[Adaptive matching]Simultaneous adaptive matching of primaries and multiples

SUMMARY

In this chapter, I develop a method to match estimates of primaries and multiples to data containing both. The method works with prestack or poststack data in either data space or image space and addresses the issue of cross-talk (leakage) between the estimates of the primaries and the estimates multiples. I pose the problem as a non-linear optimization in which non-stationary filters are computed in micro-patches to simultaneously match the estimates of the primaries and the multiples to the data, in a least-squares sense. I apply the method iteratively by updating the estimates of the primaries and the multiples after the least-squares solution is found. Only a few of these iterations are needed. The computer cost is a negligible fraction of the cost of computing the estimate of the multiples with convolutional methods such as SRME. I show, with several synthetic and real data examples, that the matched estimates of both primaries and multiples have little cross-talk. I also apply the method to the separation of spatially-aliased ground-roll and body waves and show that most residual ground-roll contaminating the estimate of the body waves can be eliminated. This method is applied in chapter 6 to adaptively match and subtract the multiple estimate from a 3D real dataset from the Gulf of Mexico.

INTRODUCTION

Most methods to attenuate multiples perform, in one way or another, two complementary but clearly distinguishable steps: first, estimate a model for the multiples, and second, adaptively match and subtract the estimate of the multiples from the data to get the estimate of the primaries. As described in chapter 1, Surface Related Multiple Elimination (**?????**) uses the auto-convolution of the data to estimate the multiples whereas moveout-based methods use filtering in either the frequency-wavenumber or a Radon-transform domain (**???**) to estimate the multiple model. Whatever the method, the estimate of the multiples is likely to be contaminated with residual primary energy and to have errors in amplitude, phase and frequency content. After adaptive subtraction, the estimated primaries are likely to suffer from undesired residual multiple energy, or weakened primaries, or both (**?**).

In this chapter I assume that the multiple model has already been estimated by some method and concentrate on the adaptive subtraction step to get the estimated-matched primaries. I present a new adaptive matching algorithm that simultaneously matches estimates of the primaries and the multiples to the data. In contrast, the most standard algorithms adaptively match the multiples only. Matching also the estimate of the primaries help constrain the matching of the multiples thus reducing the leak of residual multiples (so-called cross-talk) on the estimated primaries.

The new adaptive-matching algorithm estimates, in the least-squares sense, non-

stationary filters (**?**) that simultaneously match both the estimates of the primaries and the multiples to the data. These filters act on micro-patches *i.e* small, overlapping pieces of data (**?**) and can handle inaccuracies in the estimated multiples in terms of both amplitudes and kinematics. Once the solution to the least-squares problem is computed, I iteratively re-estimate the multiple and primary models until the residual (the sum of the matched primaries and multiples minus the data) is close to zero. In my experience, as few as three to five iterations of the least-squares inversion ("outer" iterations) seem sufficient.

I apply this new method to two synthetic datasets contaminated with multiples. In the first test, I match kinematically perfect estimates of primaries and multiples contaminated with 40% of cross-talk and show that for this simple case the method produces a cross-talk-free result. Then I apply the method to an inaccurate estimate of both the primaries and the multiples obtained via migration-demigration as described in **?**. Even with a poor initial estimate of both primaries and multiples, with strong cross-talk on both, the matched results are very good, with little cross-talk. To illustrate the method with stacked data, I apply it to a migrated section of the Sigsbee model. Here the multiples were estimated with an image space version of SRME (**?**). The results show that the method attenuated most of the multiples and produced a largely multiple-free estimate of the primaries.

The method performs well with real data, as I demonstrate by applying it to match the estimated multiples computed in the previous chapter. I adaptively matched and subtracted the multiple estimate of each individual ADCIGs and then stacked the estimated primaries to form an angle stack of primaries only. The method performed very well and the multiples were nicely attenuated in the angle stack.

Finally, to illustrate that the method may have applications beyond the matching of primaries and multiples, I apply it to a different problem, namely the separation of ground-roll and body-waves. I use a shot gather from a land dataset contaminated with strong, spatially-aliased, ground-roll and show that most of the residual ground-roll can be attenuated in the final estimate of the body waves. In a way, this is a more challenging problem because the non-stationarity characteristics of the ground-roll and the body waves are different. The requirements of filter lengths and patch sizes to match the data are therefore different for the ground-roll and the body-waves. I chose to preserve the body waves even if that meant allowing some residual ground-roll.

DESCRIPTION OF THE METHOD

Conceptually, the first step of the new method is to form the convolutional matrices of both the estimated multiples $\mathbf{M}$ and the estimated primaries $\mathbf{P}$. In practice, these are huge matrices that are not explicitly formed but are replaced by equivalent linear operators (**?**). Next, I compute non-stationary filters in micro-patches (that is, filters that act locally on overlapping two-dimensional partitions of the data) to match the

estimated multiples and the estimated primaries, to the data containing both. I compute the filters by solving the following linear least-squares inverse problem:

$$\begin{bmatrix} \mathbf{M} & \mu\mathbf{P} \end{bmatrix} \begin{bmatrix} \mathbf{f_m} \\ \mathbf{f_p} \end{bmatrix} \approx \mathbf{d} \tag{1}$$

$$\epsilon\mathbf{A} \begin{bmatrix} \mathbf{f_m} \\ \mathbf{f_p} \end{bmatrix} \approx \mathbf{0} \tag{2}$$

where $\mathbf{f_m}$ and $\mathbf{f_p}$ are the matching filters for the multiples and primaries respectively, μ is a parameter to balance the relative importance of the two components of the fitting goal (**?**), $\mathbf{d}$ is the data (primaries and multiples), $\mathbf{A}$ is a regularization operator, (in my implementation a Laplacian operator), and ϵ is the usual parameter to control the level of regularization.

Once convergence is achieved, each filter is applied to its corresponding convolutional matrix, and new estimates for $\mathbf{M}$ and $\mathbf{P}$ are computed:

$$\mathbf{M}_{i+1} \leftarrow \mathbf{M}_i\mathbf{f}_{\mathbf{m}i} \tag{3}$$

$$\mathbf{P}_{i+1} \leftarrow \mu\mathbf{P}_i\mathbf{f}_{\mathbf{p}_i}. \tag{4}$$

Here i represents the index of the outer iteration of the linear problem described by Equations 1 and 2. Notice that I hold μ constant although it could be changed from iteration i to iteration $i+1$. Also notice that the regularization operator A and the regularization parameter ϵ in Equation 2 could be different for $\mathbf{f_m}$ and $\mathbf{f_p}$. I have chosen to keep them the same to limit the number of adjustable parameters. This choice worked very well in all my tests. The updated versions of the convolutional matrices $\mathbf{M}_{i+1}$ and $\mathbf{P}_{i+1}$ are plugged into equations 1 and 2 and the process repeated until the cross-talk has been eliminated or significantly attenuated.

EXAMPLES WITH SYNTHETIC DATA

As a first example, I will consider the synthetic dataset shown in panel (a) of Figure 1. There are two primaries (black) and four multiples (white). The traveltimes of both primaries and multiples were computed analytically from a three flat-layer model: water layer, a sedimentary layer and a half space. The estimates of the multiples (b) and primaries (c) were computed by adding 40% of the primaries to the multiples and 40% of the multiples to the primaries, respectively. The goal is to simulate a situation in which the kinematics of the estimates of primaries and multiples are both correct but there is strong cross-talk (leakage) between them.

