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Short Note

Moveout analysis with flattening

Robert G. Clapp

INTRODUCTION

Moveout analysis is an important component of many data processing steps. The most basic application is Normal Moveout (NMO), where curvature is related to velocity. Radon based multiple removal techniques take advantage of the differing moveout of primaries and multiples (Hampson, 1986). Migration velocity analysis often measures curvature in common reflection point gathers in either the offset domain or angle domain.

Moveout analysis involves testing potential trajectories by applying a series of moveout functions. A new domain is created where one (or more) of the axes is now a moveout parameter. The creation of this domain can be setup as an adjoint operation, inverse problem ((Lumley et al., 1994; Guitton and Symes, 1999), or in terms of semblance analysis. Often we want to choose a single parameter at each time (or depth) that accurately represents the moveout at the time (or depth). Unfortunately this a non-linear problem. Toldi (1985) and Symes and Carazzone (1991) discuss ways of linearizing the problem. The problem becomes more complicated if we wish to describe moveout by more than a single parameter. The volume formed by scanning over multiple moveout parameters results in very large model spaces. Previous authors have suggested sparse inversion techniques (Alvarez, 2006), or successive scanning.

Another approach to the problem is to use dip information to gain moveout information. Wolf et al. (2004) suggested applying a rough NMO correction then estimating the median of the implied v_{rms} from the dip information at a given zero offset traveltime τ. Guitton et al. (2004) built more directly on the flattening work of Lomask and Claerbout (2002); Lomask and Guitton (2006); Lomask (2006). Guitton et al. (2004) τ-based tomography problem (Clapp, 2001) based on the time-shifts calculated from flattening the data. The advantage of this formulation is that picking becomes unnecessary. The problem with these approaches, when applied to moveout analysis, is the non-linear nature of flattening can easily lead to unrealistic local minima and may not converge to a satisfactory result.

In this paper I also take advantage of the power of flattening while attempting to avoid its pitfalls by limiting the model space. The first approach is to set up an inverse problem from the time shifts needed to flatten a series of Common Reflection Point (CRP). I first invert for a single parameter at each depth, and then two parameters. In the second approach I set up a non-linear inverse problem that relates dips directly to velocity. Both techniques show promise, but additional work is needed.

CHARACTERIZING RESIDUAL MOVEOUT

In migration velocity analysis there is always a debate on the best method to describe the moveout seen in CRP gathers. One approach is to measure and invert the moveout at many depths in each offset/angle of the CRP gather independently. Using this approach, complex residual moveouts can be accurately described. The downside is that these estimates are more prone to noise (cycle skipping for example). The other extreme is to use a single parameter (at many depth locations) that best describes the moveout as a function of offset or angle. A single parameter is more robust but in complex situations may not accurately describe moveout. In addition, selecting the parameter usually involves selecting the moveout with the maximum semblance at a given depth, a non-linear problem that can often lead to unrealistic solution if not properly handled (Clapp et al., 1998).

Often a good compromise between robustness and flexibity is to describe moveout with two parameters. Unfortunately, selecting these two linked parameters is more problematic than the single parameter approach. One approach is to scan over both parameters at all desired depths, for every CRP, and pick the maximum. In addition to being costly, this approach makes picking a consistent and spatially realistic model very challenging. A potentially better approach is outlined in Harlan (1998). He suggests a dual scanning approach: scan over the first-order term fitting the outer offsets, then scanning over the second-order term to best fit the middle offsets. This approach is more efficient than scanning over the entire model space. The dual scanning approach amounts to linearizing the problem arround the first order term, with all of the associated linearization drawbacks. In additional spatial consistency is also problematic.

A general weakness of the scanning approach is that moveout is being determined from semblance amplitude. Flattening offers an interesting alternative to the scanning approach. Flattening inverts for a time shift field (moveout). By incorporating an operator that estimates moveout parameters from time shift field, arbirtrary moveout descriptions can be estimted from dip.

