do not rely on 'infinite' bandwidth.

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