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Optimized implicit finite-difference migration for TTI media: A 2D synthetic dataset

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ABSTRACT

I review optimized implicit finite-difference migration for tilted TI media. The implicit finite-difference scheme is designed by fitting the dispersion relation with rational functions using least-squares optimization. I apply the method to a synthetic dataset. The result shows that the algorithm can handle laterally varying tilted TI media.

INTRODUCTION

Implicit finite-difference methods that are adapted to strongly laterally varying media and guarantee stability, have been one of the most attractive methods for isotropic media. Traditional implicit finite-difference migration methods are based on the truncation of the Taylor series of the dispersion relation. For anisotropic media, phase-shift plus interpolation (PSPI) methods (Rousseau, 1997; Ferguson and Margrave, 1998) or explicit finite-difference methods (Uzcategui, 1995; Zhang et al., 2001a,b; Baumstein and Anderson, 2003; Shan and Biondi, 2005; Ren et al., 2005) are usually chosen for migration because the dispersion relation of anisotropic media is very complex and it is difficult to derive a Talyor series for the implicit finite-difference scheme with high accuracy. However, TTI (tilted TI) media are not circularly symmetric and a 2D convolution operator is required instead of the McClellan transformations (Hale, 1991) to implement the explicit finite-difference scheme (Shan and Biondi, 2005). Although Lloyd's algorithm can be used to reduce the number of reference velocity and anisotropy parameters in PSPI (Tang and Clapp, 2006), too many reference wavefields are required to achieve decent accuracy in a strongly laterally varying TTI medium.

Lee and Suh (1985) approximate the square-root operator with rational functions and optimize the coefficients by least-squares function fitting. This method improves the accuracy of the finite-difference scheme without increasing the computational cost. Under the weak anisotropy assumption, Ristow and Ruhl (1997) design an implicit finite-difference scheme for VTI (transversely isotropic with a vertical symmetry axis) media. Liu et al. (2005) apply a phase-correction operator (Li, 1991) in the Fourier domain in addition to the implicit finite-difference operator for VTI media and improve the accuracy. Shan (2006b) approximates the dispersion relation of VTI media with rational functions and obtains the coefficients for the finite-difference scheme by using the weighted least-squares optimization. Similarly, Shan (2006a) designs implicit-finite difference scheme for TTI media by fitting the dispersion relation with rational functions and shows impulse responses in a homogeneous medium.

In this paper, I review the optimized implicit finite-difference method for TTI media and apply it to a 2D synthetic dataset to verify the methodology in laterally varying media.

OPTIMIZED FINITE-DIFFERENCE FOR TTI MEDIA

The dispersion relation of TTI media can be characterized by a quartic equation as follows:

$$d_4 S_z^4 + d_3 S_z^3 + d_2 S_z^2 + d_1 S_z + d_0 = 0, \tag{1}$$

where the coefficients d_0, d_1, d_2, d_3, and d_4 are defined as follows:

$$\begin{aligned}
d_0 &= (2+2\varepsilon\cos^2\varphi - f)S_x^2 - 1 - \left[(1-f)(1+2\varepsilon\cos^2\varphi) + \tfrac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi\right] S_x^4,\\
d_1 &= [2\varepsilon(1-f)\sin 2\varphi - f(\varepsilon-\delta)\sin 4\varphi]S_x^3 - 2\varepsilon\sin 2\varphi S_x,\\
d_2 &= [f(\varepsilon-\delta)\sin^2 2\varphi - 2(1-f)(1+\varepsilon) - 2f(\varepsilon-\delta)\cos^2 2\varphi]S_x^2 + (2+2\varepsilon\sin^2\varphi - f),\\
d_3 &= [f(\varepsilon-\delta)\sin 4\varphi + 2\varepsilon(1-f)\sin 2\varphi]S_x,\\
d_4 &= f - 1 + 2\varepsilon(f-1)\sin^2\varphi - \tfrac{f}{2}(\varepsilon-\delta)\sin^2 2\varphi,
\end{aligned}$$

where ε and δ are Thomsen anisotropy parameters (Thomsen, 1986) and φ is the tilting angle of the media. Theoretically, equation 1 can be solved analytically, but there is no explicit analytical expression for its solution. The solid line in Figure 1 shows how the dispersion relation looks, given the anisotropy parameters $\varepsilon = 0.4$, $\delta = 0.2$ and the tilting angle $\varphi = 30°$. Note that S_z is not a symmetric function of S_x. And S_z has two branches when $S_x > 0.8$. One of them represents the up-going waves and the other one represents the down going waves. Therefore, in a TTI medium waves may overturn even though it is homogeneous.