FLATTENING REVIEW

The basic idea behind flattening (Lomask, 2006) is that the gradient measured at a time (or depth) horizon $\boldsymbol{\tau}$ is equal to the dip $\mathbf{p}$ measured at each point of the horizon.

$$\nabla \boldsymbol{\tau}(x,y,t) \quad = \quad \mathbf{p}(x,y,\boldsymbol{\tau}). \tag{1}$$

In order to obtain smoothness between horizons a regularization term is added to the problem. Defining the 3-D gradient operator as $\nabla = [\frac{\partial}{\partial x} \ \ \frac{\partial}{\partial y} \ \ \frac{\partial}{\partial t}]^T$ a new system of equations can be built,

$$\mathbf{W}_\epsilon \nabla = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \epsilon\mathbf{I} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial t} \end{bmatrix} \quad , \tag{2}$$

where $\mathbf{I}$ is the identity matrix and ϵ is a scaling parameter. The residual is defined as

$$\mathbf{r} \quad = \quad \mathbf{W}_\epsilon \nabla \boldsymbol{\tau} - \mathbf{p} \quad = \quad \begin{bmatrix} \frac{\partial \boldsymbol{\tau}}{\partial x} \\ \frac{\partial \boldsymbol{\tau}}{\partial y} \\ \epsilon \frac{\partial \boldsymbol{\tau}}{\partial t} \end{bmatrix} \quad - \quad \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ \mathbf{0} \end{bmatrix}. \tag{3}$$

The dips need to be measured along the horizon, making the problem non-linear. A Gauss-Newton approach can be used with linearizing about the current estimated horizon volume. Again following the approach of (Lomask, 2006), we

iterate {

$$\mathbf{r} = [\mathbf{W}_\epsilon \nabla \boldsymbol{\tau}_k - \mathbf{p}(x, y, \boldsymbol{\tau}_k)] \tag{4}$$

$$\Delta \boldsymbol{\tau} = ((\mathbf{W}_\epsilon \nabla)^T \mathbf{W}_\epsilon \nabla)^{-1} (\mathbf{W}_\epsilon \nabla)^T \mathbf{r} \tag{5}$$

$$\boldsymbol{\tau}_{k+1} = \boldsymbol{\tau}_k + \Delta \boldsymbol{\tau} \tag{6}$$

} ,

where the subscript k denotes the iteration number.

Two different approaches can be used for the linearized step (equation 5) The most efficient is to solve the problem a direct inverse in Fourier domain (Lomask, 2003). When space-domain weighting or model restriction (Lomask and Guitton, 2006) is needed, a space-domain conjugate gradient approach is warranted.

In general we deal with 2-D angle or offset gathers. The standard approach is to solve a 2-D flattening problem where $\boldsymbol{\tau}$ is a function of time/depth and offset. We revert to a 2-D gradient operator, and solve each CMP/CRP gather independently.

POST-FLATTENING INVERSION

There are two general approaches to calculating moveout parameters using the flattening methodology. The first approach is to perform parameter estimation in two phases. First, solve for the *non-linear* $\boldsymbol{\tau}$ field, then construct a linear problem to find the moveout parameters that best fit the $\boldsymbol{\tau}$ field.

The flattening algorithm provides a time-shift $\boldsymbol{\tau}$ field that is function of depth z, offset h, and CRP $\mathbf{x}$. As a first test we want to estimate moveout of a volume migrated using downward continuation migration. Biondi and Symes (2003) demonstrated that residual moveout Δz can be approximated (assuming zero geologic dip) as a function of angle θ and depth z through

$$\Delta z = z\rho \tan(\theta)^2, \tag{7}$$

where ρ is the moveout paremeter. We can estimate $\rho(z,\mathbf{z})$ as a global inverse problem. Defining the above moveout equation above as $\mathbf{B}$mwe obtain the objective function Q,

$$Q(\boldsymbol{\rho}) = |\boldsymbol{\tau} - \mathbf{B}\boldsymbol{\rho}|^{\mathbf{2}}. \tag{8}$$

We can ensure spatial smoothness by introducing a roughener $\mathbf{A}$ to the objective function to obtain,

$$Q(\boldsymbol{\rho}) = |\boldsymbol{\tau} - \mathbf{B}\boldsymbol{\rho}|^{\mathbf{2}} + \epsilon^{\mathbf{2}}|\mathbf{A}\boldsymbol{\rho}|^{\mathbf{2}}, \tag{9}$$

where ϵ is scaling parameter.

