

Phase unwrapping of angle-domain common image gathers

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ABSTRACT

In this paper we adapt a phase unwrapping algorithm to estimate the depth shift in Angle-Domain Common Image Gathers (ADCIGs). We show how to set up a linear system of equations tailored to the seismic case and how to solve it by minimizing an L^0 measure via iterations of weighted least-squares problems. For this procedure a meaningful choice of initial weights is crucial.

We propose to unwrap jointly several angle gathers and show that this can overcome sampling deficiencies in the angle domain, such as those that come from processing a limited number of subsurface offsets for angle-gather generation.

INTRODUCTION

Migration velocity analysis is a class of techniques used for updating the velocity field, starting from a migrated image. These techniques are based on linking the curvature of image gathers (for instance Angle-Domain Common Image Gathers) to migration-velocity error. When this relation is linearized, it leads to a simple inversion problem. However the linearization of the wave field with the first-order Born approximation comes with an important limitation: it can handle delays only up to a fraction of the wavelength.

This problem has been given a possible solution in Sava and Biondi (2004). A viable alternative is to transform the ADCIGs to the Fourier domain and do phase unwrapping there, which is roughly equivalent to determining the delay in the original domain. This is suggested in Sava and Biondi (2003) concerning the Rytov approximation. Synthetic Aperture Radar Interferometry and Magnetic Resonance Imaging literature offers many examples of unwrapping techniques. Although they share common principles, each one must be carefully tuned to the specific application.

In this paper we adapt a phase-unwrapping algorithm to unwrap ADCIGs. We describe how to formulate the unwrapping problem, solve it, and test it on a simple synthetic case. We show that by unwrapping jointly several gathers we can overcome some sampling limitations in the angle domain.

PHASE UNWRAPPING

When a signal is delayed, the phases of its Fourier spectral components are rotated proportionally. However, due to the periodic nature of Fourier components, the observable phases are always limited to the interval $[-\pi, \pi]$; i.e. there is no record of the number of entire cycles that may have intervened. This phenomenon is usually referred to as phase ambiguity, because different delays can correspond to the same observed phase shift. Phase unwrapping is the problem of recovering the number of 2π cycles that unambiguously reconstructs the original delay.

Phase unwrapping can be approached in various ways. In this work we follow the recipe presented in Ghiglia and Romero (1996) and Ghiglia and Romero (1994), where the unwrapped phase is found as the solution of a linear system.

The general principle is that even though the unwrapped phases are usually outside the interval $[-\pi, \pi]$, differences in unwrapped phases of “neighboring” points are often included in that interval, so that they can be recovered also from the wrapped values, which are available. Thus we write a number of equations that describe differences in the unwrapped phases and rely on the solution of the system to integrate those differences.

Of course some of the original equations are wrong (they assume the phase difference to be within the interval $[-\pi, \pi]$, when in fact it is not) and conflict with others. The algorithm that solves the system will eventually have to make a decision and discard some equations, favoring others.

First we have to define a graph that represents the equations we will use. Then we describe the algorithm for the solution of the system.

GRAPH AND LINEAR SYSTEM

In our domain, the signal is a function of angle (α), vertical wavenumber (kz) and midpoint inline position (x). Each equation we include in our system connects two points, so that each equation corresponds to a link and the entire system to a graph. A simple cartesian grid was used in the angle- kz plane. Each point is connected to its four neighbors, so for example the point $A(\alpha, kz, x)$ is connected to $A(\alpha, kz \pm 1, x)$ and to $A(\alpha \pm 1, kz, x)$. Points at the boundary of the domain have fewer connections.

To increase the robustness of the unwrapping procedure we do not consider each gather independently but connect several gathers in the inline direction, presuming continuity along that axis too. So $A(\alpha, kz, x)$ is also connected to $A(\alpha, kz, x \pm 1)$, raising to six the number of equations in which a given point typically appears.

An example of the basic equation is the following:

$$\phi(\alpha, kz, z) - \phi(\alpha - 1, kz, z) = [\varphi(\alpha, kz, z) - \varphi(\alpha - 1, kz, z)]_{2\pi} \quad (1)$$

where the other cases are straightforward. The expression $[\cdot]_{2\pi}$ represents the wrapping operator, or the remainder after integer division by 2π ; ϕ are the unwrapped values and φ their wrapped, observed counterparts.