Figure 2 shows the estimated multiples after one, two and three outer iterations of the algorithm. The corresponding results for the estimated primaries are shown in Figure 3. In both figures we see that the cross-talk is substantially reduced after the first outer iteration and is completely eliminated after the third. Notice the hole in the top multiple and the bottom primary in the final estimates. This is actually

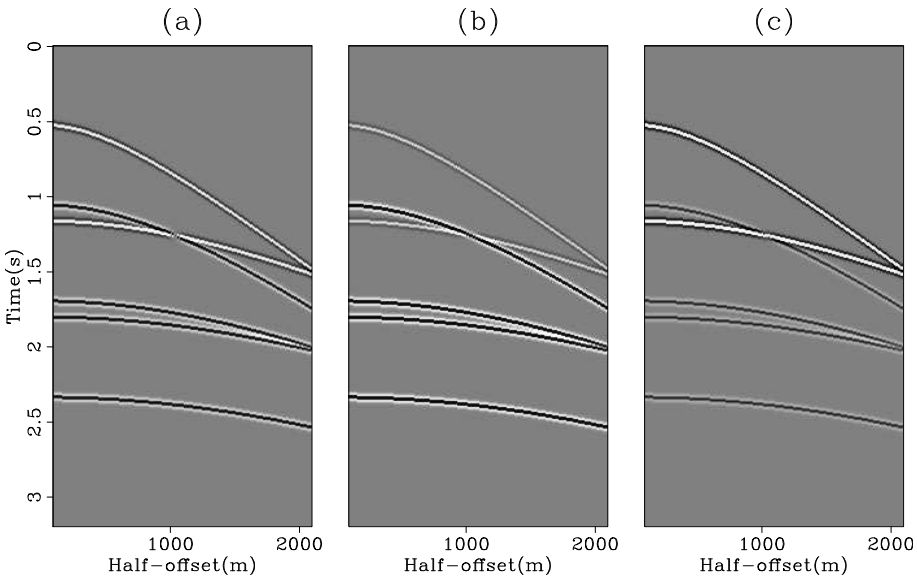

Figure 1: Synthetic CMP gather (a) showing two primaries (black) and four multiples (white) from a three flat-layer model. The initial estimates of multiples (b) and primaries (c) are contaminated with 40% cross-talk.

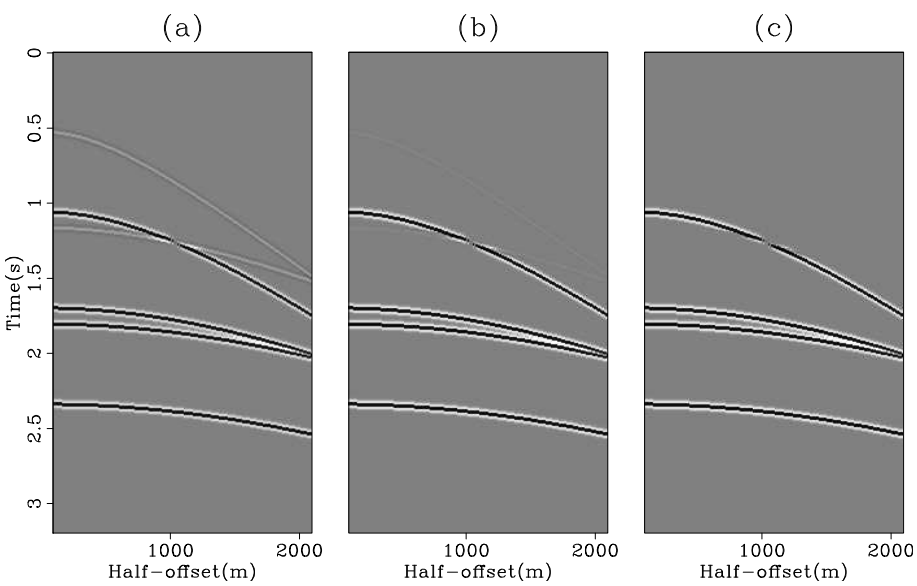

Figure 2: Matched estimates of multiples after one (a), two (b) and three (c) outer iterations of the algorithm.

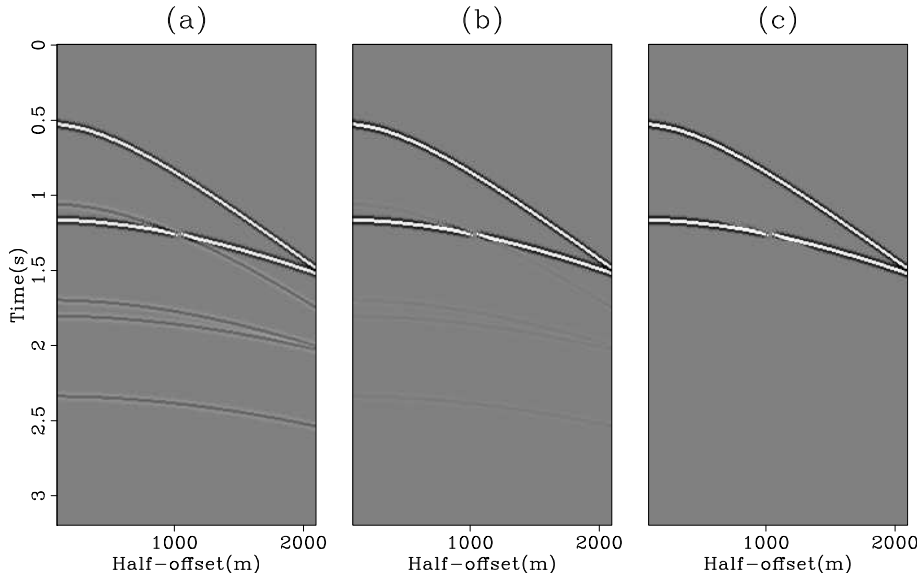

Figure 3: Matched estimates of primaries after one (a), two (b) and three (c) outer iterations of the algorithm.

present in the data (panel (a) in Figure 1) and is an artifact because both primaries and multiples were modeled with the same amplitude and opposite polarity.

Consider now the more realistic situation of kinematic and offset-dependent amplitude errors in the original estimates of both primaries and multiples, as well as noise as shown in Figure 4. The multiple and primary estimates were obtained via migration-demigration as described in **?**. These are imperfect estimates with cross-talk on primaries and multiples and other noises.

Panel (a) of Figures 5 and 6 show the results after one outer iteration, whereas panels (b) and (c) of the same figures show the results after three and five outer iterations respectively. There is still some localized cross-talk from the multiples into the primaries.

The next example uses the well-known Sigsbee model (**?**) to illustrate the method in the image space. For this example, therefore, "data" means the migrated image with primaries and multiples. This dataset has the advantage that, along with the modeled data (primaries and multiples), there exists a related dataset without the free surface multiples (http://www.delphi.tudelft.nl/SMAART/S2Breadme.htm). Panel (a) of Figure 7 shows the modeled data, panel (b) the data without free surface multiples (b), and panel (c) the estimated free surface multiples in the image space computed with an image space version of SRME (**?**). All panels are plotted at the exact same clip value. Notice that the estimate of the multiples is accurate only in kinematics, not in amplitudes or frequency content. The estimate of the multiples was computed with an image space version of SRME (**?**).