To test the methodology I migrated a line from a 3-D North Sea dataset. Figure 1 displays two cross-sections of the migrated data (left) and the $\boldsymbol{\tau}$ field (right) calculated from the volume. A moveout field $\boldsymbol{\rho}$ is then calculated from the $\boldsymbol{\tau}$ field using a conjugate gradient algorithm to minimize equation 9. Figure 2 shows the resulting moveout field. The inversion approach has an additional advantage, it easy to assess where the moveout parameterization effectively described the time shifts and where it failed. Figure 3 shows the result of stacking the absolute value of the residual over the offset plane. Areas of high amplitude represent areas where a single parameter did not accurately describe $\boldsymbol{\tau}$.

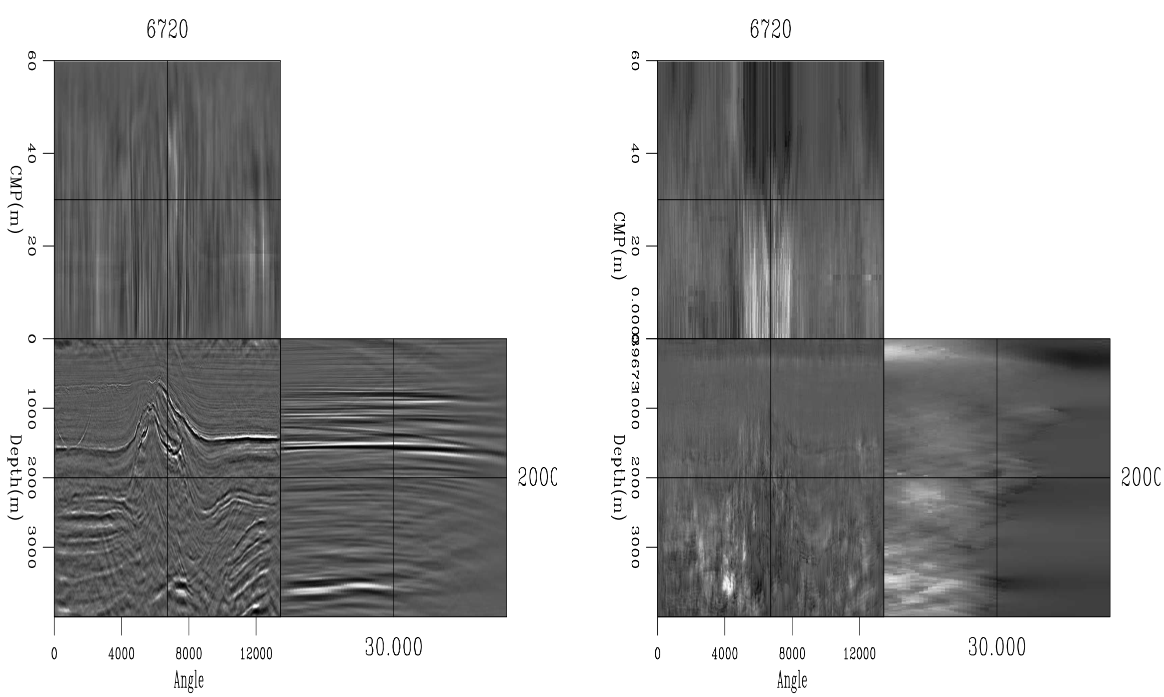

Figure 1: The left panel shows three cross-sections of the migrated image (depth, inline, angle). The right panel shows the time shifts calculated from the volume. bob3-data [ER]