The system is not complete without some boundary equations that serve as a phase reference. We set to zero the zero-angle phases of a reference gather for all the considered wavenumbers.

The whole system can be written in matrix form:

$$\mathbf{G}\Phi = \mathbf{d}, \quad (2)$$

where \mathbf{G} is the graph incidence matrix plus border equations, Φ is the unknown vector of unwrapped phases and \mathbf{d} is a function of the observed phases (the wrapped differences). \mathbf{G} is a very sparse matrix with typically two non-zero entries per row.

L^0 SOLUTION AND WEIGHTED ITERATIONS

The solution of the above system of equations (2) can be found by minimizing a chosen indicator. Given the particular nature of the unwrapping problem, the L^0 measure is considered a good choice. The point is that we are not looking for a smooth solution that tries to accomodate all equations (like the L^2 norm does); we instead want the algorithm to make hard choices between alternatives and to produce a solution that satisfies, with no approximation, the highest possible number of equations. Ghiglia and Romero (1996) describe a way to minimize the L^0 measure via successive steps that are computed solving weighted least squares problems. Ghiglia and Romero's algorithm is more general and provides a way to minimize any L^p measure, with p in $[0, 2]$. An application of the L^1 -norm is found in Lomask (2006).

The following is the outline of the suggested algorithm, (setting $p = 0$ for our specific case):

- Set up the initial weights, \mathbf{W}_0 .
- Set $i = 0$.
- Until i has reached the maximum number of iterations, repeat the following steps:

1. Solve (to convergence) the Weighted Least Square (WLS) system:

$$\mathbf{G}^T \mathbf{W}_i \mathbf{G} \Phi_i = \mathbf{G}^T \mathbf{W}_i \mathbf{d}. \quad (3)$$

2. Compute new weights according to the formula

$$W_{i+1}(n) = \frac{\epsilon_0}{\epsilon_0 + |\mathbf{g}(n)\Phi_i - d(n)|^{2-p}} W_i(n). \quad (4)$$

3. Increase i by 1.
- End.

\mathbf{W}_i is a diagonal matrix with elements $W_i(n)$, the weights for each equation. The vector $\mathbf{g}(n)$ is the n^{th} row of \mathbf{G} , so that $\mathbf{g}(n)\Phi_i$ is a scalar and ϵ_0 an adequately small value. For efficiency reasons the WLS step is implemented by preconditioned conjugate gradient.

With this iterative mechanism and this particular choice of weights, each equation which is not satisfied at a given iteration is almost ignored for the next iteration, provided that more trusted equations exist that involve the same points.

Thus the choice of the initial weights is critical to yielding good results. We preliminarily used the amplitude information as a measure for the phase reliability: each equation was given a weight proportional to the harmonic average between the amplitudes of the two points involved.