In contrast with the previous examples, in this case I do not have an independent

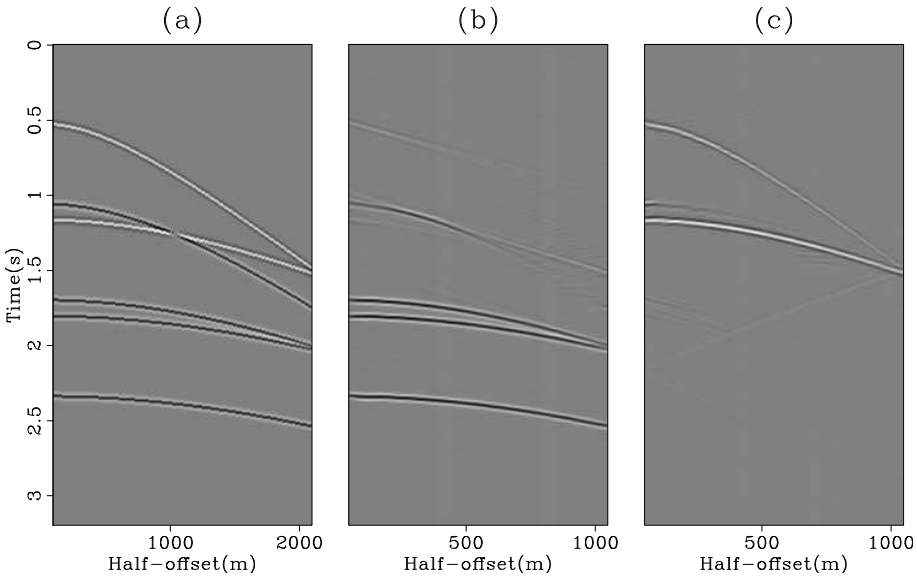

Figure 4: Original CMP gather (a), initial estimate of the multiples (b) and initial estimate of the primaries (c).

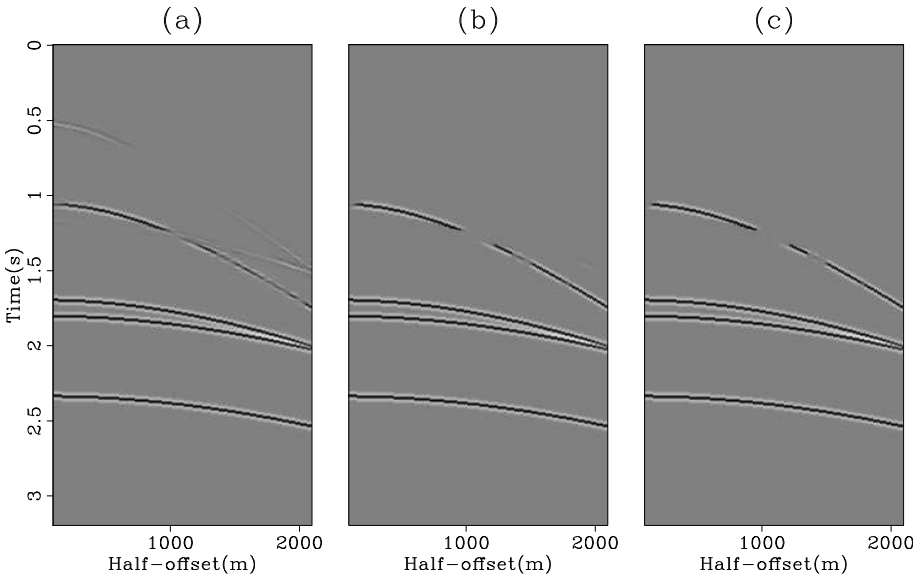

Figure 5: Matched estimates of multiples after one (a), three (b) and five (c) outer iterations of the algorithm.

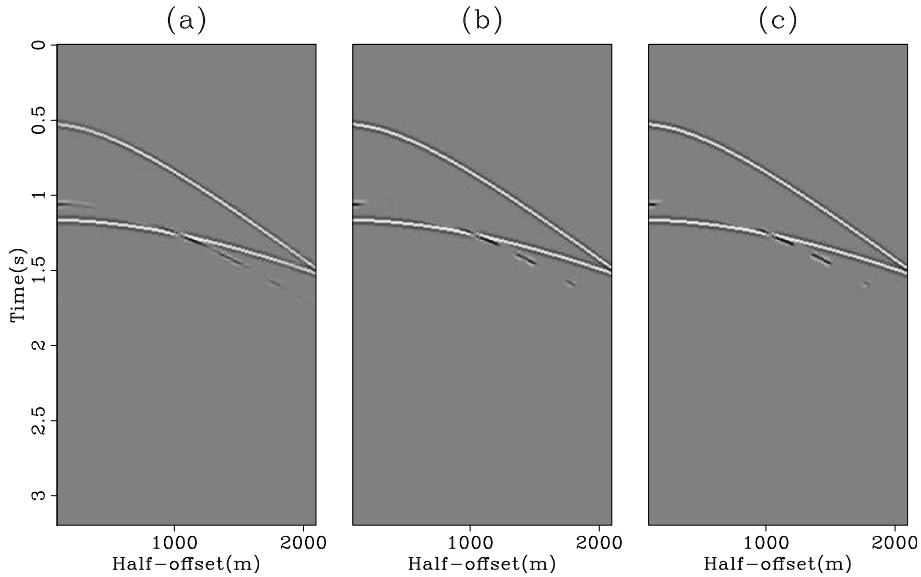

Figure 6: Matched estimates of primaries after one (a), three (b) and five (c) outer iterations of the algorithm.

initial estimate of the primaries. I could subtract the estimate of the multiples from the data, but the corresponding estimate of the primaries is too distorted. Using such a poor primary estimate actually hurts the chances of matching the multiples to the data. Another option is to use the data itself as the initial estimate of the primaries. I found, however, that a better alternative is to do a first iteration setting $\mu = 0$, meaning only the multiples need to be matched. Once matched, the multiples are subtracted from the data to get the estimate of the primaries for the next iteration.

Figures 8 and 9 show a close-up view of the matched primaries and multiples, respectively, after one, two and three outer iterations. After the first iteration, the most obvious multiples contaminating the estimate of the primaries have been attenuated (compare panels (a) of Figures 7 and 8) but strong residual multiple energy remains. The second iteration helps attenuate the multiples further, although it is hard to appreciate in these small figures. See, for example the multiple inside the salt and in the bottom right corner of panel (b). The third iteration cleans up most of the noise, although it also weakens the subsalt primaries.

On the estimate of the multiples, again the first iteration extracts the most significant multiples and the second iteration locally corrects the amplitudes. The third iteration actually hurts the estimate of the multiples because the effect of the regularization term becomes significant as the match of both the primaries and the multiples to the data improves. The net result is an estimate of the primaries that is close to the primaries in the original image. Because of the need for regularization, the estimate of the multiples, however, is weaker than it should.

