Optimized implicit finite-difference migration for tilted TI media-2D synthetic data examples

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INTRODUCTION

Implicit finite-difference method, adaptive to strongly lateral variation and guarantee stability, has been one of the most attractive methods for isotropic media. Traditional finite-difference migration methods are based on the truncation of Taylor series of the dispersion relation. For anisotropic media, PSPI or explicit finite-difference methods are usually used for migratin because the dispersion relation is very complex and it is difficult to derive the Talyor series for finite-difference. Lee and Suh (1985) approximate the square-root equation with rational functions, and optimize the coefficient with least-squares. This method improves the accuracy with the same computational cost. Under the weak anisotropy assumption, Ristow and Ruhl (1997) design an implicit scheme for VTI media. Liu et al. (2005) apply a phase-correction operator (Li, 1991) after the finite-difference operator for VTI media and improve the accuracy. Shan (2006b) approximates the VTI dispersion relation with rational functions and obtains the coefficients using weighted least-squares optimization. Similarly, Shan (2006a) design implicit-finite difference for TTI media by fitting the dispersion relation with rational function and show impulse response in homogeneous media.

In this paper, I review an optimized finite-difference for TTI media and apply it to synthetic datasets to verify the algorithm.

OPTIMIZED FINITE-DIFFERENCE FOR TILTED TI MEDIA

The dispersion relation of tilted TI media can be characterized by a quartic equation as follows:

$$d_4 S_z^4 + d_3 S_z^3 + d_2 S_z^2 + d_1 S_z + d_0 = 0, \tag{1}$$

where the coefficients d_0, d_1, d_2, d_3, and d_4 are as follows:

$$d_0 = (2 + 2\varepsilon\cos^2\varphi - f)S_x^2 - 1 - \left[(1-f)(1+2\varepsilon\cos^2\varphi) + \frac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi\right]S_x^4,$$

$$d_1 = [2\varepsilon(1-f)\sin 2\varphi - f(\varepsilon-\delta)\sin 4\varphi]S_x^3 - 2\varepsilon\sin 2\varphi S_x,$$

$$d_2 = [f(\varepsilon-\delta)\sin^2 2\varphi - 2(1-f)(1+\varepsilon) - 2f(\varepsilon-\delta)\cos^2 2\varphi]S_x^2 + (2 + 2\varepsilon\sin^2\varphi - f),$$

$$d_3 = [f(\varepsilon-\delta)\sin 4\varphi + 2\varepsilon(1-f)\sin 2\varphi]S_x,$$

$$d_4 = f - 1 + 2\varepsilon(f-1)\sin^2\varphi - \frac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi.$$

Therotically, equation 1 can be solved analylitically, but there is no explicit expression for its solution. Therefore, it is difficult to obtain the Taylor series. Figure **??** show how the dispersion relation looks, when the anisotropy parameters $\varepsilon = 0.4$, $\delta = 0.2$ and the tilting angle $\theta = 30°$.

Generally, the Padé approximation suggests that if the function $S_z(S_r) \in C^{n+m}$, then $S_z(S_r)$ can be approximated by a rational function $R_{n,m}(S_r)$:

$$R_{n,m}(S_r) = \frac{P_n(S_r)}{Q_m(S_r)}, \tag{2}$$

where

$$P_n(S_r) = \sum_{i=0}^{n} a_i S_r^i$$

and

$$Q_m(x) = \sum_{i=0}^{m} b_i S_r^i$$

are polynomials of degree n and m, respectively. The coefficients a_i and b_i can be obtained either analytically by Taylor-series analysis or numerically by least-squares fitting.

S_z is an even function of S_x for isotropic and VTI media. In contrast, S_z is not an symmetric function S_x for a tilted TI medium. It's well known that an general function can be decomposed into an even function and an odd function. We can approximate the even part with the even rational functions, such as S_x^2, S_x^4 and approximate the odd part with odd rational functions, such as S_x, S_x^3 To make the finite-difference scheme keep ??? stability, I approximate the dispersion relation of TTI media with rational functions in the shape as follows:

$$S_z(S_x) \approx S_{z0} + \frac{a_1 S_x^2 + c_1 S_x}{1 + b_1 S_x^2} + \frac{a_2 S_x^2 + c_2 S_x}{1 + b_2 S_x^2}, \tag{3}$$

where $S_{z0} = S_z(0)$ and the coefficients $c_1, b_1, a_1, c_2, b_2, a_2$ can be estimated by least-squares methods. They are functions of the anisotropy parameters ε, δ and the tilting angle ϕ. When these parameters vary laterally, the coefficients $c_1, b_1, a_1, c_2, b_2, a_2$ also vary laterally. It is too expensive to run least-squares estimation for each grid point during the wavefield extrapolation. They can be calculated and stored in a table before the wavefield extrapolation. During the wavefield extrapolation, given the anisotropy parameters ε, δ, and the tilting angle ϕ, we search for these coefficients from the table and put them into the finite-difference algorithm. Given the coefficients found from the table, the finite difference algorithm in TTI media is similar to the isotropic media.