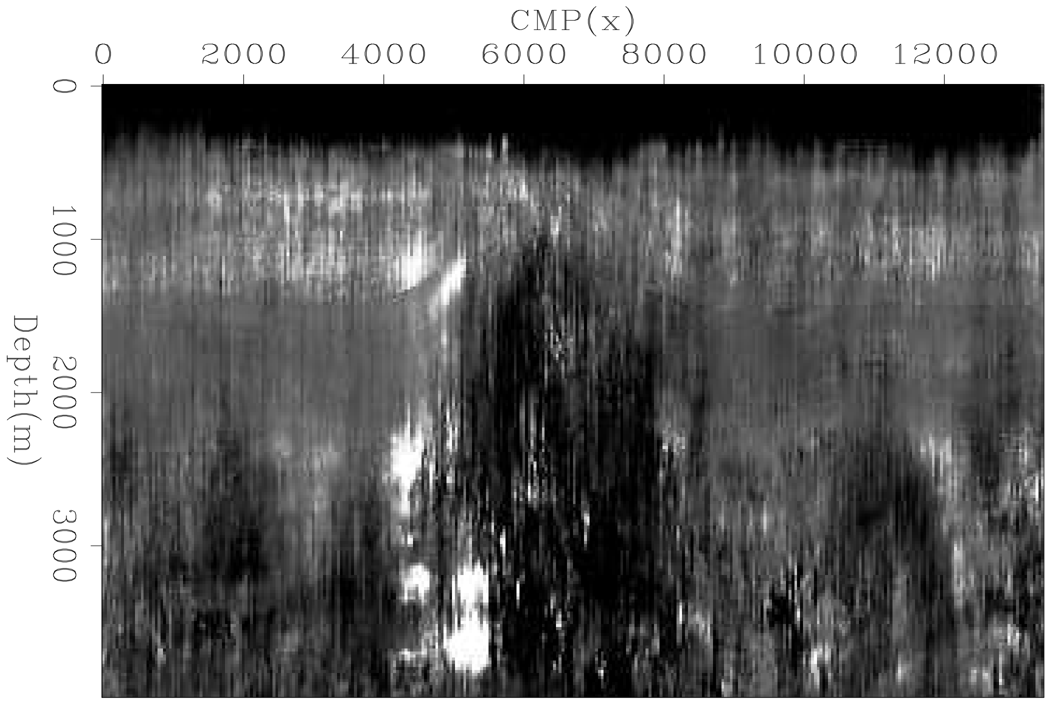

Figure 2: The result of inverting for the moveout parameter ρ from the time shifts shown in the right panel of Figure 1. bob3-rho1 [ER]

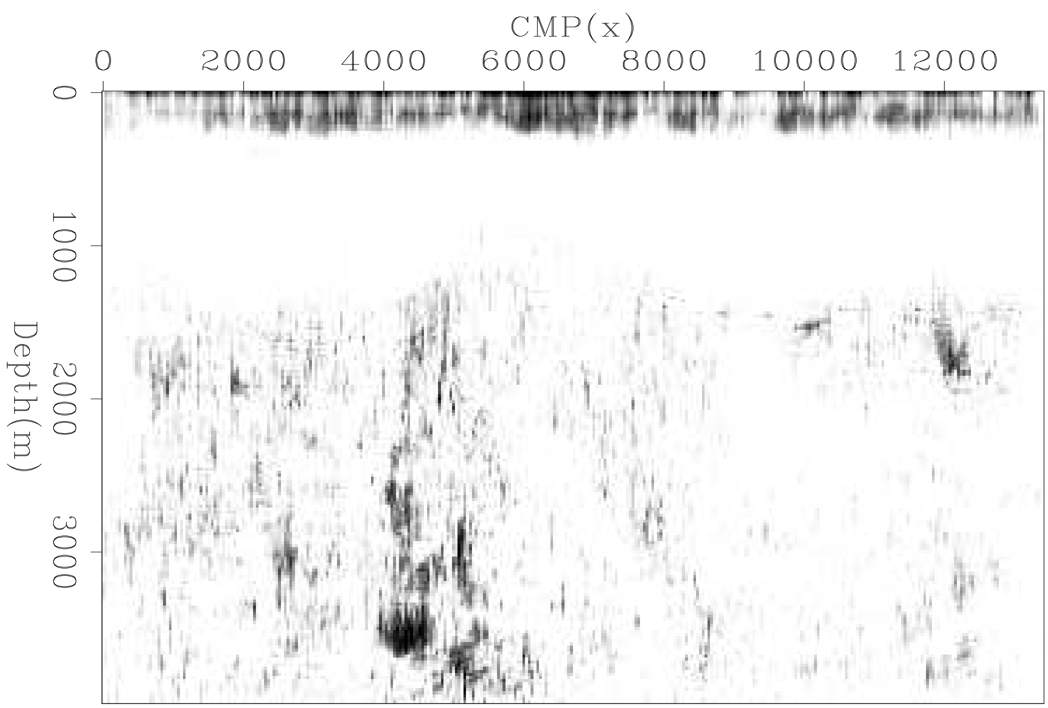

Figure 3: The spatial error fitting error associated with the time shifts shown in Figure 1 and the moveout parameter shown in Figure 2. bob3-resid1 [ER]

