Traveltime of refracted rays

In this Appendix I derive equations **??** and **??**. From equation **??** we have:

$$t_{s_1} \cos\alpha_s + \rho\tilde{t}_{s_2} \cos\beta_s = t_{r_1} \cos\alpha_r + \rho\tilde{t}_{r_2} \cos\beta_r, \tag{A-1}$$

and, from the imaging condition (the sum of the traveltime of the extrapolated rays at the image point has to be equal to the traveltime of the multiple) we have

$$t_{s_2} + t_{r_2} = \tilde{t}_{s_2} + \tilde{t}_{r_2}. \tag{A-2}$$

Solving those two equations for $\tilde{t}_{s_2}$ and $\tilde{t}_{r_2}$ we get

$$\tilde{t}_{s_2} = \frac{t_{r_1} \cos\alpha_r - t_{s_1} \cos\alpha_s + \rho(t_{s_2} + t_{r_2}) \cos\beta_r}{\rho(\cos\beta_s + \cos\beta_r)}, \tag{A-3}$$

$$\tilde{t}_{r_2} = \frac{t_{s_1} \cos\alpha_s - t_{r_1} \cos\alpha_r + \rho(t_{s_2} + t_{r_2}) \cos\beta_s}{\rho(\cos\beta_s + \cos\beta_r)}. \tag{A-4}$$

It is interesting to check these equations in two particular cases:

1. For a specular multiple from a flat water-bottom, we have $\alpha_s = \alpha_r$, $\beta_s = \beta_r$, $t_{s_1} = t_{s_2} = t_{r_2} = t_{r_1}$ and therefore we get $\tilde{t}_{s_2} = t_{s_2}$ and $\tilde{t}_{r_2} = t_{r_2}$ as the geometry of the problem requires. Notice that this is true for any ρ.

2. For a specular water-bottom multiple migrated with water velocity ($\rho = 1$), we have $\beta_s = \alpha_s$ and $\beta_r = \alpha_r$. Furthermore, since the multiple behaves as a primary, $(t_{s_1} + t_{s_2}) \cos\alpha_s = (t_{r_1} + t_{r_2}) \cos\alpha_r$ and we again get $\tilde{t}_{s_2} = t_{s_2}$ and $\tilde{t}_{r_2} = t_{r_2}$.

Image Depth in ADCIGs

Figure A-1 shows the basic construction to compute the image depth in ADCIGs based on the image depth in SOCIGs. Triangles ABD and CBD are congruent since they have one side common and the other equal because $|AB| = |BC| = h_\xi$. Therefore, $\theta = \pi/2 - \beta_r + \delta$. Also, triangles AED and FCD are congruent because $|AD| = |CD|$ and also $|AE| = |CF|$ (**?**). Therefore, the angle δ in triangle DCF is the same as in triangle AED. We can compute δ from the condition

$$\begin{aligned} \theta + \delta + \beta_s &= \frac{\pi}{2}, \\ \frac{\pi}{2} - \beta_r + \delta + \delta + \beta_s &= \frac{\pi}{2}, \\ \delta &= \frac{\beta_r - \beta_s}{2}. \end{aligned}$$

The depth of the image point in the ADCIG, from triangle ABC, is therefore

$$z_{\xi_\gamma} = z_\xi + z^* = z_\xi + (\mathrm{sign}(h_\xi)) h_\xi \cot\left(\frac{\pi}{2} - \beta_r + \delta\right). \tag{A-5}$$

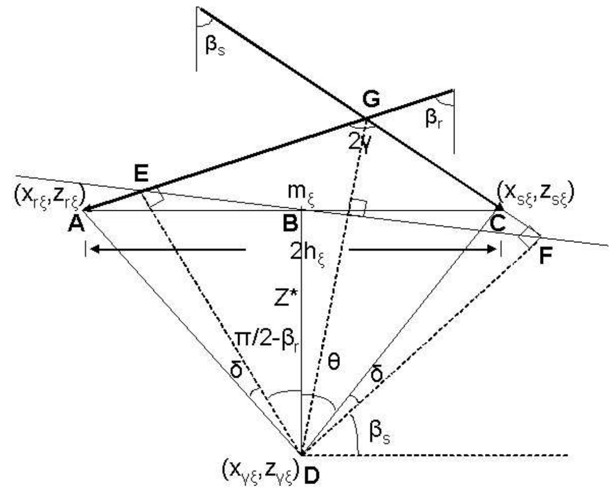

Figure A-1: Sketch to show the computation of the image depth in an ADCIG.

Replacing the expression for δ we get, after some simplification (and taking $\text{sign}(h_\xi) = -1$)

$$z_{\xi_\gamma} = z_\xi + z^* = z_\xi - h_\xi \tan\left(\frac{\beta_r + \beta_s}{2}\right) = z_\xi - h_\xi \tan(\gamma). \tag{A-6}$$

Residual Moveout in ADCIGs

In this appendix I show that, for a flat reflector, the residual moveout of the multiples in ADCIGs reduces to the tangent-squared expression derived by Biondi and Symes (2004) for the residual moveout of under-migrated primaries:

$$\Delta \mathbf{n}_{\mathrm{RMO}} = (\rho - 1)\tan^2\gamma z_0 \mathbf{n}. \tag{A-7}$$

Start with Equation **??**

$$z_{\xi_\gamma} = \frac{z_{\xi_\gamma}(0)}{1+\rho}\left[1 + \frac{\cos\gamma(\rho^2 - (1-\rho^2)\tan^2\gamma)}{\sqrt{\rho^2 - \sin^2\gamma}}\right], \tag{A-8}$$

where $z_{\xi_\gamma}(0)$ is the normal-incidence migrated-depth, (*i.e.* z_0) in the previous equations.

There is an important and unfortunate difference in notation here, however, because ρ in equation A-7 is the ratio of the migration to the true *slowness* whereas ρ in equation A-8 is the ratio of the migration to the true *velocity*. Therefore, in order to get a better idea of how the approximation for the RMO of the multiples (accounting for ray bending at the reflector interface) relates to that of the primaries (neglecting ray bending), I rewrite equation A-8 replacing ρ by $1/\rho$ and $z_{\xi_\gamma}(0)$ with z_0 to get:

$$z_{\xi_\gamma} = \left[\rho + \frac{\cos\gamma(1 - (\rho^2 - 1)\tan^2\gamma)}{\sqrt{1 - \rho^2\sin^2\gamma}}\right]\frac{z_0}{1+\rho}. \tag{A-9}$$

Since $\Delta n_{\mathrm{RMO}} = z_0 - z_{\xi_\gamma}$ we get:

$$\Delta n_{\mathrm{RMO}} = \left[1 - \frac{\cos\gamma(1 - (\rho^2 - 1)\tan^2\gamma}{\sqrt{1 - \rho^2 \sin^2\gamma}}\right] \frac{z_0}{1+\rho}. \tag{A-10}$$

For small γ, $\sin\gamma \approx 0$ and $\cos\gamma \approx 1$, therefore

$$\Delta n_{\mathrm{RMO}} = (\rho^2 - 1)\tan^2\gamma \frac{z_0}{1+\rho} = (\rho - 1)\tan^2\gamma z_0. \tag{A-11}$$

This is the same as equation A-7 save for the unit vector $\mathbf{n}$. This result is intuitively appealing because it shows that the approximation of neglecting ray bending at the reflecting interface deteriorates as the aperture angle increases which is when the ray bending is larger.