2D SYNTHETIC DATA EXAMPLE

Figure **??**, **??**, **??** show the velocity (the velocity in a direction paralleling to the symmetry axis) mode l, the anisotropy parameter ε and the tilting angle of the media. There is an anisotropic layer, which includes VTI part and TTI parts with the tilting angle of 30°, 45° and 60°. Figure **??** shows the image obtained by an isotropic migration. Figure **??** shows the image obtained by anisotropic optimized fini te-difference migration. In Figure **??**, the dipping reflectors ar e not at right position and the flat reflector bellow the anisotropic layer does not focus. These features are fixed in Figure **??** by anisotropic migration.

CONCLUSION

ACKNOWLEDGMENTS

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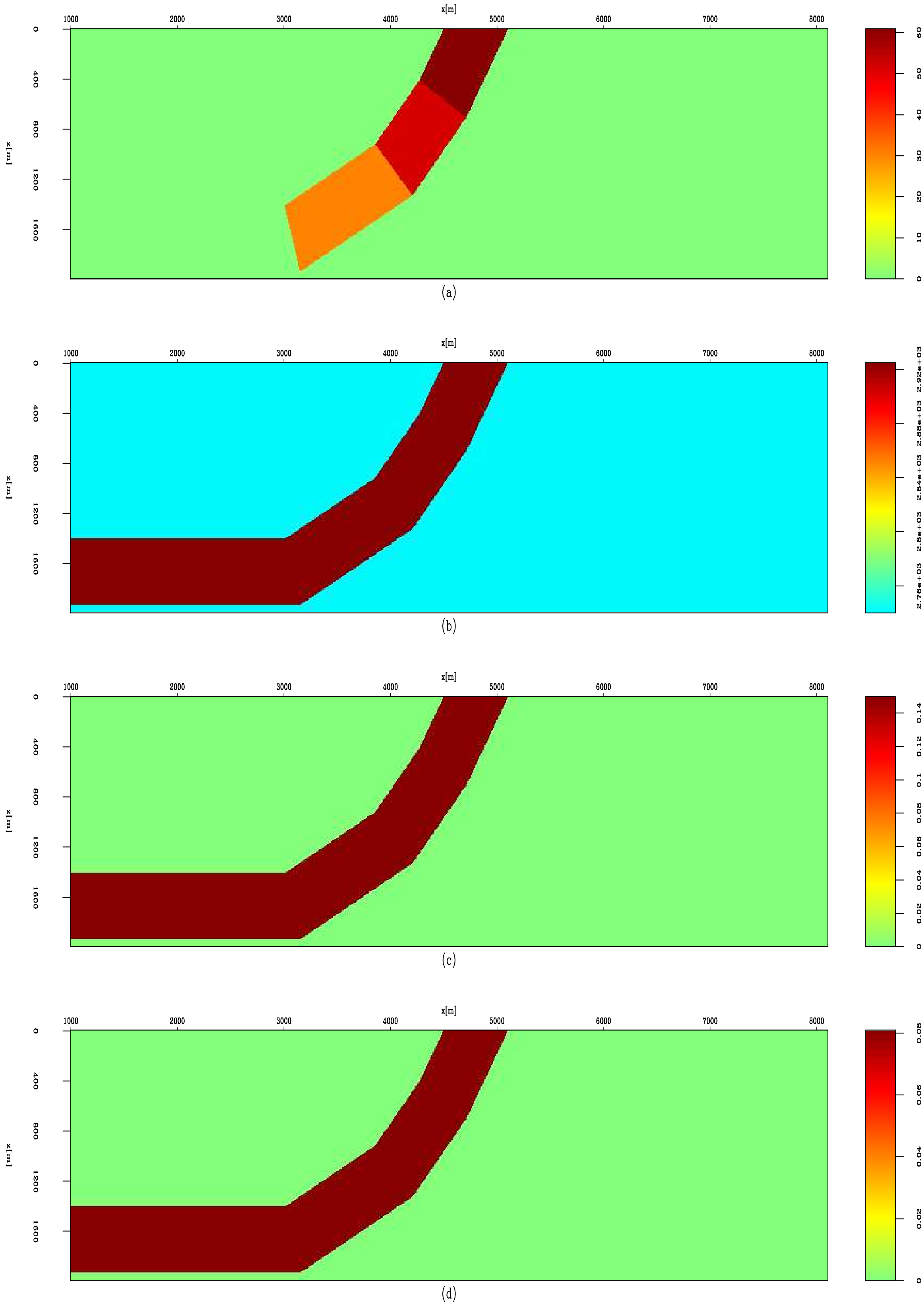
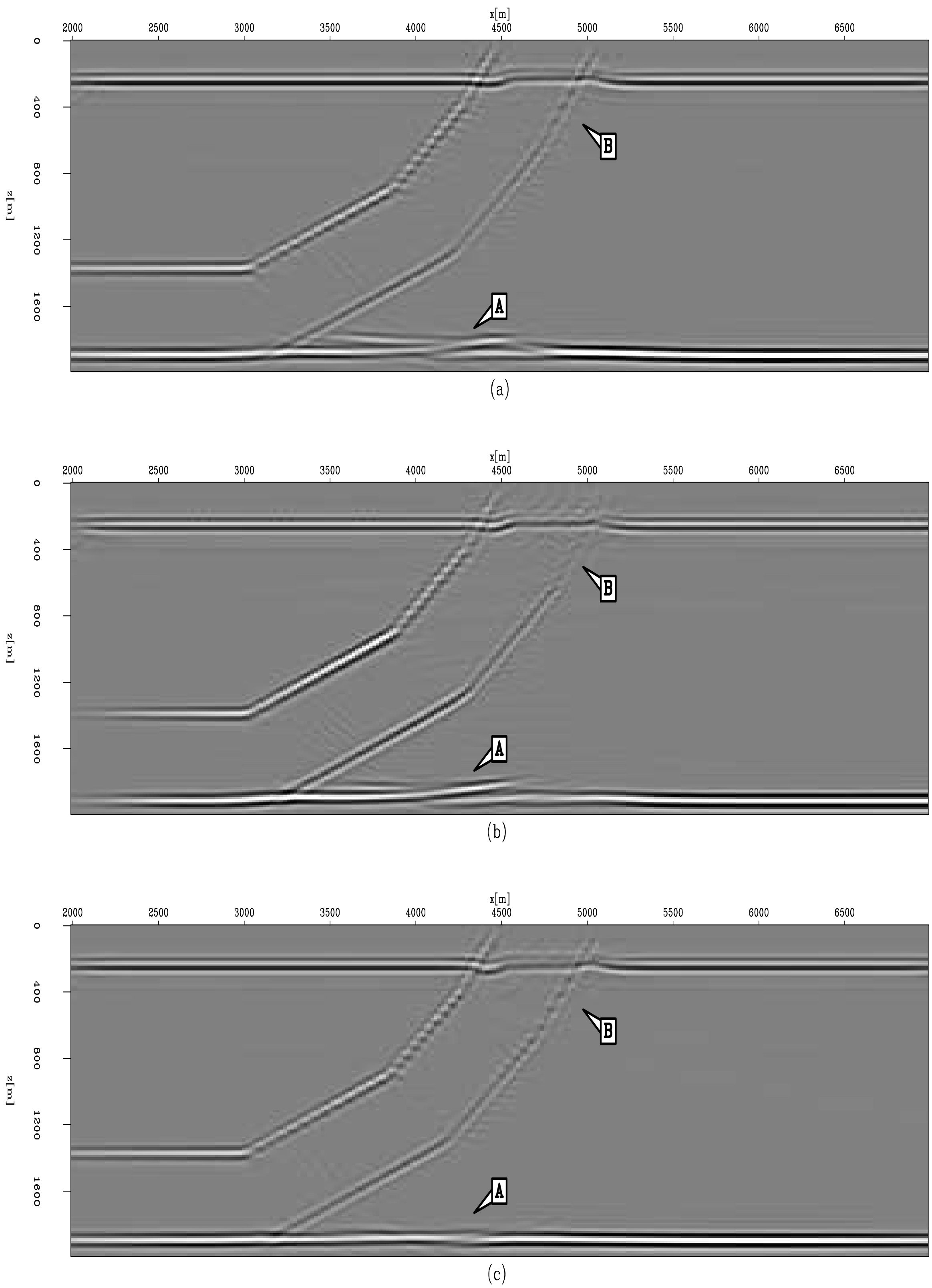


Figure 1: Velocity model $\boxed{\text{model}}$ [ER]



(a)

(b)

(c)

Figure 2: Isotropic migration image [ER]