Conventional implicit finite-difference methods are designed by truncating the Taylor series of the dispersion relation. The dispersion relation for TTI media is so complex that it is difficult to derive an analytical Taylor series used for an implicit finite-difference scheme.

Generally, the Padé approximation suggests that if the function $S_z(S_x) \in C^{n+m}$, then $S_z(S_x)$ can be approximated by a rational function $R_{n,m}(S_x)$:

$$R_{n,m}(S_x) = \frac{P_n(S_x)}{Q_m(S_x)}, \tag{2}$$

where

$$P_n(S_x) = \sum_{i=0}^{n} a_i S_x^i$$

and

$$Q_m(x) = \sum_{i=0}^{m} b_i S_x^i$$

are polynomials of degree n and m, respectively. The coefficients a_i and b_i can be obtained either analytically by Taylor-series analysis or numerically by least-squares fitting.

S_z is an even function of S_x for isotropic and VTI media. In contrast, S_z is not an symmetric function of S_x for TTI media. It's well known that an general function can be decomposed into an even function and an odd function. We approximate the even part of the dispersion with the even rational functions, such as S_x^2, S_x^4 and approximate the odd part with odd rational functions, such as S_x, S_x^3.

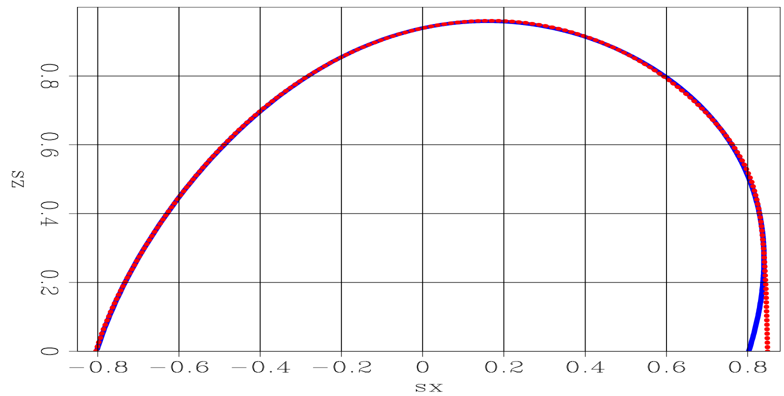

Figure 1: Comparison of the true and approximate dispersion relations for a TTI medium with $\varepsilon = 0.4$, $\delta = 0.2$ and $\varphi = 30°$: the solid line is the true dispersion relation for TTI media; the dashed line is the approximate dispersion relation for finite-difference scheme. The dispersion relation for the finite-difference scheme is very close to the true one for negative S_x. When the phase-angle is close to 90° or more than 90° for the positive S_x, the dispersion for the finite-difference scheme diverge from the true one. guojian2-dispersion [ER]

Considering the stability of the finite-difference scheme, I approximate the dispersion relation of TTI media with rational functions as follows:

$$S_z(S_x) \approx S_{z0} + \frac{a_1 S_x^2 + c_1 S_x}{1 + b_1 S_x^2} + \frac{a_2 S_x^2 + c_2 S_x}{1 + b_2 S_x^2}, \tag{3}$$

where $S_{z0} = S_z(0)$ and the coefficients $a_1, b_1, c_1, a_2, b_2, c_2$ are estimated by least-squares optimization. They are functions of the anisotropy parameters ε, δ and the tilting angle φ. Figure 1 compares the true dispersion relation with the approximate dispersion relation. The solid line is the true dispersion relation (equation 1) and the dashed line is the approximate dispersion relation for the finite-difference scheme (equation 3). The dispersion relation for the finite-difference scheme is very close to the true one for the negative S_x. When the phase-angle is close to 90° or more than 90° for positive S_x, the dispersion for the finite-difference scheme diverges from the true one. Figure 2 shows the relative dispersion error defined as follows:

$$E(S_x) = \frac{S_z^{fd}(S_x) - S_z^{true}(S_x)}{S_z^{true}(S_x)}, \tag{4}$$

where $S_z^{true}(S_x)$ is the value of S_z calculated from equation 1 and $S_z^{fd}(S_x)$ is the value of S_z from equation 3 using the coefficients from the least-squares estimation.

