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A generalization of wave-equation migration velocity analysis

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ABSTRACT

Wave-equation migration velocity analysis is derived from wavefield-continuation migration techniques. The velocity model is updated by optimizing certain properties of the migrated images. Different migration velocity analysis optimization criteria exist, of which two commonly used are fitting a target image and minimizing the differential semblance of migrated images. Both techniques are special cases of a general family of optimization functions. Fitting a target image is an attractive technique because we can guide the solution in the desired direction. However, we can only progress in small steps with the target image being kept within the Born approximation with respect to the reference image. Minimizing differential semblance is an attractive technique, too, because we are operating with small differences of nearby offsets which are likely within the Born approximation. However, this method is not directly guided toward the solution and aliasing or any remnants of coherent noise, like multiples or converted waves, can cause it to diverge.

INTRODUCTION

Migration velocity analysis (MVA) is one of the most important problems of seismic imaging (Claerbout, 1999), and yet it remains one without a conventional solution. Many techniques have been devoted to solving this problem and, generally speaking, they fall into two broad categories: methods which directly use traveltimes computed using the eikonal equation, and methods which use the entire recorded wavefields. The methods in the first category are usually known by the name of *traveltime tomography* (Stork, 1992; Clapp, 2001), while the methods in the second category are known by the names of *wave-equation tomography* (Tarantola, 1984; Woodward, 1992) or *wave-equation migration velocity analysis* (Biondi and Sava, 1999; Sava and Fomel, 2002; Stolk and Symes, 2002).

The wave-equation MVA techniques are, in theory, superior to the traveltime-based MVA methods since they make use of the entire recorded data and not only of picked traveltimes at selected events. Some of those methods are also better able to account for multipathing occurring in complicated geological situations, a goal that is difficult to achieve with ray-traced traveltimes. Moreover, wave-equation techniques are more accurately describing wave propagation, since they are not based on high frequency asymptotic assumptions.

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However, none of the wave-equation velocity analysis methods has yet been accepted as a practical solution to exploration problems. Part of the reason is cost, which remains high, despite the continually decreasing cost of computing hardware. In addition, many of those velocity analysis techniques become unstable if the data are polluted with coherent noise, if the recorded offsets are too short or if enough low frequencies are not available in the band of the data (Pratt, 1999).

Wave-equation MVA (WEMVA) is different from wave-equation tomography (WET) with respect to the domain in which each one computes residuals: WET operates in the data space, and estimates velocity by fitting the recorded data, while WEMVA operates in the image space, and estimates velocity by improving the quality of the migrated images. As for the traveltime tomography methods, estimating velocity in the migrated image space is a much more robust approach and more likely to converge to geologically meaningful solutions.

The usual property used for optimization is that of flat events measured along angle-domain common-image gathers. Optimal flatness in the angle-domain is equivalent to optimal focusing at zero-offset (Stolk and Symes, 2002), therefore explicit conversion to the angle-domain is not necessary. Similarly, we could use focusing along the spatial axes as well as focusing along offset in order to estimate migration velocity (Sava and Etgen, 2002)

In this paper, we generalize the wave-equation migration velocity analysis technique to include both the target image fitting method of Biondi and Sava (1999) and the differential semblance optimization method of Stolk and Symes (2002) in a unified framework. We show that both methods are just special cases of a more general technique. We discuss these two members of this general class of problems, and we point out that other more or less optimal methods exist.

In the following sections, we present in detail our generalization of the WEMVA method, followed by an example and a brief discussion of the results.

THEORY OF WAVE-EQUATION MVA

This section presents in detail our generalization of WEMVA. Throughout this section, we use the following notation conventions: we write $\mathbf{A}[x]$ when we mean $\mathbf{A}$ operates on x, and $\langle a,b\rangle$ when we mean the inner product of the vectors a and b.

We begin with a brief review of wavefield extrapolation, followed by a discussion of image transformations to the optimization domain, the objective functions and their gradient with respect to velocity. In the end, we restate our main results in the familiar SEP notation using linear fitting goals.