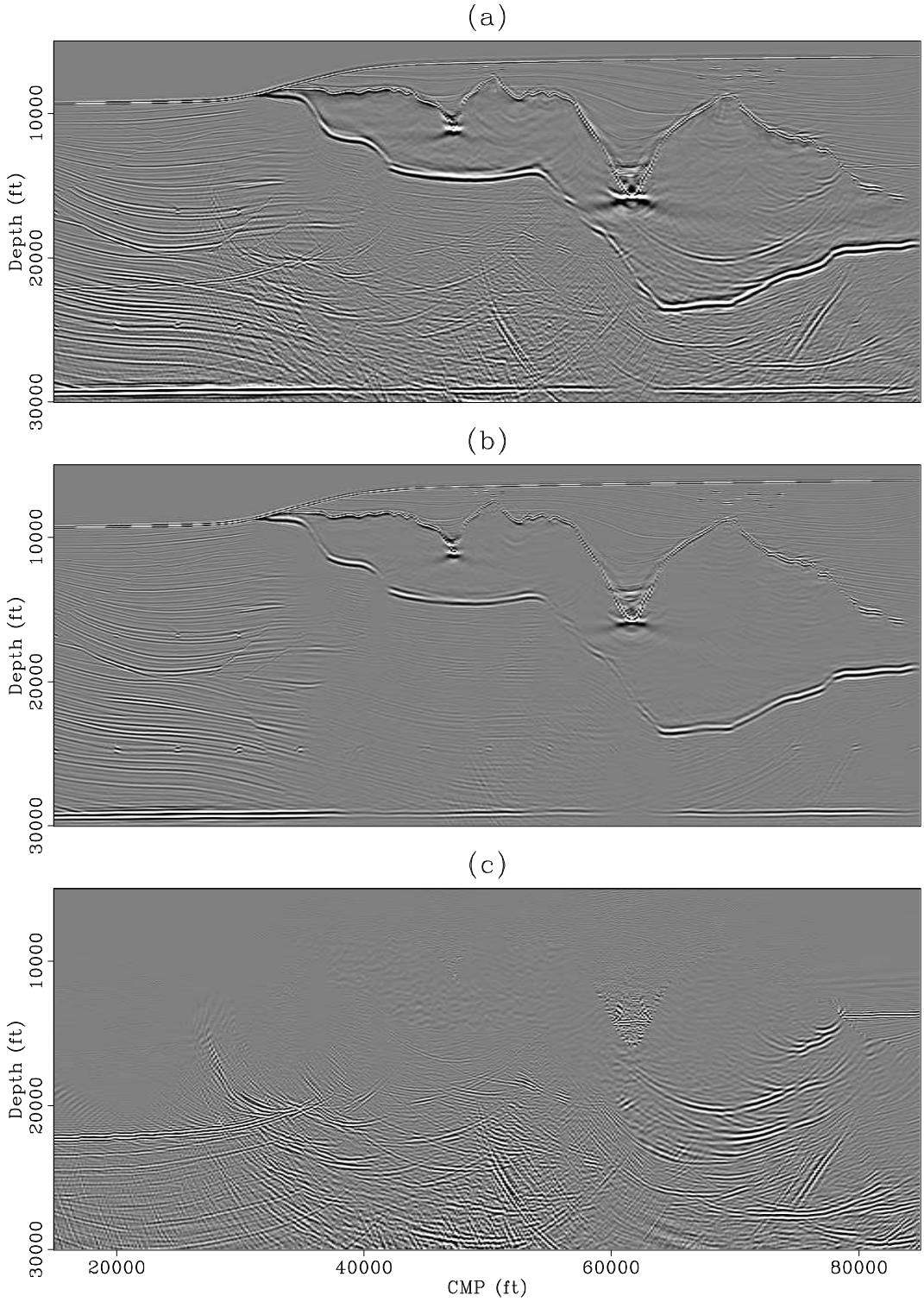

Figure 7: Sigsbee migrated dataset. Data (a), migrated model without surface multiples (b) and initial estimates of multiples (c).

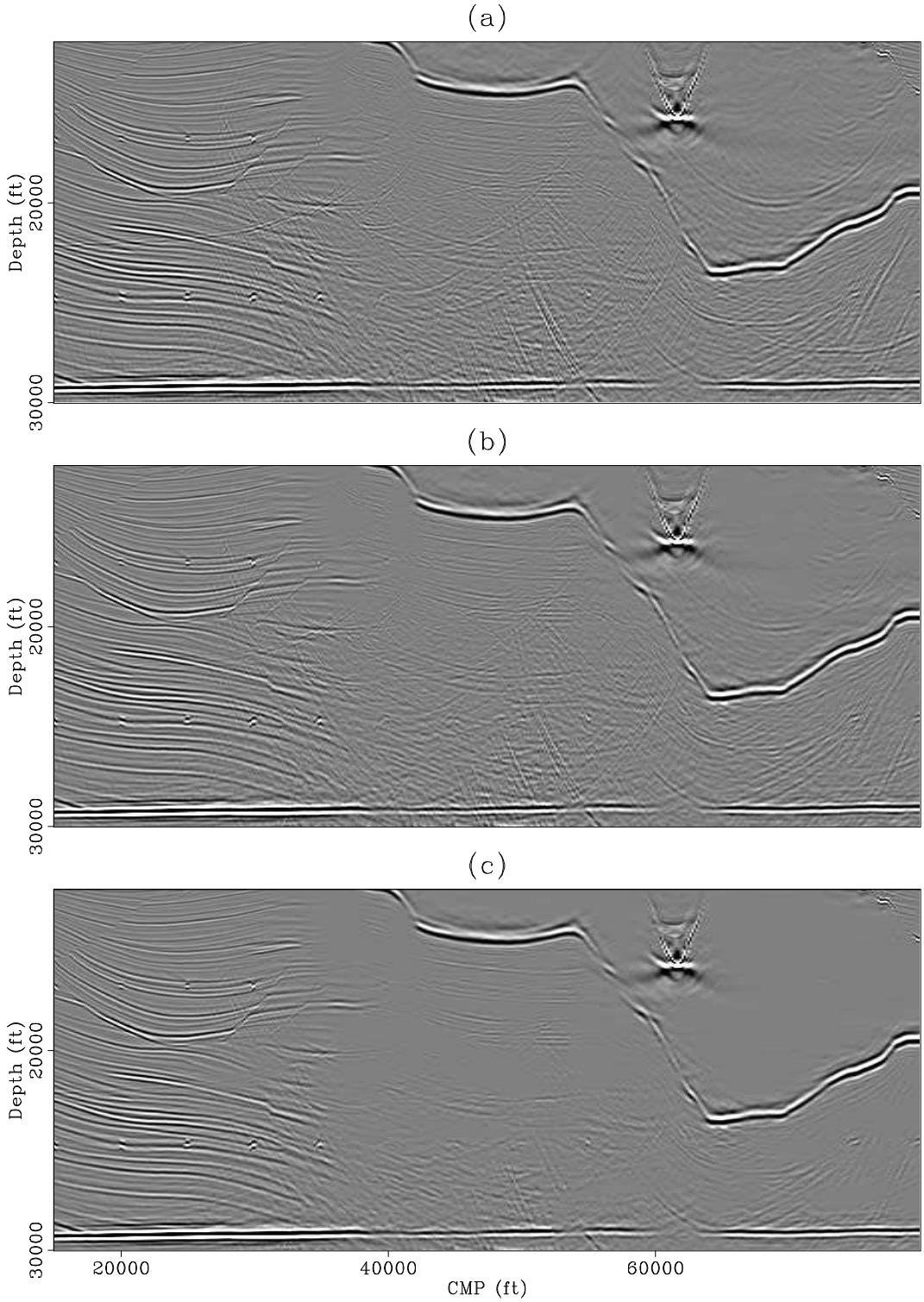

Figure 8: Estimated primaries after one (a), two (b) and three (c) outer iterations of the algorithm.