For a laterally varying medium, the anisotropy parameters vary laterally. As a consequence, the coefficients for the finite-difference scheme vary laterally. It is too expensive to

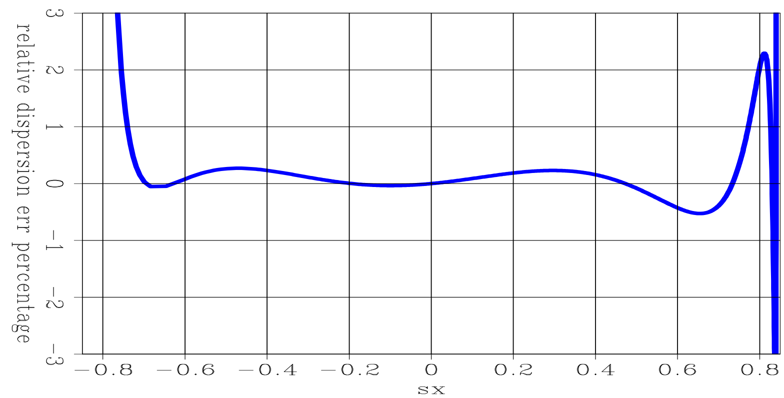

Figure 2: Relative dispersion relation error of finite-difference approximation for a TTI medium with $\varepsilon = 0.4$, $\delta = 0.2$ and $\varphi = 30°$. guojian2-err [ER]

estimate these coefficients for each discrete grid during the wavefield extrapolation. After estimating the minimum and maximum value of the anisotropy parameters and the tilting angle, I compute the coefficients for the anisotropy parameters and the tilting angle in these ranges and store them in a table before the migration. During the wavefield extrapolation, given the anisotropy parameters ε, δ, and the tilting angle φ, I search the coefficients in the table and put them into the finite-difference scheme. Given the coefficients found from the table, the finite difference algorithm in TTI media is the same as the isotropic media. The table of coefficients is small, and the computation cost for table-searching is trivial compared to that of solving the finite-difference equation. Therefore, the cost of the optimized implicit finite-difference for TTI media is similar to that of the conventional finite-difference methods for isotropic media.

2D SYNTHETIC DATA EXAMPLE

Figure 3 shows an anisotropic model with a thrust sheet embedded in the isotropic background (Fei et al., 1998). Figure 3(a) shows the tilting angle of the thrust, Figure 3(b)-(d) show the velocity, the anisotropy parameters ε and δ of the model, respectively. This model represents the thrust shale layer usually seen in the Canadian Foothills. In the thrust sheet, the anisotropy parameter ε is 0.224, the anisotropy parameter δ is 0.10, and the velocity (in the direction paralleling to the symmetry axis) is 2925m/s. The background velocity is 2740m/s. The tilting angles of the anisotropic layer are 0°, 30°, 45° and 60°. There are 86 shots recorded with a split-spread geometry.

Figure 4 compares images of the synthetic dataset. Figure 4(a) is the image migrated by using an isotropic migration, Figure 4(b) is the image obtained by an anisotropic migration regarding the model as VTI media (Shan, 2006b), and Figure 4(c) is the image obtained by an anisotropic migration for TTI media. In Figure 4(a) and (b), the low boundary of the thrust sheet are not at the right position and the flat reflector does not focus at the right position in the area below the thrust sheet (at "A"). These features are imaged well in Figure 4(c) by the anisotropic migration for TTI media.

At "B", the low boundary of the 60° thrust sheet is better imaged in the isotropic migration (Figure 4(a)), compared to the migration for VTI media (Figure 4(b)). For the high-angle energy in a TTI medium with a large tilting angle, the velocity of the waves is close to the velocity in the symmetry-axis direction. When we regard the medium as a VTI medium, for the high-angle energy we use the velocity close to the velocity in the direction normal to the symmetry axis. In contrast, we use the velocity paralleling the symmetry axis in the isotropic migration. That is why the low boundary of the 60° thrust sheet at "B" is better imaged by the isotropic migration compared to the anisotropic migration for VTI media.