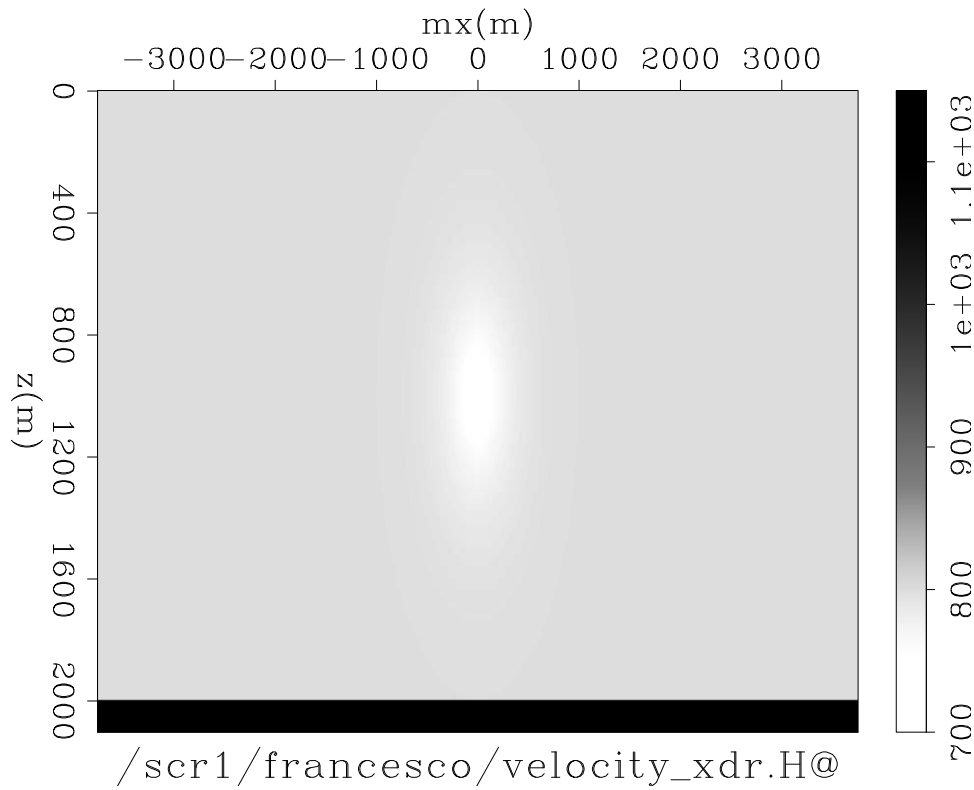


Figure 1: The velocity used for modeling the seismic data.

EXAMPLE

For a first test we create a model with a negative Gaussian anomaly in an constant velocity background (see Fig.1) and migrate (incorrectly) the modeled data using a

constant velocity model. After migration we apply an offset-to-angle transformation using 33 offsets. The result is seen in Figure 2. As expected, the angle gathers show some deviation from being flat. This curvature can ideally be used to correct the migration velocity and improve the focusing. Notice the jump at near angles because of the insufficient angle sampling, a consequence of the number of processed offsets.