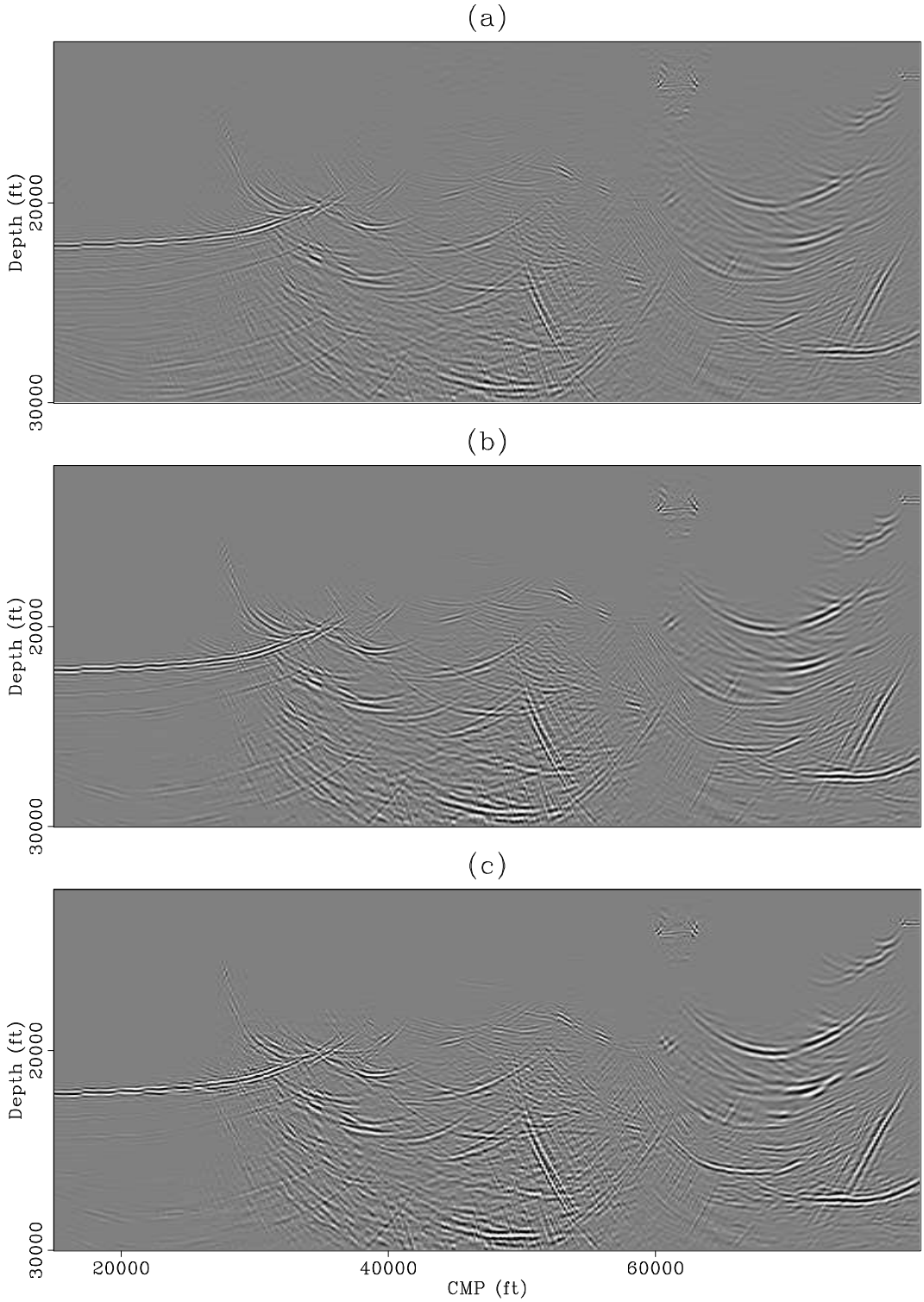

Figure 9: Estimated multiples after one (a), two (b) and three (c) outer iterations of the algorithm.

EXAMPLES WITH REAL DATA

I will now illustrate the method with real data. First, I will match and subtract the multiple model for the real dataset from the Gulf of Mexico introduced in Chapter **??** and then I will show an example of attenuating spatially-aliased ground-roll.

Gulf of Mexico data

I start with the Angle-Domain Common-Image Gather (ADCIG) shown in Figure 10. Panel (a) is the initial data, panel (b) the estimated multiples and panel (c) the estimated primaries.

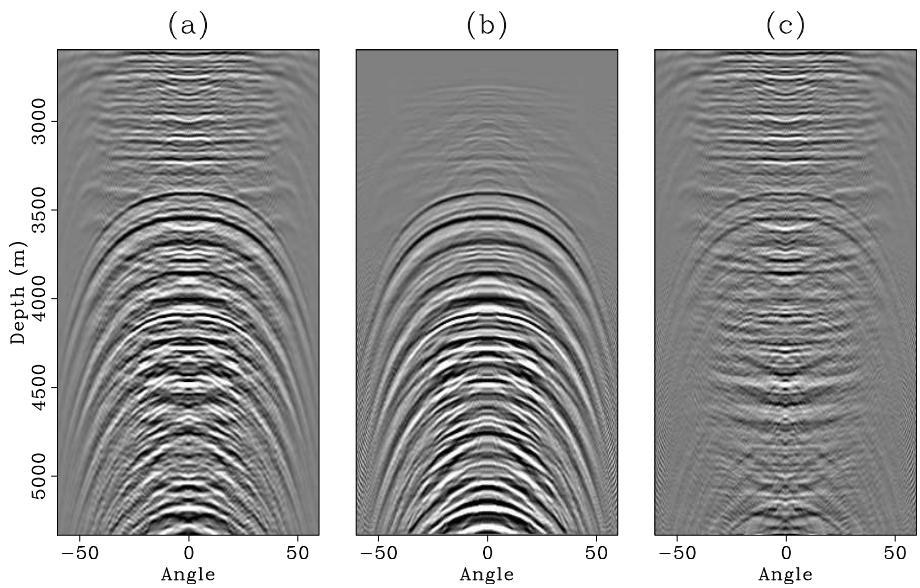

Figure 10: ADCIG from the Gulf of Mexico line of chapter 2 (a), initial estimate of the multiples (b), and the primaries (c). Note the crosstalk on both panels.

The estimate of the multiples was obtained with Radon transform in the image space presented in chapter 2 (without the apex-shift) and the estimate of the primaries was obtained simply by subtracting it from the data. Notice the residual primary energy just below 3000 m in the estimate of the primaries. Note also the residual energy from the multiples in the estimate of the primaries.

Figure 11 shows the ADCIG after one, five and ten outer iterations. The first iteration attenuates the strongest residual multiples (compare panel (a) of Figure 11 with panel (c) of Figure 10). Subsequent iterations further reduce the residual multiples. Also, although hard to see in the hard copy, the primary energy that contaminated the estimate of multiples below 3000 m has been mapped back to the primaries. Figure 12 shows the corresponding results for the multiples. Notice again that the residual primary energy has been severely attenuated.