Rather than solving for a single moveout parameter at each location, we can solve for multiple moveout parameters simultaneously. To test this approach I introduced a new operator $\mathbf{C}$ that estimates the moveout parameter $\boldsymbol{\mu}$ by searching for higher order moveout anomalies. For $\mathbf{C}$ I chose an arbitrary moveout function,

$$\Delta z = \mu z \tan(\theta)^4 \tag{10}$$

that attempts to see if a higher polynomial of the same form as $\mathbf{C}$ to help to describe the moveout. The optimization goal of equation (9) becomes

$$Q(\boldsymbol{\rho},\boldsymbol{\mu}) = |\boldsymbol{\tau} - \mathbf{B}\boldsymbol{\rho} - \mathbf{C}\boldsymbol{\mu}|^2 + \epsilon^2 |\mathbf{A}\boldsymbol{\rho}|^2 + \epsilon^2 |\mathbf{B}\boldsymbol{\mu}|^2. \tag{11}$$

Figure 4 shows the resulting $\boldsymbol{\rho}$ (left) and $\boldsymbol{\mu}$ (right) fields. Note how similar the $\boldsymbol{\rho}$ field is to the one in Figure 2, indicating that a two-stage estimation approach would have yielded a similar result. Figure 5 shows the resulting residual. Note the decrease in some areas compared to Figure 3, but still showing areas where the moveout is significantly more complex.

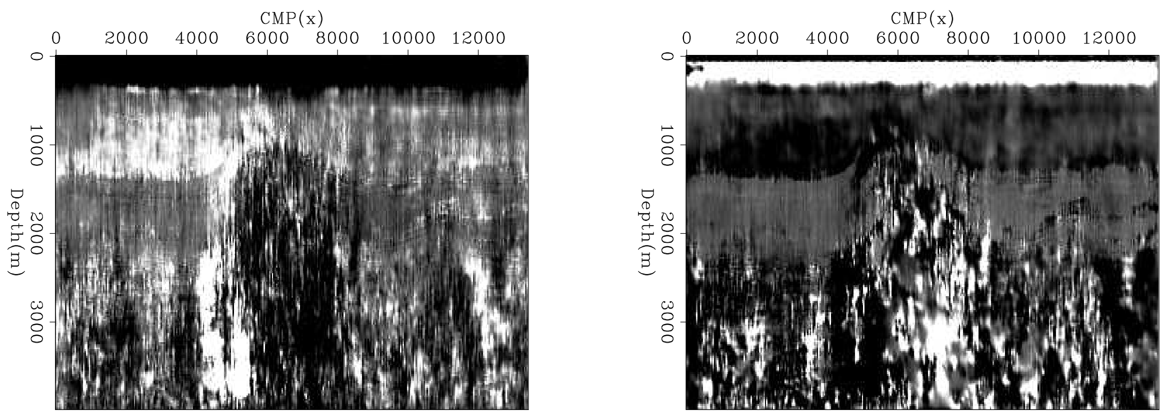

Figure 4: The result of inverting for both ρ (left panel) and μ (right panel). Note the similarity to the single parameter estimation shown in Figure 2. bob3-rho2 [ER]

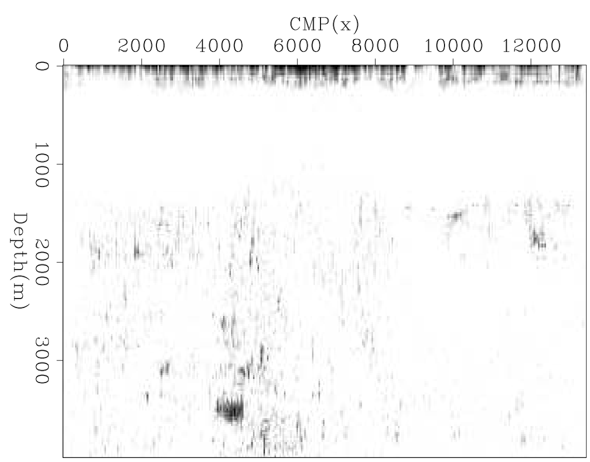

Figure 5: The fitting error associated with the two parameter fitting shown in Figure 4. bob3-resid2 [ER]

The methodology of this section assumed that the $\boldsymbol{\tau}$ field was accurate. The non-linear nature means this assumption is problematic, particularly when we are far from the correct solution. In the context of the moveout problem, this means we are far from flat the defacto starting guess.