Imaging by wavefield extrapolation

Imaging by wavefield extrapolation (WE) is based on recursive continuation of the wavefields $\mathcal{U}$ from a given depth level to the next by means of an extrapolation operator $\mathbf{E}$:

$$\mathcal{U}_{z+\Delta z} = \mathbf{E}_z \left[\mathcal{U}_z \right]. \tag{1}$$

This recursive relation can also be explicitly written in matrix form as

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ -\mathbf{E}_0 & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E}_1 & \mathbf{1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{E}_{n-1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \mathcal{U}_0 \\ \mathcal{U}_1 \\ \mathcal{U}_2 \\ \vdots \\ \mathcal{U}_n \end{pmatrix} = \begin{pmatrix} \mathcal{D}_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

or in a more compact notation as:

$$(\mathbf{1} - \mathbf{E}) \, \mathcal{U} = \mathcal{D}, \tag{2}$$

where the vector $\mathcal{D}$ stands for the recorded data, $\mathcal{U}$ for the extrapolated wavefield, $\mathbf{E}$ for the extrapolation operator and $\mathbf{1}$ for the identity operator.

The wavefield at every depth level $\mathcal{U}_z$ is imaged using an imaging operator $\mathbf{I}_z$:

$$\mathcal{R}_z = \mathbf{I}_z \left[\mathcal{U}_z \right], \tag{3}$$

where $\mathcal{R}_z$ stands for the image at some depth level. We can write the same relation in compact matrix form as:

$$\mathcal{R} = \mathbf{I} \mathcal{U}, \tag{4}$$

where $\mathcal{R}$ stands for the image, and $\mathbf{I}$ stands for the imaging operator which is applied to the extrapolated wavefield $\mathcal{U}$.

Wavefield perturbations

A perturbation of the wavefield at some depth level can be derived from the background wavefield by a simple application of the chain rule to Equation (1):

$$\delta \mathcal{U}_{z+\Delta z} = \mathbf{E}_z \left[\delta \mathcal{U}_z \right] + \delta \mathbf{E}_z \left[\mathcal{U}_z \right]. \tag{5}$$

This is also a recursive equation which can be written in matrix form as

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ -\mathbf{E}_0 & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E}_1 & \mathbf{1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{E}_{n-1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \delta\mathcal{U}_0 \\ \delta\mathcal{U}_1 \\ \delta\mathcal{U}_2 \\ \vdots \\ \delta\mathcal{U}_n \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \delta\mathbf{E}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \delta\mathbf{E}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \delta\mathbf{E}_{n-1} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathcal{U}_0 \\ \mathcal{U}_1 \\ \mathcal{U}_2 \\ \vdots \\ \mathcal{U}_n \end{pmatrix},$$

or in a more compact notation as:

$$(\mathbf{1} - \mathbf{E}) \, \delta \mathcal{U} = \delta \mathbf{E} \mathcal{U}, \tag{6}$$

where the operator $\delta\mathbf{E}$ stands for a perturbation of the extrapolation operator $\mathbf{E}$.

Biondi and Sava (1999) show that, at every depth level, we can write the operator $\delta\mathbf{E}$ as a chain of the extrapolation operator $\mathbf{E}$ and a scattering operator $\mathbf{S}$ applied to the slowness perturbation δs_z:

$$\delta\mathbf{E}_z\left[\mathcal{U}_z\right] = \mathbf{E}_z\left[\mathbf{S}_z\left[\delta s_z\right]\right]. \tag{7}$$

The expression for the wavefield perturbation $\delta\mathcal{U}$ becomes

$$\delta\mathcal{U}_{z+\Delta z} = \mathbf{E}_z\left[\delta\mathcal{U}_z\right] + \mathbf{E}_z\left[\mathbf{S}_z\left[\delta s_z\right]\right], \tag{8}$$

which is also a recursive relation that can be written in matrix form as

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ -\mathbf{E}_0 & \mathbf{1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{E}_1 & \mathbf{1} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{E}_{n-1} & \mathbf{1} \end{pmatrix} \begin{pmatrix} \delta\mathcal{U}_0 \\ \delta\mathcal{U}_1 \\ \delta\mathcal{U}_2 \\ \vdots \\ \delta\mathcal{U}_n \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{E}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{E}_{n-1} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{S}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{S}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{S}_n \end{pmatrix} \begin{pmatrix} \delta s_0 \\ \delta s_1 \\ \delta s_2 \\ \vdots \\ \delta s_n \end{pmatrix},$$

or in a more compact notation as:

$$(\mathbf{1} - \mathbf{E})\,\delta\mathcal{U} = \mathbf{ES}\delta s. \tag{9}$$

The vector δs stands for the slowness perturbation.