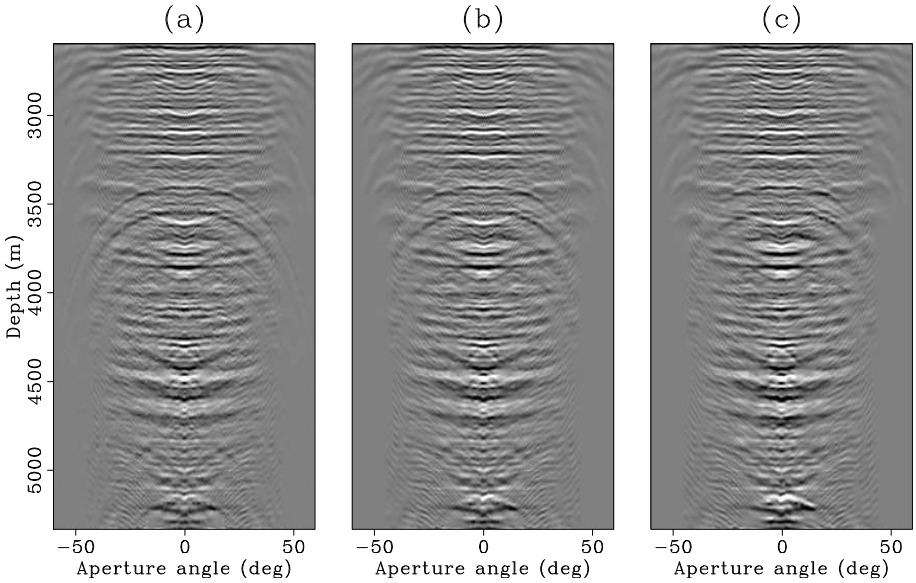

Figure 11: Estimated primaries after one (a), five (b) and ten (c) outer iterations. Notice how the residual multiples decrease with the outer iterations although are not completely eliminated.

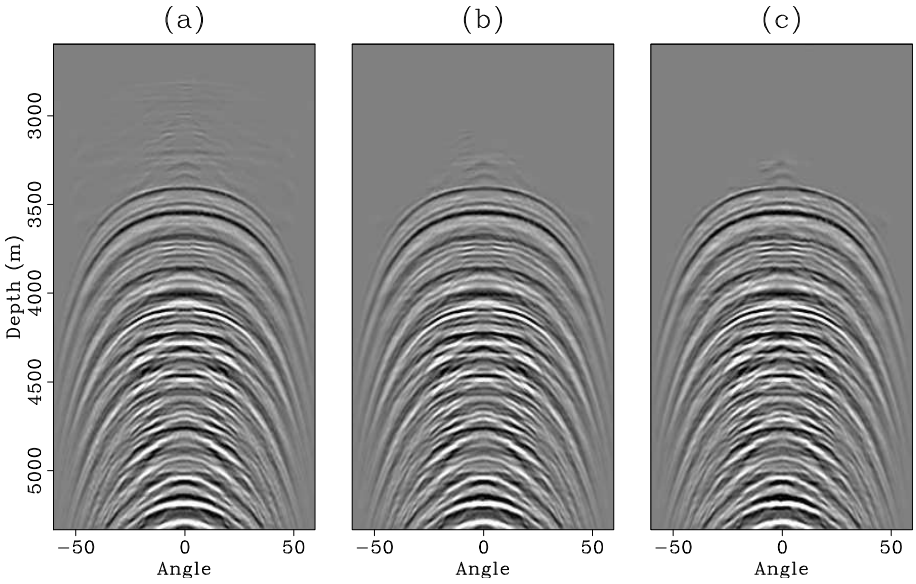

Figure 12: Estimated multiples after one (a), five (b) and ten (c) outer iterations. Here too, the residual primaries decrease and almost disappear after the 10th outer iteration.

To show the impact of the better matching of the multiples in the angle stack, I applied the same steps to all the 900 ADCIGs in the seismic line. Figure 13 shows the angle stack of the data (primaries and multiples) and the angle stack of the initial estimates of the multiples and the primaries. Recall that this initial estimate of the primaries was obtained by direct subtraction of the estimate of the multiples from the data (without adaptive subtraction). All the panels are plot at the exact same clip value. Note that although most multiples have been attenuated some multiple energy remain below the salt body.

In order to concentrate the comparison of the different estimates of the multiples and primaries to the region where the multiples are present, I windowed the data to below 2600 m. Figure 14 shows the comparison between the initial estimate of the multiples (a windowed version of panel (c) in Figure 13) plot at a lower clip (panel (a)) and the results of applying the matching algorithm after one and five outer iterations (panels (b) and (c) respectively). The first outer iteration didn't improve much, but after five outer iterations the result is much better with the specular multiples largely reduced in amplitude. The diffracted multiples still remain because the Radon filtering did not account for the apex-shift in this example.

Ground-roll

As a final example, consider the problem of separating spatially-aliased ground-roll from body waves in land data. This a more challenging application of the algorithm because the body waves have curvature that changes rapidly with both offset and time so to match it I need small filters in relatively small patches. The ground-roll, on the other hand, has little global curvature (although it may have strong local curvature due to aliasing) and matching it is more successful with large filters in large patches. A refinement to the method could use different regularization operators or at least different regularization parameters ϵ on the non-stationary filters for the primaries and the multiples. For the sake of simplicity, I used the same regularization for both.

Figure 15 shows the original shot as well as the initial estimates of the body waves and the ground-roll. The ground-roll estimate was computed simply by high-cut filtering the data to 24 Hz using a Butterworth filter with six poles. I allowed significant energy from the body waves to leak into the estimate of the ground-roll to illustrate the problem described in the previous paragraph. Similarly, the estimate of the body waves was computed by low-cut filtering the data to 18 Hz also with a Butterworth filter with 6 poles. Since I don't want to reduce the low frequency components of the signal too much, I allowed strong ground-roll to leak into the estimate of the body waves. The purpose is to eliminate this ground-roll without hurting the signal and ideally, mapping back some of the body-waves from the estimate of the ground-roll.

Figure 16 shows the estimate of the body-waves after one, five and 10 outer iterations of the proposed algorithm. Even after just the first iteration, most of the ground-roll has been eliminated and after five iterations it is almost gone. For this