CONCLUSION

I present the optimized implicit finite-difference method for wavefield extrapolation in TTI media. The 2D synthetic dataset shows that the algorithm is stable and works for laterally varying media.

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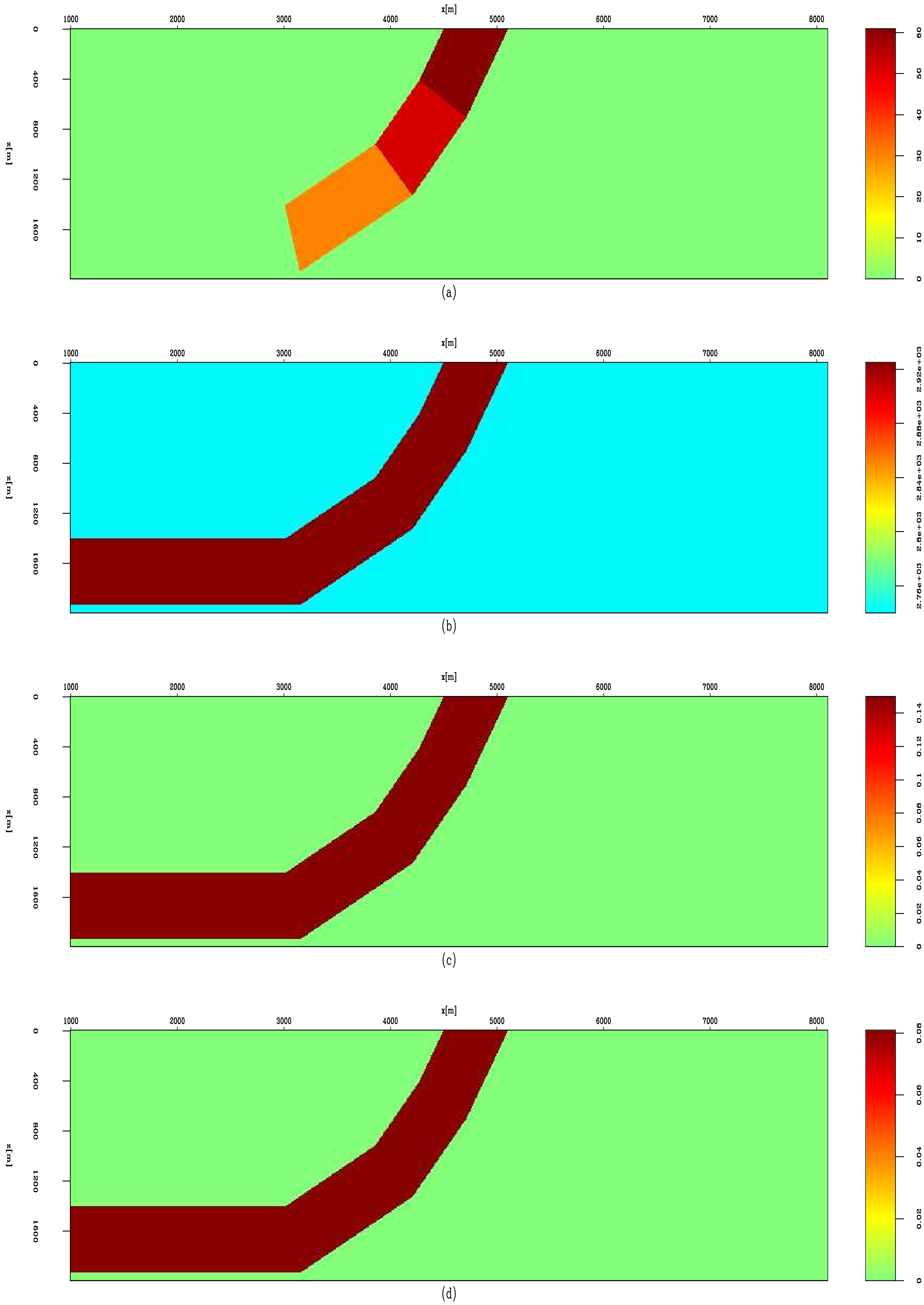