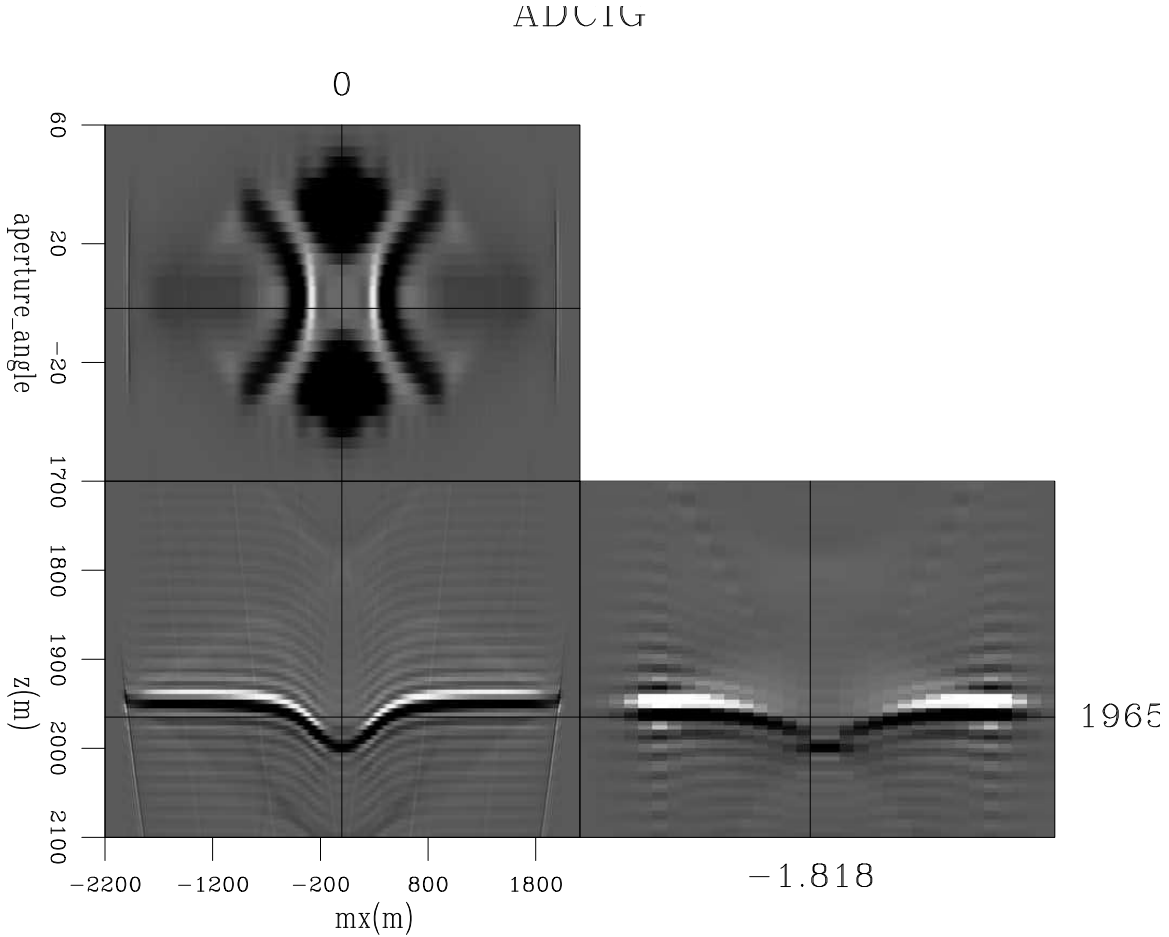


Figure 2: An Angle-Domain Common Image Gather computed using 33 offsets.

We pick 33 gathers equally spaced in the inline direction, from a position where the presence of the anomaly is unfelt to directly under the anomaly. After windowing, we transform the z axis so that for each gather we have now a kz -angle panel instead of the original z -angle panel. Applying the described unwrapping procedure, we obtain the result shown in Figure 3, which refers to the gather right under the anomaly. The left image is the original wrapped phase, referred to the 0 angle for visualization convenience. The right image is the corresponding unwrapped phase. Please note that the wrapped phase field is not devoid of ambiguity, i.e. the integration path is not irrelevant. However the algorithm is able to cut the phase at approximately the right position, at low angles for high frequencies. This is possible because of the choice of initial weights and the linking of several gathers together.

To confirm the result, we apply the same algorithm to the case where we have

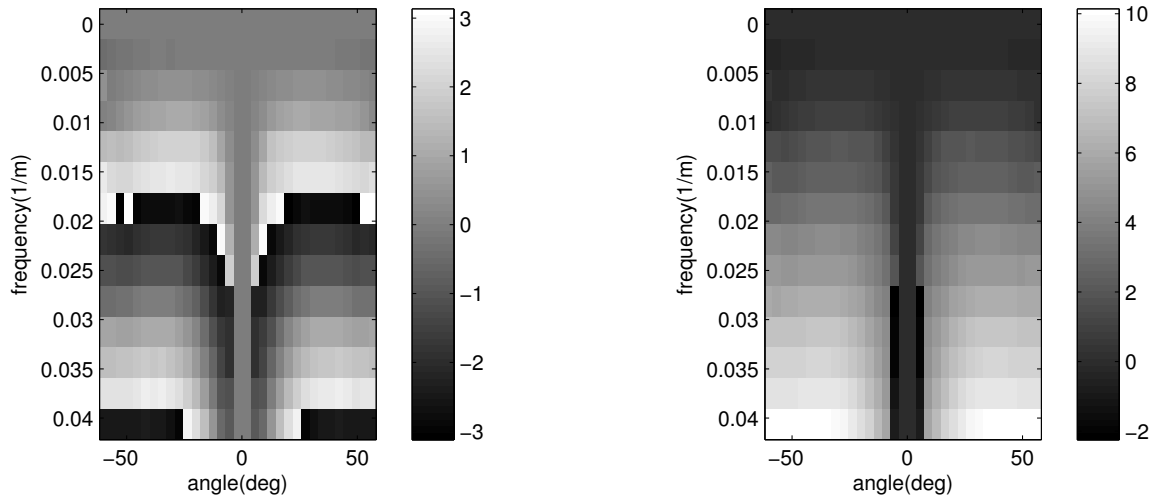


Figure 3: Wrapped (left) and unwrapped (right) phase for an ADCIG computed using 33 offsets. Phase measured in radians.

computed a larger number of offsets, 65 instead of 33. This increases the resolution in the angle domain (see Fig. 4), and the discontinuity disappears from the z -angle domain. The same happens in the kz -angle domain (see Fig. 5, left), where we no longer see a jump in the wrapped phase. The unwrapped phase is comparable to the previous one.