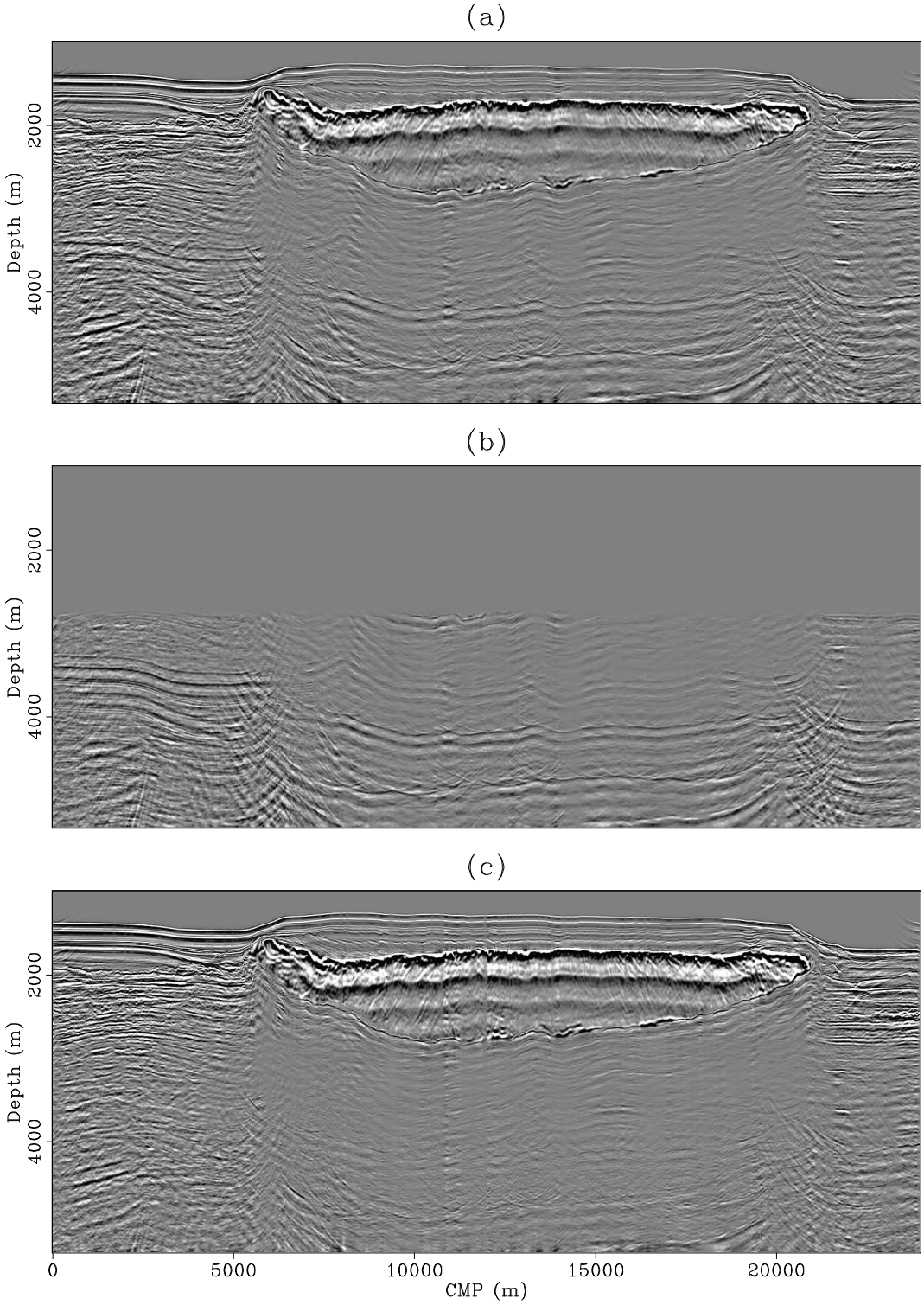

Figure 13: Comparison of angle stacks for the data (panel (a)), the initial estimate of the multiples (panel (b)) and the initial estimate of the primaries (panel (c)).

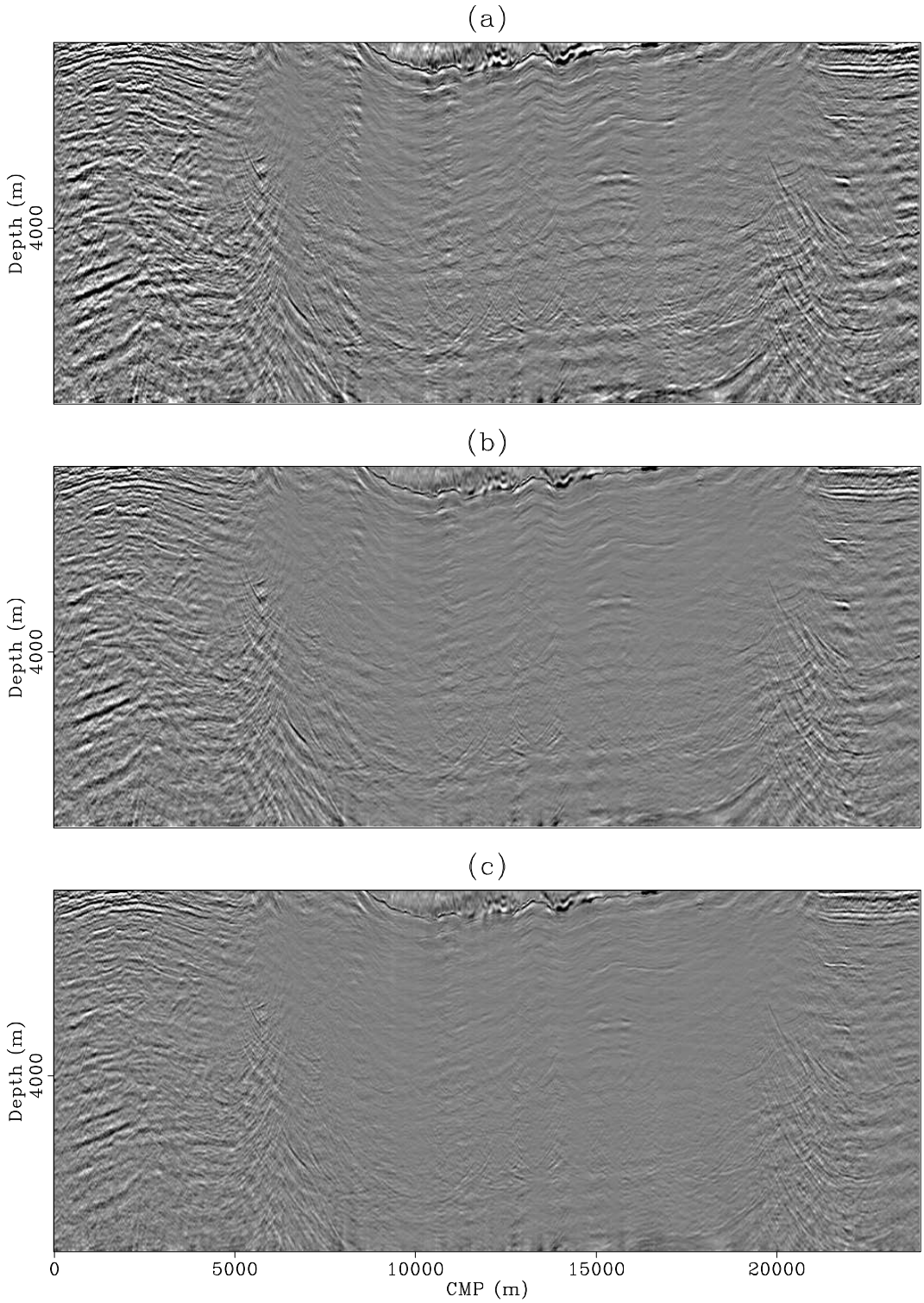

Figure 14: Comparison of windowed angle stacks for the initial estimate of the primaries (panel (a)), the estimate of the primaries after one outer iteration (panel (b)) and after five outer iterations (panel (c)).

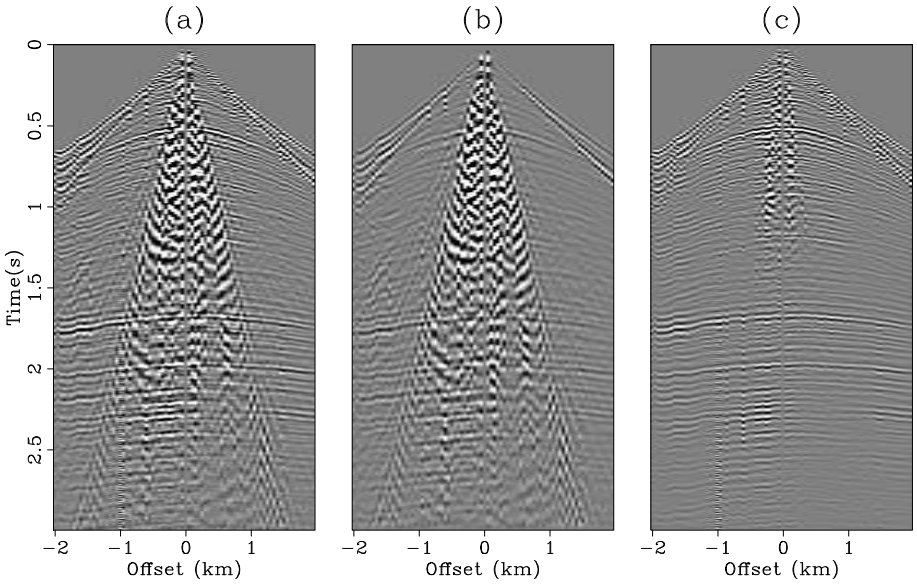

Figure 15: Land shot gather with strong ground-roll (a), initial estimate of ground-roll (b), and body waves (c).