If we introduce the notation

$$\mathbf{G} = (\mathbf{1} - \mathbf{E})^{-1}\,\mathbf{ES}, \tag{10}$$

we obtain a relation between a slowness perturbation and the corresponding wavefield perturbation:

$$\delta\mathcal{U} = \mathbf{G}\delta s. \tag{11}$$

Image transformation

Migration velocity analysis is based on estimating the velocity that optimizes certain properties of the migrated images. In general, measuring such properties involves making a transformation to the extrapolated wavefield by some function f, followed by imaging:

$$\mathcal{P}_z = \mathbf{I}_z\left[f_z(\mathcal{U}_z)\right]. \tag{12}$$

In compact matrix form, we can write this relation as:

$$\mathcal{P} = \mathbf{I}f(\mathcal{U}). \tag{13}$$

The image $\mathcal{P}$ is subject to optimization from which we derive the velocity updates.

Examples of transformation functions are:

- $f(x) = x - t$ where t is a known target. A WEMVA method based on this criterion optimizes

  $$\mathcal{P}_z := \mathbf{I}_z\left[\mathcal{U}_z - \mathcal{T}_z\right], \tag{14}$$

  where $\mathcal{T}_z$ stands for the target wavefield. For this method, we can use the acronym TIF standing for *target image fitting* (Biondi and Sava, 1999; Sava and Fomel, 2002).

- $f(x) = Dx$ where D is a known operator. A WEMVA method based on this criterion optimizes
$$\mathcal{P}_z := \mathbf{I}_z \left[\mathbf{D}_z \left[\mathcal{U}_z\right]\right]. \tag{15}$$
If $\mathbf{D}$ is a differential semblance operator, we can use the acronym DSO standing for *differential semblance optimization* (Symes and Carazzone, 1991; Stolk and Symes, 2002).

In general, both examples presented above belong to a family of affine functions that can be written as
$$\mathcal{P}_z := \mathbf{I}_z \left[\mathbf{A}_z \left[\mathcal{U}_z\right] - \mathbf{B}_z \left[\mathcal{T}_z\right]\right], \tag{16}$$
or in compact matrix form as
$$\mathcal{P} := \mathbf{I}\left(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}\right), \tag{17}$$
where the operators $\mathbf{A}$ and $\mathbf{B}$ are known and take special forms depending on the optimization criterion we use. For example, $\mathbf{A} = \mathbf{1}$ and $\mathbf{B} = \mathbf{1}$ for TIF, and $\mathbf{A} = \mathbf{D}$ and $\mathbf{B} = \mathbf{0}$ for DSO. $\mathbf{1}$ stands for the identity operator, and $\mathbf{0}$ stands for the null operator.

Objective function

With the definition in Equation (17), we can write the optimization function J as:

$$J(s) := \frac{1}{2}\sum_{z,\vec{m},\vec{h}} \left|\mathcal{P}_z\right|^2 = \frac{1}{2}\sum_{z,\vec{m},\vec{h}} \left|\mathbf{I}_z \left[\mathbf{A}_z \left[\mathcal{U}_z\right] - \mathbf{B}_z \left[\mathcal{T}_z\right]\right]\right|^2, \tag{18}$$

where s is the slowness function, and $z, \vec{m}, \vec{h}$ stand respectively for depth, and the midpoint and offset vectors. In compact matrix form, we can write the objective function as:

$$J(s) := \frac{1}{2}\left|\mathbf{I}\left(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}\right)\right|^2, \tag{19}$$

which takes special forms depending on our choice of the operators $\mathbf{A}$ and $\mathbf{B}$:

WEMVA by TIF	WEMVA by DSO
$J(s) = \frac{1}{2}\left|\mathbf{I}\left(\mathcal{U} - \mathcal{T}\right)\right|^2$	$J(s) = \frac{1}{2}\left|\mathbf{I}\left(\mathbf{D}\mathcal{U}\right)\right|^2$

Gradient

Optimization of the objective function in Equation (19) requires computation of its gradient with respect to slowness. The objective function J can be rewritten using the inner product as:

$$J(s) = \frac{1}{2}\left\langle \mathbf{I}\left(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}\right), \mathbf{I}\left(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}\right)\right\rangle. \tag{20}$$

A perturbation of the function J is related to a perturbation of the wavefield by the relation:

$$\delta J(s) = \left\langle \mathbf{I}\left(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}\right), \mathbf{I}\mathbf{A}\delta\mathcal{U}\right\rangle. \tag{21}$$