When dispersion effects can be ignored, it is possible to derive from the unwrapped phases a single number representing the delay for a given gather and a given angle. We interpolate lines into reliable unwrapped phase values, again using amplitude as a reliability criterion. The slopes of the lines correspond to the z -domain delays. Figures 6 and 7 display these delays in terms of samples for the two cases, with 33 and 65 offsets. For visualization purposes we subtracted the average delay for each gather, so that the effect of the anomaly is more clearly visible. A mask is used because midpoints have different angular coverage.

Phase unwrapping makes it possible to treat different wavenumbers independently, i.e. to take advantage of the information carried by the dispersion. Even in this simple example we can actually see some dispersion effects. Figure 8 shows the delay predicted by the single wavenumber for all gather-angle pairs, after subtraction of the “average” delay. Lower wavenumbers have a higher dispersion, but higher ones are more prone to unwrapping problems.

CONCLUSIONS

Phase unwrapping in the kz -angle domain can be used to evaluate the delay of angle gathers, a preliminary step towards velocity analysis. Simple numerical test indicates that some limitations that come from angle sampling or illumination can be overcome

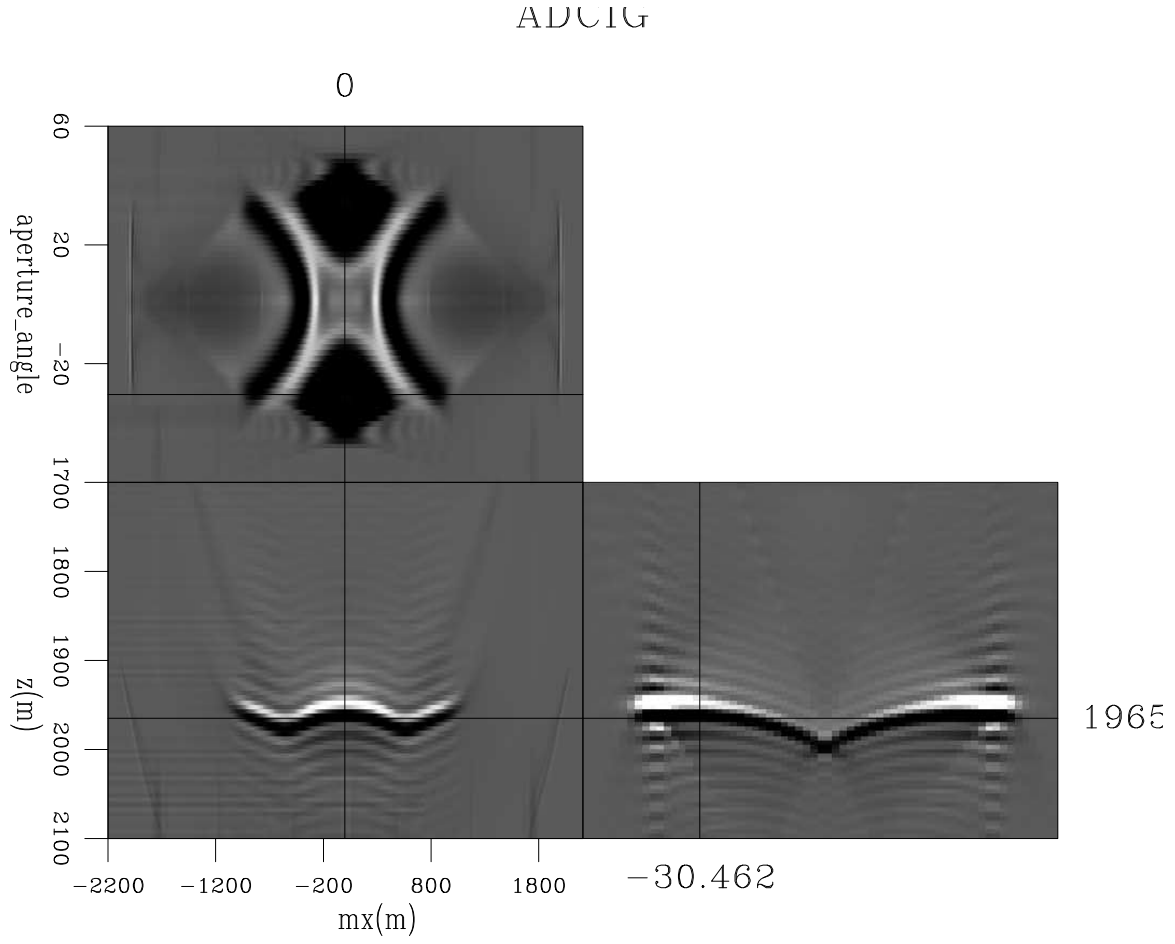


Figure 4: An ADCIG computed using 65 offsets.