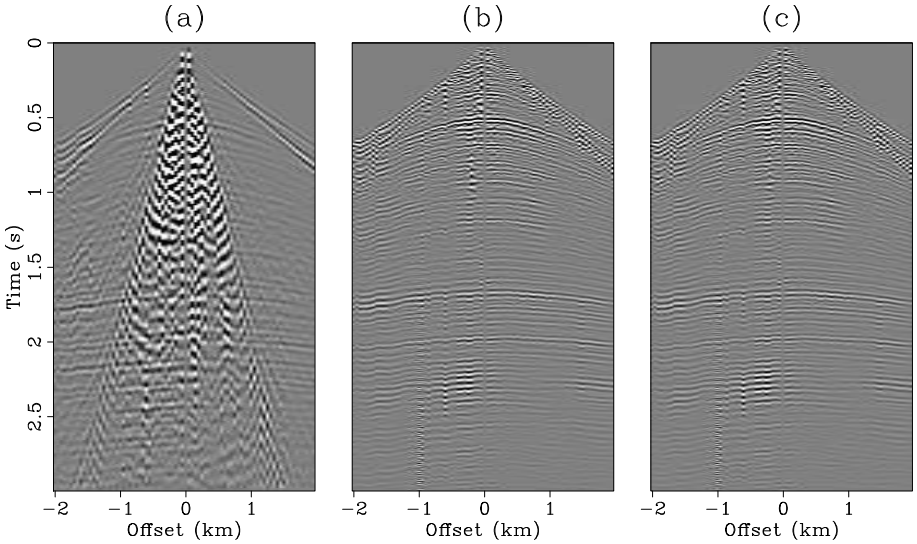

Figure 16: Estimate of body waves after one outer iteration (a), after 5 outer iterations (b) and after 10 outer iterations (c). Notice how after the fifth iteration the ground-roll is essentially gone.

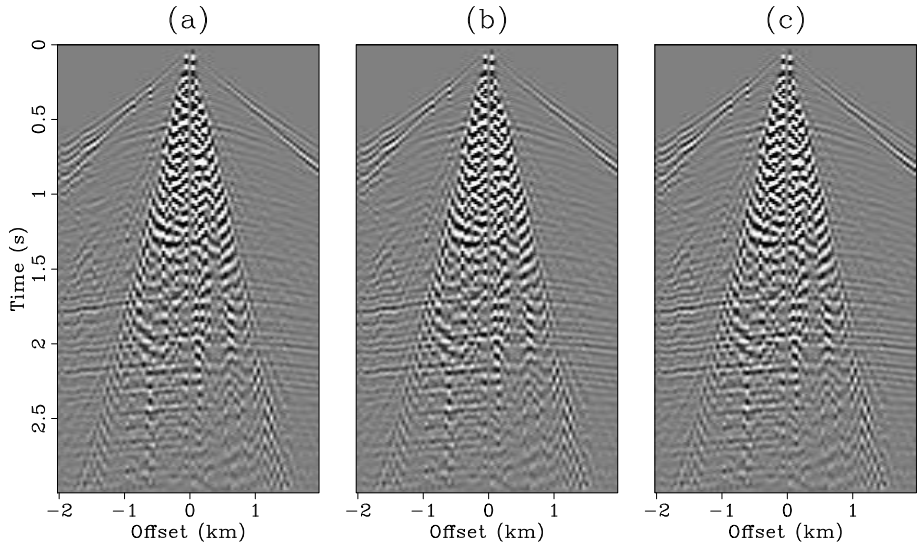

Figure 17: Estimate of ground-roll after one outer iteration (a), after 5 outer iterations (b) and after 10 outer iterations (c). Some of the body waves have been removed in panel (c) but much still remains.

example I used just two patches in time and one in offset. Figure 17 shows similar results for the ground-roll. Since the patches were so large, the energy of the leaked body-waves were only slightly attenuated (see the reflector at about 1.7 secs). This energy was mapped back to the estimate of the body-waves.

DISCUSSION AND CONCLUSIONS

The standard approach to match the estimated multiples directly to the data and obtain the primaries by subtraction of the matched multiples often leads to weakened primaries and/or contamination with residual multiples. By exploiting the estimates of both, multiples and primaries, we prevent the matching algorithm from attempting to match weak residual primaries along with the multiples. Furthermore, we obtain simultaneous estimates of both the primaries and the multiples that are guaranteed to be consistent with the original data.

As with most inversions, the performance of the algorithm depends critically on the choice of the inversion parameters. There are no guaranteed combinations of parameters that work in every case, but we now discuss the most important:

Patch size

The size of the overlapping patches is a function of the non-stationarity of the data. Smaller patches represent rapidly changing data better but are more expensive and

likely to match small patterns of correlated noise. For the examples on multiples we used patches that were just a few samples long (less than 10 in all axes). For the ground-roll example, we used patches 200 samples long in the time axis and 80 samples long in the offset axis.

Non-stationary filter lengths

The length of the filter depends on the character of the noise model and the size of the patch. We found that, for matching the multiples, short filters (2 to 4 samples in each axes) worked well but for the ground-roll example a longer filter gave better results (27 samples in time and 2 in offset) because the nature of the noise and the signal were so different.

Balancing primaries and multiples

The parameter μ in Equation 1 can be used to give more relative weight to the estimates of the primaries or the multiples. In particular, setting it to zero reduces the algorithm to the more standard multiples-only matching. We found that this is not a critical parameter and a value of one works well and was used in all the examples in this paper.

Regularization

As with any inversion problem, the choice of regularization is important to impose constraints of the admissible models. For the multiple matching problem, since the character of both the primaries and the multiples is similar, it is best to use the same regularization for estimating the primary and multiple non-stationary filters. In an example like the ground-roll attenuation, where the character of the signal and the noise is so different, it may be better to use different regularization operator A (or at least a different level of regularization ϵ) for the estimate of the two filter banks. We used the same regularization for the noise and the signal in all of our examples.

Number of outer iterations

The algorithm tends to converge rather quickly, so only a few outer iterations are required (two or three) to get close to a reasonably good answer. For the ground-roll example 10 outer iterations were used.

It should be emphasized that the algorithm, as presented, is independent of the method employed to obtain the initial estimates of the multiples and the primaries. It should also be stressed that the algorithm does not rely on explicit knowledge of

the moveouts of the primaries or the multiples. It only relies on the fact that the data is the sum of the multiples and the primaries. The method can be used not only to match primaries and multiples but in general to match estimates of noise and signal to data containing both. I showed an example with the separation of ground-roll and body-waves with land data, but other applications may also be possible.

ACKNOWLEDGEMENTS

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