Appendix A

HEMNO Equivalence with Levin and Shah's Equations

In this appendix, I prove that the HEMNO equation is equivalent to Levin and Shah's traveltime equation (Levin and Shah, 1977) in the limit of small dip angle. They show that in a constant velocity medium with dipping target reflector and multiple generator, the moveout equation of the "S102G" pegleg multiple (see Figure 2.3) is:

$$t^2 = \left[\tau^* \cos\theta + \tau \cos\phi\right]^2 + \left[\frac{x \cos(\phi + \theta)}{V} - \tau^* \sin\theta - \tau \sin\phi\right]^2, \tag{A.1}$$

where ϕ and θ are the dip angle (in radians) of the multiple generator and target reflector, respectively. τ^* and τ are the zero-offset traveltimes to the two reflectors, x is offset, and V is the medium velocity. For small dip angles (i.e., less than 5 degrees), we can make the small angle approximation for angles ϕ, θ, and $\phi + \theta$ to update equation (A.1) accordingly:

$$t^2 = \left[\tau^* + \tau\right]^2 + \left[\frac{x}{V} - \tau^*\theta - \tau\phi\right]^2. \tag{A.2}$$

Multiplying out the squares in equation (A.2) and collecting terms gives:

$$t^2 = \left[\tau^* + \tau\right]^2 + \frac{x^2}{V^2} - 2\frac{\theta\tau^* x}{V} - 2\frac{\phi\tau x}{V} + (\tau^*\theta)^2 + (\tau\phi)^2. \tag{A.3}$$

The θ^2 and ϕ^2 terms are negligible for small angles, so we can ignore these terms and further simplify equation (A.3):

$$t^2 = \left[\tau^* + \tau\right]^2 + \frac{x^2}{V^2} - 2\frac{(\theta\tau^* + \phi\tau)x}{V}. \tag{A.4}$$

I will now show that the HEMNO equation (2.27) is equivalent to the Levin/Shah equation (A.1) under the constant velocity and small dip angle assumptions. First I make some preliminary definitions. In a constant-velocity medium, the expression for x_p, equation (2.21), simplifies to:

$$x_p = \frac{\tau}{\tau + \tau^*}x. \tag{A.5}$$

Then $x - x_p$, which will be needed later, simplifies to:

$$x - x_p = \frac{\tau^*}{\tau + \tau^*}x \tag{A.6}$$

Since the reflectors in this derivation are assumed planar and the velocity is assumed constant, using equations (A.5) and (A.6), we can directly write the (two-way) zero offset traveltime to the seabed and subsea reflection at any midpoint as a function of the corresponding zero-offset traveltimes at the midpoint location, y_0:

$$\begin{aligned}
\tau^*(y_0 - x_p/2) &= \tau^*(y_0) - \frac{x_p \sin\phi}{V} \\
&\approx \tau^*(y_0) - \frac{\phi\ \tau(y_0)\ x}{(\ \tau(y_0) + \tau^*(y_0)\ )\ V} && \text{(A.7)} \\
\tau(y_0 - (x - x_p)/2) &= \tau(y_0) - \frac{(x - x_p)\sin\theta}{V} \\
&\approx \tau(y_0) - \frac{\theta\ \tau^*(y_0)\ x}{(\ \tau(y_0) + \tau^*(y_0)\ )\ V}, && \text{(A.8)}
\end{aligned}$$

where the small angle approximation was employed as before. Substituting the zero-offset traveltimes (A.7) and (A.8) into the HEMNO equation (2.27) yields:

$$t^2 = \left[\tau(y_0)+\tau^*(y_0)-\frac{(\ \phi\ \tau(y_0)+\theta\ \tau^*(y_0)\ )\ x}{(\ \tau(y_0)+\tau^*(y_0)\ )\ V}\right]^2+\frac{x^2}{V^2} \tag{A.9}$$

$$\approx \left(\tau(y_0)+\tau^*(y_0)\right)^2-2\frac{(\ \phi\ \tau(y_0)+\theta\ \tau^*(y_0)\ )\ x}{V}+\frac{x^2}{V^2}. \tag{A.10}$$

Equation (A.10) is equivalent to equation (A.4). Therefore, we have proven the equivalence of the moveout equations of the true and approximate raypaths shown in Figure 2.7, subject to the small dip angle approximation. As before, ϕ^2 and θ^2 terms were dropped in going from equation (A.9) to equation (A.10). Although explicit seabed and subsea reflector dip angles, ϕ and θ, are contained in equation (A.10), they were introduced only to show equivalence to equation (A.4). Locally-planar reflectors are not required to implement equation (2.27).

Appendix B

Derivation of Snell Resampling Operator

In the following appendix, I derive the Snell resampling operation, equation (2.21). The graphical basis for the derivation is Figure 2.4. Since the pegleg multiple and primary in the figure have the same emergence angle, θ, the stepout, or spatial derivative, of the traveltime curves of the two events is the same at x and x_p. First we compute the stepout of the primary event, starting from the standard NMO equation:

$$t_p^2 = \tau + \frac{x_p^2}{V^2} \tag{B.1}$$

$$\frac{d}{dx_p}\left(t_p^2\right) = 2t_p \frac{dt_p}{dx_p} = \frac{2x_p}{V^2} \tag{B.2}$$

$$\frac{dt_p}{dx_p} = \frac{x_p}{t_p V^2}. \tag{B.3}$$

Using equations (2.18) and (2.20), we can similarly compute the stepout of the corresponding j^{th}-order pegleg multiple:

$$\frac{dt_m}{dx} = \frac{x}{t_m V_{eff}^2}. \tag{B.4}$$

Finally, we compute x_p as a function of x by squaring equations (B.3) and (B.4), setting them equal, and substituting traveltime equations (2.18) and (B.1) for t_m and t_p, respectively:

$$\frac{x_p^2}{t_p^2 V^4} = \frac{x^2}{t_m^2 V_{eff}^4}. \tag{B.5}$$

$$x^2\left[V^4\tau^2 + x_p^2 V^2\right] = x_p^2\left[V_{eff}^4(\tau + j\tau^*)^2 + x^2 V_{eff}^2\right] \tag{B.6}$$

$$x_p^2 = \frac{x^2\tau^2 V^4}{(\tau + j\tau^*)^2 V_{eff}^4 + x^2(V_{eff}^2 - V^2)}. \tag{B.7}$$

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