Figure 3: The velocity model and anisotropy parameters: (a) the tilting angle of the TTI medium; (b) the velocity paralleling the symmetry axis; (c) the anisotropy parameter ε; (d) the anisotropy parameter δ. guojian2-model [ER]

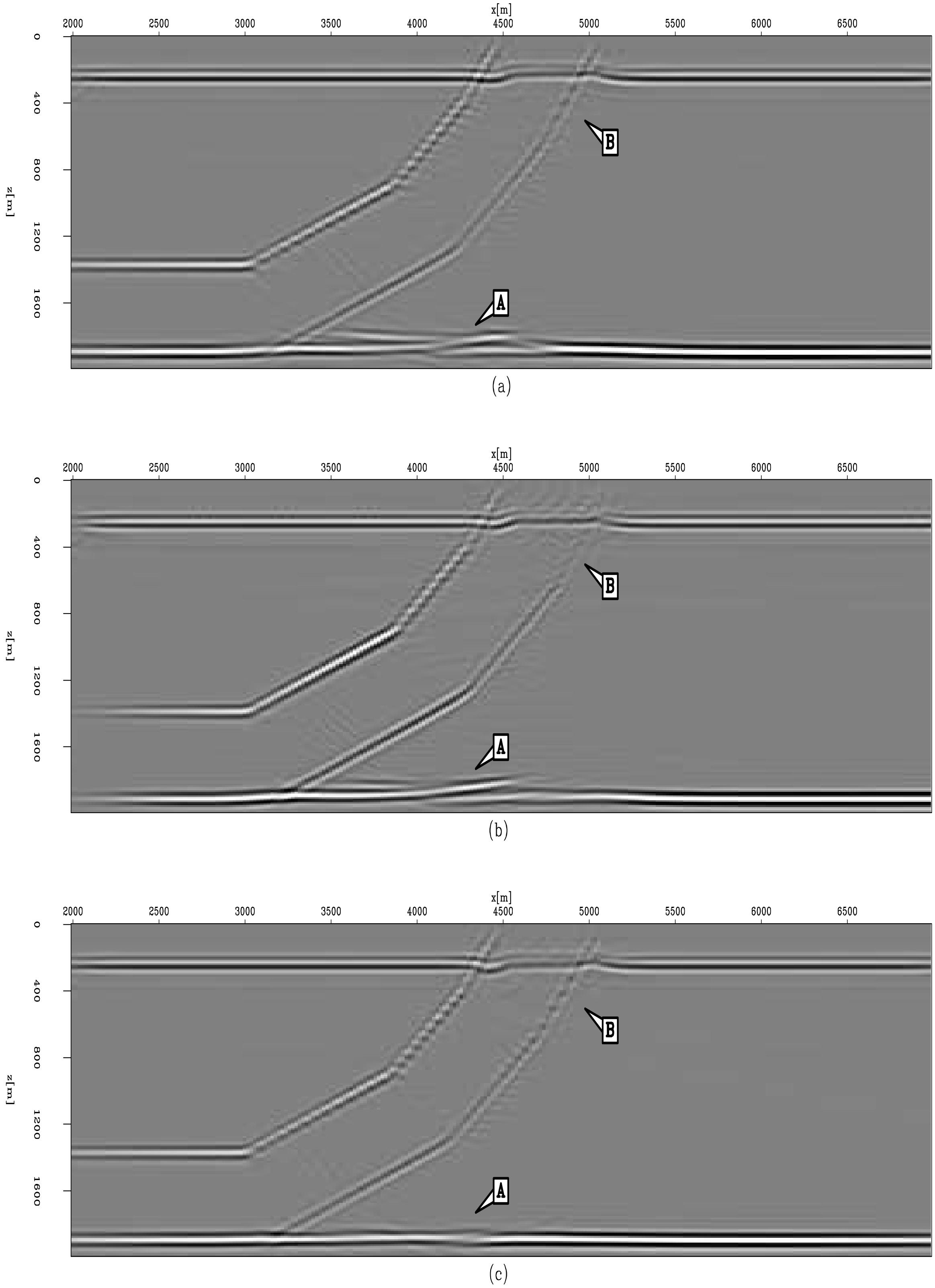

Figure 4: Image comparisons: (a) the image obtained by the isotropic migration; (b) the image obtained by anisotropic migration for VTI media; (c) the image obtained by anisotropic migration for TTI media. guojian2-image [CR]