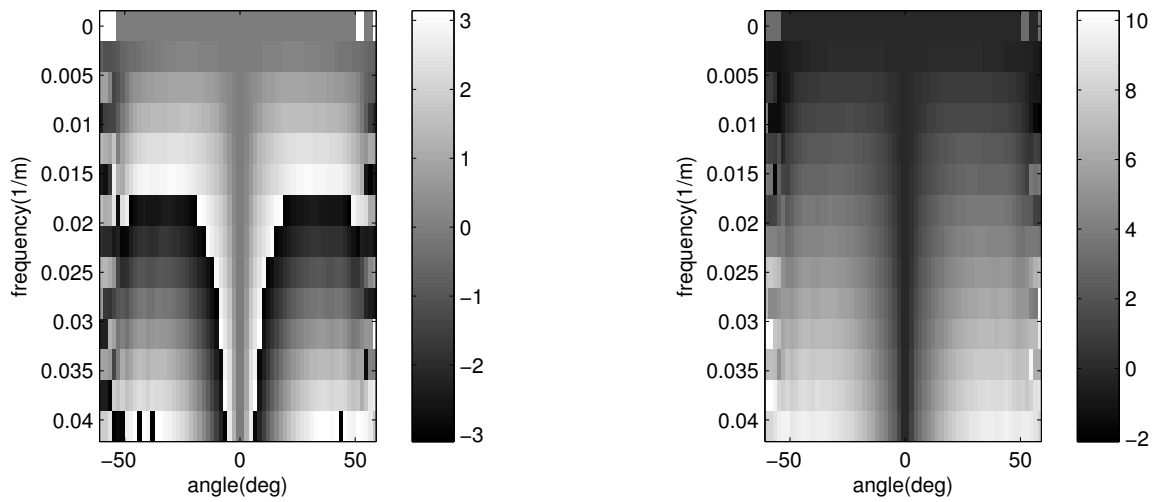


Figure 5: Wrapped (left) and unwrapped (right) phase for an ADCIG computed using 65 offsets.