If we replace $\delta \mathcal{U}$ from Equation (11) we obtain:

$$\delta J(s) = \langle \mathbf{I}(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}), \mathbf{IAG}\delta s \rangle, \tag{22}$$

therefore the gradient of the objective function can be written as

$$\nabla_s J = \mathbf{G}^* \mathbf{A}^* \mathbf{I}^* \mathbf{I}(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}). \tag{23}$$

Following the definition of the operator **G**, we can write

$$\mathbf{G}^* = \mathbf{S}^* \mathbf{E}^* \left[(\mathbf{1} - \mathbf{E})^{-1} \right]^* = \mathbf{S}^* \mathbf{E}^* \left[(\mathbf{1} - \mathbf{E})^* \right]^{-1}. \tag{24}$$

Finally, the expression for the gradient of the objective function with respect to slowness becomes

$$\nabla_s J = \mathbf{S}^* \mathbf{E}^* \left[(\mathbf{1} - \mathbf{E})^* \right]^{-1} \mathbf{A}^* \mathbf{I}^* \mathbf{I}(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}) \tag{25}$$

which takes special forms depending on our choice of the operators **A** and **B**:

WEMVA by TIF	WEMVA by DSO
$\nabla_s J = \mathbf{S}^* \mathbf{E}^* \left[(\mathbf{1} - \mathbf{E})^* \right]^{-1} \mathbf{I}^* \mathbf{I}(\mathcal{U} - \mathcal{T})$	$\nabla_s J = \mathbf{S}^* \mathbf{E}^* \left[(\mathbf{1} - \mathbf{E})^* \right]^{-1} \mathbf{D}^* \mathbf{I}^* \mathbf{ID}\mathcal{U}$

The gradient in Equation (25) is computed using the adjoint state method, which can be summarized by the following steps:

1. Compute by downward continuation the wavefield

$$\mathbf{A}^* \mathbf{I}^* \mathbf{I}(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}). \tag{26}$$

2. Compute by upward continuation the adjoint state wavefield

$$\mathcal{W} = \left[(\mathbf{1} - \mathbf{E})^* \right]^{-1} \mathbf{A}^* \mathbf{I}^* \mathbf{I}(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}), \tag{27}$$

   i.e. solve the adjoint state system

$$(\mathbf{1} - \mathbf{E})^* \mathcal{W} = \mathbf{A}^* \mathbf{I}^* \mathbf{I}(\mathbf{A}\mathcal{U} - \mathbf{B}\mathcal{T}). \tag{28}$$

3. Compute the gradient

$$\nabla_s J = \mathbf{S}^* \mathbf{E}^* \mathcal{W}. \tag{29}$$

Linearization

Minimizing the objective function J in Equation (19) involves solving a non-linear least-squares problem using the gradient given by Equation (25).

Alternatively, we can linearize the wavefield $\mathcal{U}$ with respect to a reference wavefield $\mathcal{U}^r$

$$\mathcal{U} = \mathcal{U}^r + \delta\mathcal{U} = \mathcal{U}^r + \mathbf{G}\delta s \tag{30}$$

and optimize

$$J(s) = \frac{1}{2}\left|\mathbf{I}\left(\mathbf{A}\mathcal{U}^r - \mathbf{B}\mathcal{T} + \mathbf{A}\mathbf{G}\delta s\right)\right|^2, \tag{31}$$

which is a linear least-squares problem.

Equation (31) can also be represented by fitting goals using the usual SEP terminology as:

$$-\mathbf{I}\left(\mathbf{A}\mathcal{U}^r - \mathbf{B}\mathcal{T}\right) \approx \mathbf{IAG}\delta s, \tag{32}$$

which takes special forms depending on our choice of the operators **A** and **B**:

WEMVA by TIF	WEMVA by DSO
$-\mathbf{I}(\mathcal{U}^r - \mathcal{T}) \approx \mathbf{IG}\delta s$	$-\mathbf{I}(\mathbf{D}\mathcal{U}^r) \approx \mathbf{IDG}\delta s$

EXAMPLE

Our synthetic example, Figure 1, is represented by a simple horizontal reflector (top panel) embedded in a velocity model with smooth lateral velocity variation (middle panel). A small perturbation introduced in the velocity model creates an image perturbation which does not violate the Born approximation (bottom panel).

We compute the gradient of the objective function in Equation (19) using Equation (25) particularized both for the target image fitting (TIF) and for differential semblance optimization (DSO) criteria. Figure 2 shows the ideal image perturbation (top panel), the gradient for TIF (middle panel), and the gradient for DSO (bottom panel).