LIMITING FLATTENING MODEL SPACE

The two-stage approach of the last section is only applicable when the $\boldsymbol{\tau}$ estimation is able to fully describe the moveout in the gather. When it cannot, another approach must be found. The left panel of Figure 6 is a synthetic CMP gather created by bandpassing random numbers, and then spraying them out with adjoint of NMO. The right panel of Figure 6 shows the result of estimating time shifts (7 Gauss-Newton steps) and then applying those time shifts to flatten the data. Note that the flattening approach has failed in several areas.

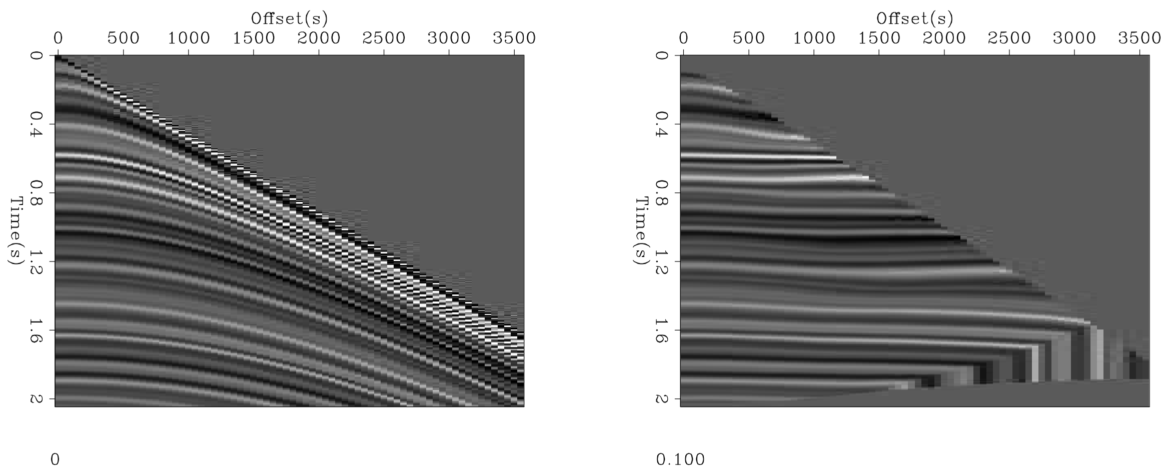

Figure 6: The left panel is a synthetic CMP gather. The right panel shows the result of flattening the CMP gather using the standard approach. Note the waviness of several reflectors due to the the non-linear nature of the flattening technique. bob3-syn [ER]

Estimation of the time shifts is problematic because the problem is inherently non-linear. One successful strategy is to try to start with an initial guess that is as close as possible to the correct solution. Another is to limit the model space to feasible candidates. In this simple case we know that the moveout is governed by the NMO equation. We can linearize the NMO equation that relates time shifts τ, zero offset time t_0, offset h, and slowness s through

$$\tau = \sqrt{t_0^2 + h^2 s^2} - t_0 \tag{12}$$

around our initial slowness $\mathbf{s_0}$. We obtain an equation,

$$\Delta\tau = \frac{h^2 s_0}{\sqrt{t_0^2 + h^2 s_0^2}} h^2 s_0 \Delta s, \tag{13}$$

that relates $\boldsymbol{\Delta}\tau$ to $\boldsymbol{\tau}\mathbf{s}$. The implied operator $\mathbf{H}$ then helps to form the linearized optimization equation,

$$Q(\boldsymbol{\Delta} s) = |\mathbf{W}_\epsilon \nabla \mathbf{H} \Delta \mathbf{s} - \boldsymbol{\Delta}\mathbf{p}|^{\mathbf{2}}. \tag{14}$$

In practice we need to add an additional weighting operator $\mathbf{W_0}$ which accounts for areas effected wavelet stretch and for reflections that exist at zero offset, but not at larger offsets. As a result we must use a space-domain conjugate gradient scheme

$$Q(\mathbf{\Delta} s) = |\mathbf{W_0 W}_\epsilon \nabla \mathbf{H} \Delta \mathbf{s} - \mathbf{\Delta p}|^{\mathbf{2}}. \tag{15}$$

Figure 7 shows the flattened CMP gather using the slowness model space description. While not perfect, the result is significantly flatter than the alternative approach (right panel of Figure 6).