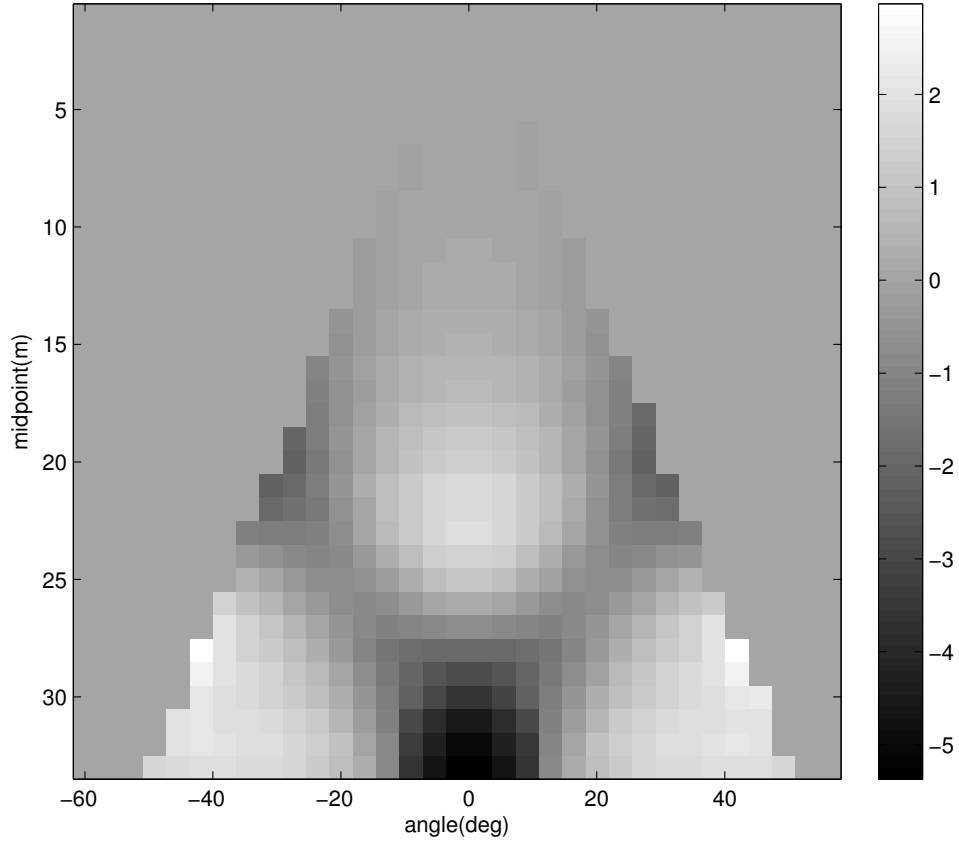


Figure 6: The delay (in samples) for a number of ADCIGs as a function of aperture angle. Gathers were computed using 33 offsets.

Figure 7: The delay (in samples) for a number of ADCIGs as a function of aperture angle. Gathers were computed using 65 offsets.

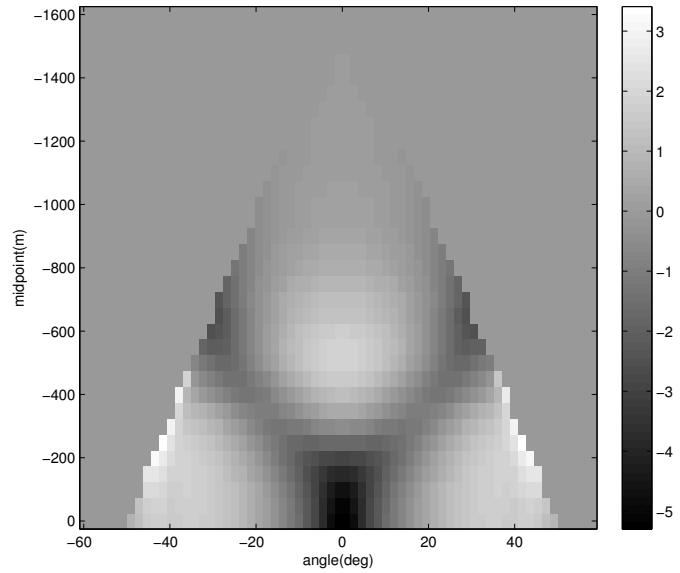
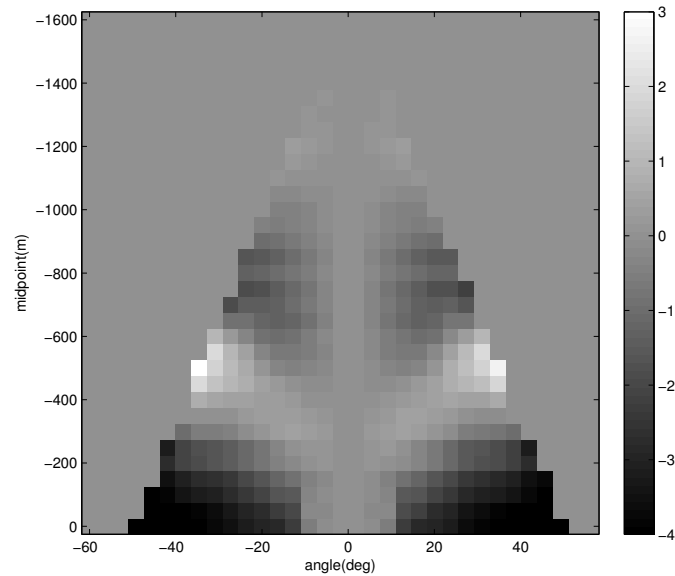


Figure 8: An example of the residual delay (converted to samples) for a given kz , after subtracting the delay identified using all wavenumbers.



by considering jointly a number of gathers from the same horizon. The application of this unwrapping technique may require more investigation about image windowing (which should ideally follow the still-unknown gather curvature) and gather picking to ensure phase continuity.

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