DISCUSSION

Not surprisingly, the two methods generally represented by Equation (17) produce significantly different results. Although our analysis in this paper does not cover all cases and possibilities, we can make several observations:

- The general form in Equation (17) is not unique, meaning that other forms of equal generality exist. Moreover, we have presented and compared just two members of our general form, although many others exist. The obvious question, for which we do not have a definite answer, is which is the optimal form? Is there such thing, or do we need to consider different forms for different situations? These questions remain the subject of future research.

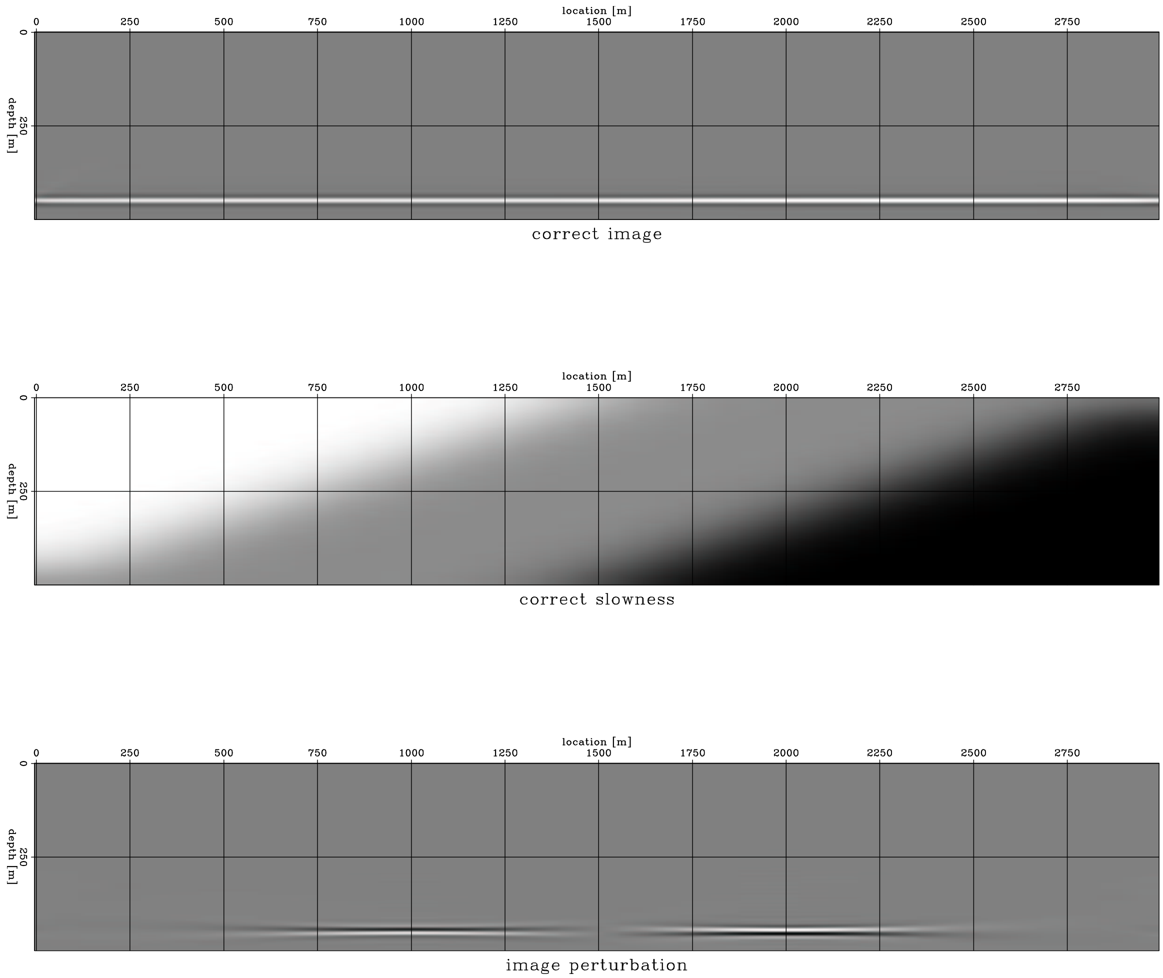

Figure 1: Reflectivity model (top), background slowness (middle), and image perturbation (bottom). paul3-imags [CR]

- The *target image fitting* (TIF) approach is an attractive alternative because it can, in principle, be driven in the desired direction given by the target image. However, if the constraints presented by the Born approximation are not observed (i.e. the target is too far from the actual image), then inversion may diverge (Sava and Fomel, 2002). We also need to create the actual target, an improved image, which is not a trivial task.