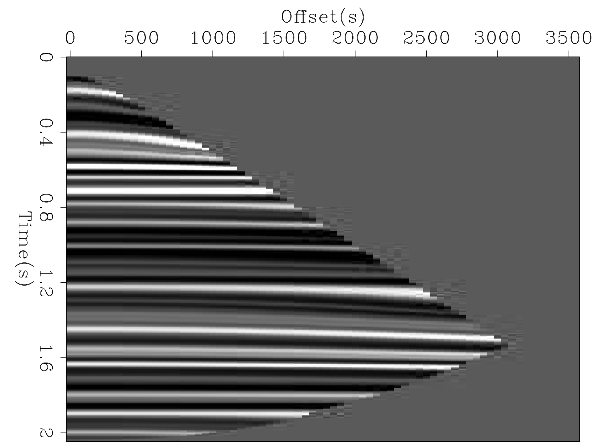

Figure 7: The flattened CMP gather using a Gauss-Newton scheme with the model space limited to hyperbolic moveout. The result is much flatter than the standard parameterization scheme show in the right panel of Figure 7. bob3-cmp [ER]

Figure 8 shows the result of applying both techniques to a CMP gather from the same North Sea dataset used in the previous section. The left panel is the raw gather, the center panel uses the conventional technique, and the right panel limits the moveout description to a single hyperbolic parameter. Note how both approaches fail at early times but the hyperbolic description provides noticably better result.

CONCLUSIONS

Flattening is used to analyze moveout. Two different approaches are used. The first approach uses time shift information generated through flattening as the 'data' in inverting for one or more moveout parameters. The second approach directly relates dips to a moveout parameter. Both approaches show promise but additional work is needed.

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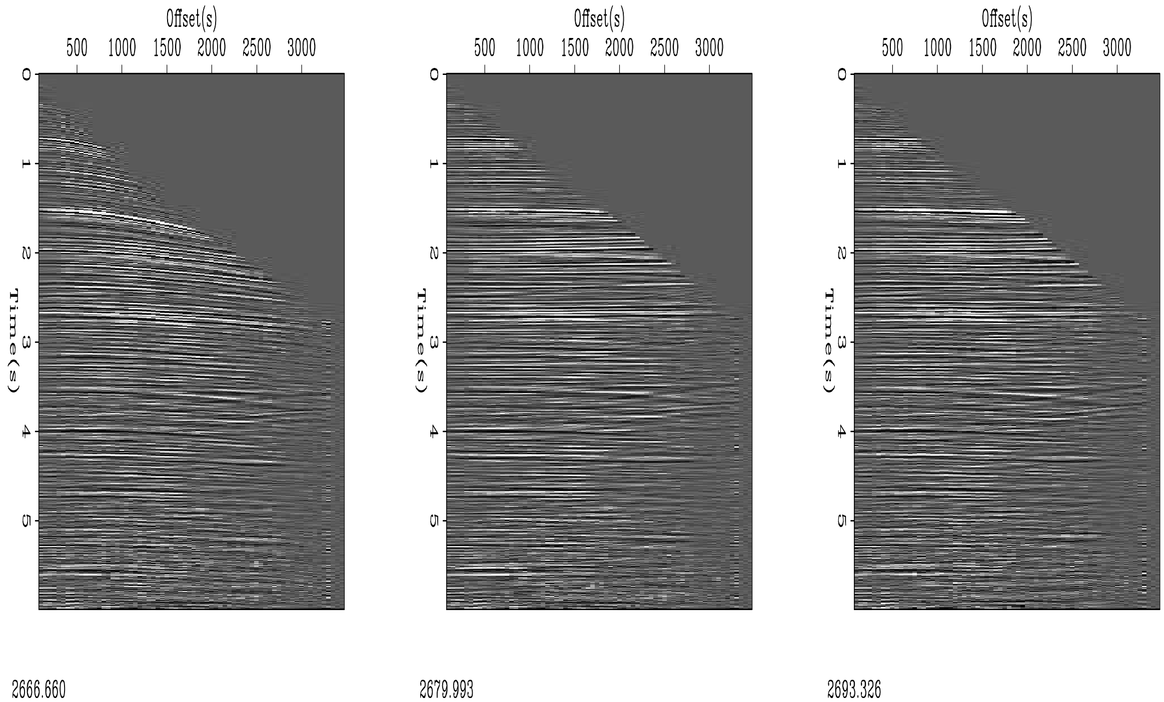

Figure 8: A CMP gather from the North Sea. The left panel shows the original gather. The center panel shows the result of using a standard parameterization for flattening. The right panel is the result using a hperbolic parameterization. bob3-elf [ER]

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