- The *differential semblance optimization* (DSO) approach is also attractive for other reasons. The objective function is smooth and unimodal, at least for certain simplified cases (Symes, 1999). However, even DSO is not guaranteed to converge when the data are aliased or when they are polluted with residual multiples or converted waves.

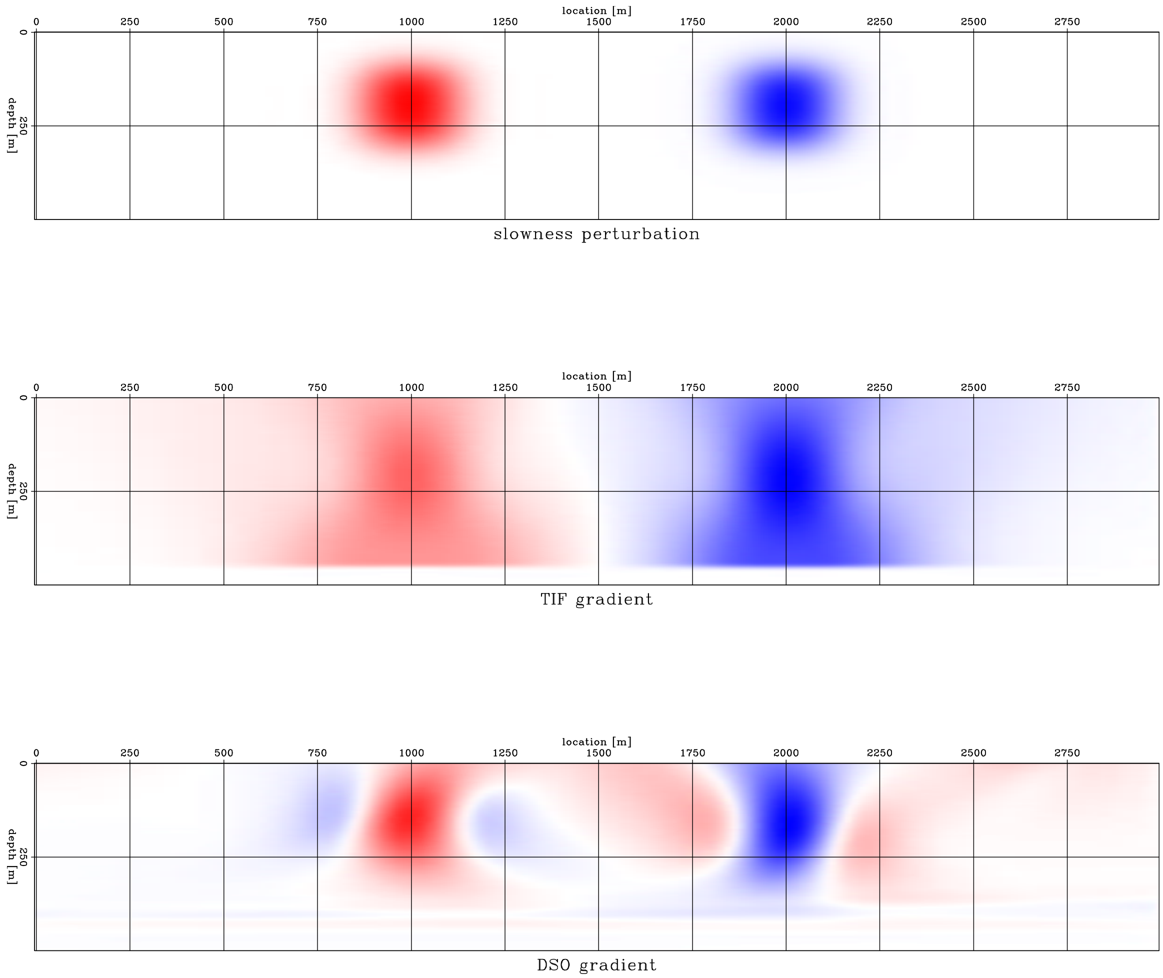

Figure 2: Slowness perturbation (top), TIF gradient (middle) and DSO gradient (bottom). paul3-slows [CR]

CONCLUSION

We present a generalization of the wave-equation migration velocity analysis technique. We show that various objective functions can be used and that each one of them has distinctive properties which make them attractive under different circumstances. Our generalization provides a framework in which we can test various optimization strategies. Those extensions, however, remain subjects for future research.

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